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OF CIVIL ENGINEERS

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Original papers and discussions of current papers should be submitted to the Manager of Technical Publications, ASCE. Authors should indicate the technical division to which the paper is referred. The final date on which a discussion should reach the Society is given as a footnote with each paper. Those who are planning to submit material will expedite the review and publication procedures by complying with the following basic requirements:

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2. A summary of approximately 50 words must accompany the paper, a 300-word synopsis must precede it, and a set of conclusions must end it.

3. The manuscript (an original ribbon copy and two duplicate copies) should be double-spaced on one side of 8½-inch by 11-inch paper. Three copies of all illustrations, tables, etc., must be included.

4. The author's full name, Society membership grade, and footnote reference stating present employment must appear on the first page of the paper.

5. Mathematics are recomposed from the copy that is submitted. Because of this, it is necessary that letters be drawn carefully, and that special symbols be properly identified. The letter symbols used should be defined where they first appear, in the illustrations or in the text, and arranged alphabetically in an Appendix.

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HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

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Note.--Part 2 of this Journal is the 1961-41 Newsletter of the Hydraulics ision.
The three preceding issues of this Journal are dated May 1961, July 1961, September 1961.

DISCUSSION

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

DUAL CHANNEL STREAM MONITOR

By Susumu S. Karaki,[1] A.M. ASCE, Earl E. Gray,[2] and Jack Collins[3]

SYNOPSIS

The dual channel stream monitor is an ultrasonic instrument developed to onitor water surface and stream bed profiles simultaneously under dynamic nditions in an alluvial channel. The instrument was developed primarily as aboratory tool to aid hydraulic research.

The stream monitor is a light-weight, portable, fully transistorized instru-ent that uses the echo ranging principle to determine distances to reflecting rfaces. Two piezoelectric transducers are used. Power requirement is 15 and the instrument will operate with either 115-v, 60-cycle AC or a self-ntained battery pack.

The range of the instrument is from 0 ft to 10 ft of flow depth with a step-wn scale at shallow depths to provide for greater accuracy in measurement. e readout is presented on a dual channel strip-chart recorder that registers ter surface and stream bed profiles separately.

Note.—Discussion open until April 1, 1962. To extend the closing date one month, a tten request must be filed with the Executive Secretary, ASCE. This paper is part he copyrighted Journal of the Hydraulics Division, Proceedings of the American Society Civil Engineers, Vol. 87, No. HY 6, November, 1961.

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Under normal flow velocities excellent results are obtained by the dua channel stream monitor.

INTRODUCTION

The dual channel stream monitor[4] is an electroacoustic instrument tha was developed for use in an alluvial channel under dynamic conditions. Hig frequency water-borne sound waves are generated and received by piezo- electric ceramic transducers. The time differential between transmission o sound and reflection of the sound wave or echo is used to indicate distances t specific reflecting surfaces.

Among the earliest applications of underwater sound waves was by mer- chant ships in detecting proximity to light ships. Another was in the use o the Fressenden oscillator for underwater communication in morse code Early research and development in ultrasonics and electroacoustics produce the fathometer, a navigational aid using the echo ranging principle to deter- mine water depth below ship hull. The fathometer has also been widely use in mapping subacqueous topographic features such as shoals and reefs. Dur- ing World War II, sonar was developed and used extensively by the Navy i anti-submarine warfare. Echo sounding instruments are currently (1961) use by various private groups. One such group is the commercial fishermen wh use ultrasonic equipment to locate schools of fish. In fact, through experience they are able to distinguish schools of various species of fish. For nearl two decades, the United States Army Corps of Engineers and the United State Bureau of Reclamation, Department of Interior (USBR), have been using modi fied sonar equipment for determining sedimentation rates in reservoirs

Although not an underwater application of ultrasonics, the medical profes sion has put into use an ultrasonic instrument called the sonoscope that make possible non-destructive observations of pathological tissue growth.

The dual channel stream monitor is an advanced version of its predecessor the model 1024 sonic depth sounder. The latter instrument is a single channe vacuum tube unit. Detailed information about the sonic depth sounder is avail able.[5]

PRINCIPLES OF OPERATION

The dual channel stream monitor is designed to convert electrical energ into underwater acoustical energy at ultrasonic frequencies and to reconve the sonic energy to electrical potential. Both generation and reception of sou are achieved through the same transducer.

The transducers are cylindrical in shape and constructed of ceramic bar ium titanate, rendered piezoelectric by permanent polarization. When a electric potential is placed across opposite sides of the piezoelectric materia

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4 "Dual Channel Stream Monitor," by E. E. Gray and S. S. Karaki, Agric. Resear Service, Beltsville, Md.; Colorado State Univ., Ft. Collins, Colo., August, 1960, Repo No. CER60SSK46.

5 "Sonic Depth Sounder for Laboratory and Field Use," by E. V. Richardson, D. Simons, and G. J. Posakony, Colorado State Univ., Ft. Collins, Colo., Report N CER60DBS11.

deformation takes place along specific axes of the material. Conversely, hen the material is subjected to stress along a certain axis, an electric po-ntial is produced across a specific section of the material. If the electric otential across the transducer is varied at a given frequency, the transducer ll vibrate at that frequency, and by properly shielding the transducer, the coustic axis is made to coincide with the cylindrical axis. The acoustic axis efers to the axis along which maximum sensitivity is achieved.

Sensitivity of the transducer is also a function of sound-wave frequency and gle of incidence relative to the acoustic axis. The transducers of the stream onitor are constructed to achieve a high directivity index, and by controlling e amplification of the electric potential generated by a narrow bandwidth of equencies along the acoustic axis, it is possible to monitor a small area of e water surface and stream bed.

Sound waves are generally classified into three regions: Frequencies of ss than 50 cycles per sec are termed sub-sonic; the audible range between cycles per sec and 15,000 cycles per sec as sonic; and the range of fre-encies greater than 15,000 cycles per sec is termed ultrasonic. The ve-city of sound in water is independent of frequency and dependent principally temperature, salinity, and ambient pressure. The interrelationship be-een these factors has been determined empirically. For fresh water, for hich the effect of salinity is negligible, the velocity of sound transmission ay be expressed by:[6]

$$c = 4625 + 7.68\,(t - 32) - 0.0376\,(t - 32)^2 + 0.018d$$

which c denotes the velocity of transmission, in feet per second; t refers to e temperature of the water, in degrees Fahrenheit; and d is the vertical pth below water surface, in feet. In the range of depths from 0 ft to 10 ft, e variation of c due to the change of depth only is 0.18 fps. Thus the last ctor in Eq. 1 can safely be ignored for the range of the stream monitor.

Transmission of sound in alluvial stream channels is affected by the pres-ce of air bubbles and solid particles in the flow. It is also affected by the sorption of sound energy by the water, although attenuation of sound inten-y due to this factor is negligible within distances of several hundred feet. fraction of sound due to a temperature gradient can generally be discounted most stream channels. Divergence of sound and consequent reduction in ensity is compensated by a highly directive transducer both in projecting d receiving sound waves. The principle effect on sound transmission is enuation and scattering of sound waves due to the presence of air bubbles d sediment particles in the flow.

If a sound beam is transmitted towards a target area containing a signifi-nt population of air bubbles or sediment particles, the summation of the flected sound intensities from individual particles may be sufficient to tuate the transducer. The reflection from an individual particle is depend-t on the size of the particle relative to the sound frequency. The reflection om an air bubble is greater than the reflection from a solid particle of the me size because acoustic impedance mismatch is much greater at a water-boundary than at a water-solid boundary. Hence, sound waves are better le to penetrate through layers of sediment particles than through air bub-

[6] "Underwater Acoustics Handbook," by V. M. Albers, The Pennsylvania State Univ. ss, University Park, Pa., 1960.

bles. The effect of densely populated layers of either solid particles or air bubbles is to indicate a distance somewhat less than the true distance to the water-solid or water-air boundary. Therefore, use of the ultrasonic stream monitor is limited to stream channels that do not contain large concentrations of air bubbles and sediment particles.

PHYSICAL DESCRIPTION OF THE INSTRUMENT

The individual units of the dual channel stream monitor, exclusive of the connecting leads, is shown in Fig. 1. The total instrument package consists of the transducers, the electronic monitoring unit, and the recorder. The two

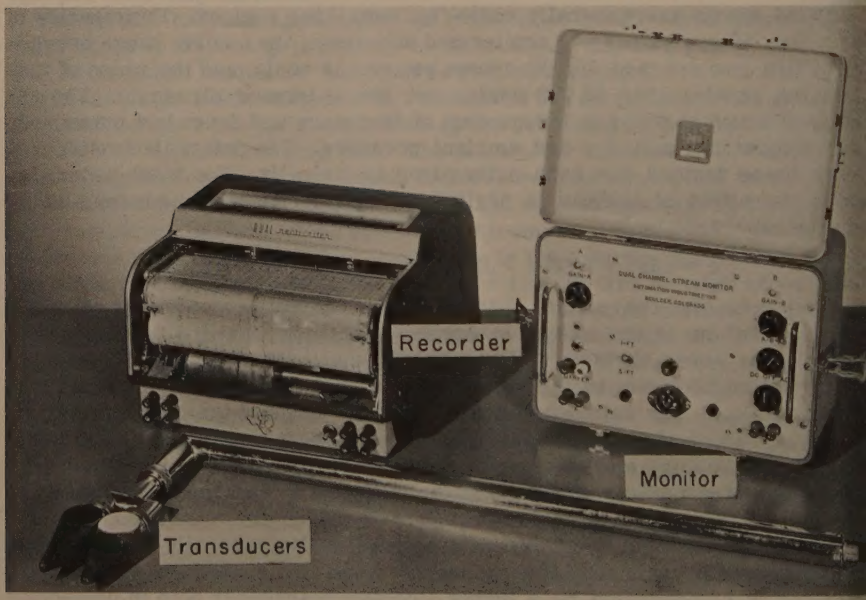

FIG. 1.–DUAL CHANNEL STREAM MONITOR

transducers are mounted on a variable-position head and attached to an L shaped probe. The length of the probe may be varied to permit immersion o the transducers to any desired depth. The probe shown in Fig. 1 is 30 in long.

The monitoring unit is encased in a metal box 9 in. wide, 13 in. long, an 9 in. deep with a handle on the lid. The battery pack, which is an auxiliar source of power supply consisting of a 45-v radio "B" battery and two 90 batteries, is contained within the case. The instrument panel is provided wit an input receptacle for a 115-v, 60-cycle AC source, together with dual cabl jacks to the recorder and appropriate controls for the two channels.

The recorder shown in Fig. 1 is a spring operated dual channel recording nilliameter.

The cables connecting the transducers to the monitor and the monitor to he recorder are co-axial RG-62U transmission cables. The cables from the ransducers to the monitor are used as part of the tuned circuitry.

CIRCUIT DESCRIPTION

The electronic circuitry of the dual channel stream monitor can be subivided into seven basic components, as depicted in the block diagram of Fig. . These components are power supply, clock, pulsers, transducers, reeivers, computers, and recorder.

Power Supply.—The power is supplied to the monitor by either a 115-v, 0-cycle AC power source or a self-contained battery pack. The power supply Fig. 3) provides three output voltages; low current unregulated 180 v; regu-

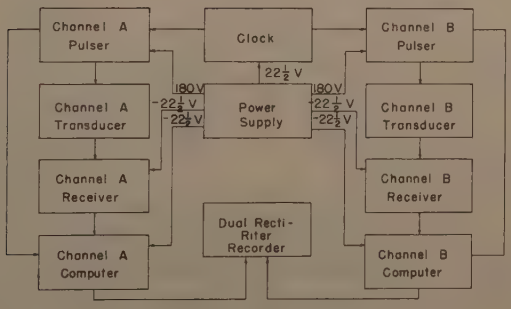

FIG. 2.—DUAL CHANNEL STREAM MONITOR BLOCK DIAGRAM

ted 22-1/2-v positive source; and a regulated 22-1/2-v negative source. ircuitry in all three power supply sections is conventional. The choice of ower source can be made by a selector switch on the instrument panel.

Clock.—The clock provides the time base with which the other components re synchronized. It is the base of time measurement between the pulse and rinciple echo. The clock consists of a unijunction transistor, relaxation oscilltor, and a bistable multivibrator. The relaxation oscillator frequency is etermined by the range scale switch, 1 kilocycle for the 1-ft range and 200 rcles per sec for the 5-ft range. The negative spike output of the relaxation scillator triggers the bistable multivibrator which, in turn, produces a square ave having half the frequency of the triggered pulse. The square wave outts are fed to the respective channel pulses, one to channel A and its mirror nage to channel B. The detailed circuitry of the clock and pulser are shown Fig. 4.

FIG. 3.—POWER SUPPLY

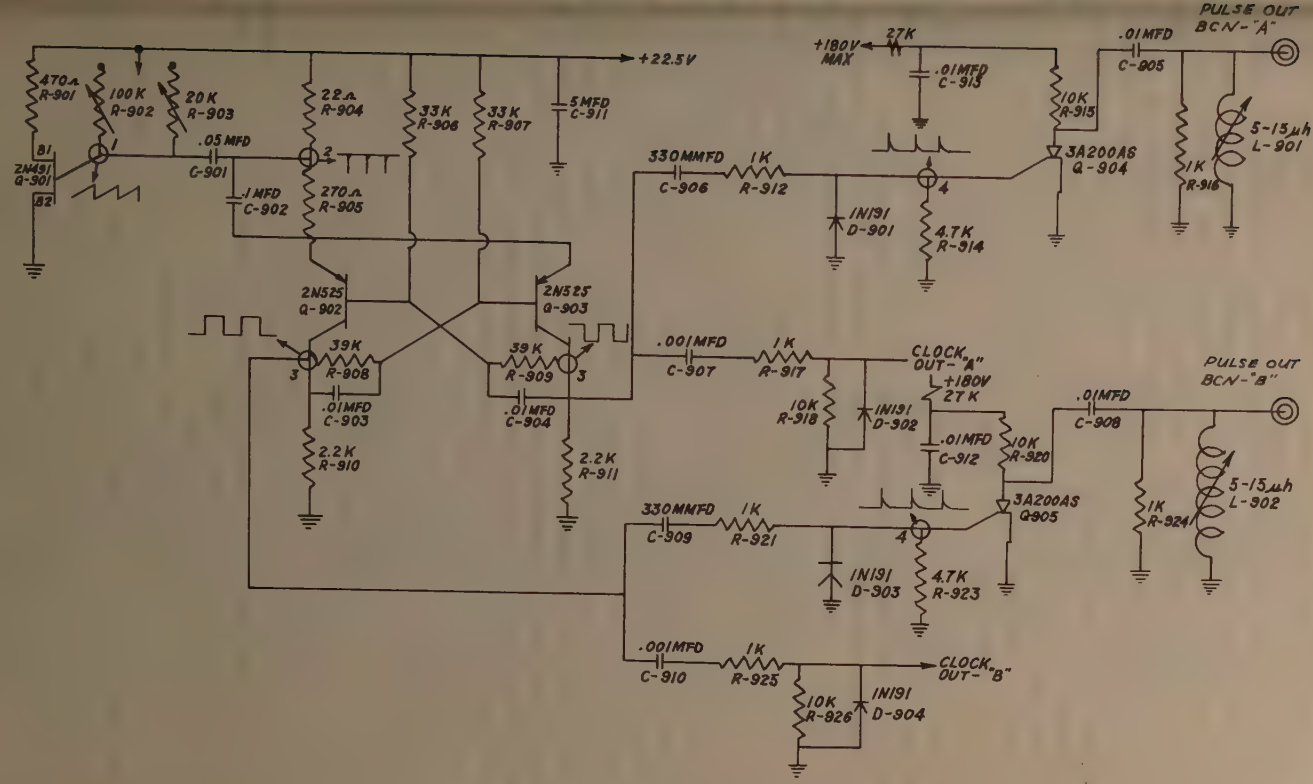

FIG. 4.—CLOCK AND PULSER

Pulser.—The pulser provides regulated bursts of high voltage to vibrate the transducer and produce sound waves. The pulser differentiates the square wave output of the clock and uses the positive spike as a trigger for a silicon controlled switch. This switch controls the charge and discharge paths for a capacitor. When the switch is open, the capacitor charges to a voltage between 0 v and 180 v as controlled by the sensitivity dial. When the switch is closed by the spike of voltage from the differentiated clock signal, the capacitor discharges through a parallel resonant circuit that consists of a resistor, a coil, and the capacitance of the coaxial transmission cable and transducer. The discharge produces a ringing in the tank circuit for a short period of time at a frequency of one megacycle and activates the transducer. By using only the positive spike from the clock, the two channels are activated alternately and interaction is greatly reduced. The resulting interrogating rates for the 1-ft and 5-ft ranges are 500 cycles per sec and 100 cycles per sec, respectively.

Receiver.—The receiver section monitors the electric potential generated by the echo returning to the transducer. It is preceded by a voltage limiter to prevent the transmitter pulse from over-driving the amplifiers. The circuitry of this component is given in Fig. 5. Following the diode limiter there are two stages of amplification that feed through an emitter follower to the detector. The detector is a standard peak detector using an R. C. time constant that is too slow to follow the R. F. signal, but fast enough to follow the signal envelope. The output of the detector is fed through another emitter follower to a video amplifier that provides the signal to the computer. The output signal consists of a pulse corresponding in time to the transmitter pulse and a second pulse corresponding in time to the echo signal.

Computer.—The computer consists of a bistable multivibrator and an emitter follower having zener diode output regulation. The computer circuitry is shown in Fig. 6. The bistable multivibrator has two trigger inputs, a pulse from the clock and the receiver outputs. The receiver output consists of two, or more pulses; the transmitter pulse and echo or echoes. The transmitter pulse and the clock pulse occur almost simultaneously such that only the first to occur, the clock pulse, serves to switch the multivibrator on. The echo, which comes a short time later, switches the multivibrator off. The output is a rectangular wave having an "on" time equal to the time lapse between the clock pulse and echo return, and a period equal to the repetition rate of the transmitter pulses. This rectangular wave is fed through the emitter follower, where peak output voltage is regulated to 6 v and, henceforth, to the recorder.

Recorder.—The recorder should be an average reading device that integrates the rectangular wave and records accordingly. Because the magnitude of the rectangular wave is regulated by the zener diode, the average value of the output is directly proportional to the "on" time and, hence, the time lapse between the clock pulse and echo return.

The total circuitry for the dual channel stream monitor consists of one clock, two pulsers, two transducers, two receivers, and two computers. The two channels operate alternately. Channel 'A' is triggered on the positive going swing of the clock output, and channel 'B' is triggered on the negative going swing of the clock output.

In describing the receiver output, reference is made to "one or more" echoes. Repeated echoes may occur because of reverberation. If the secon-

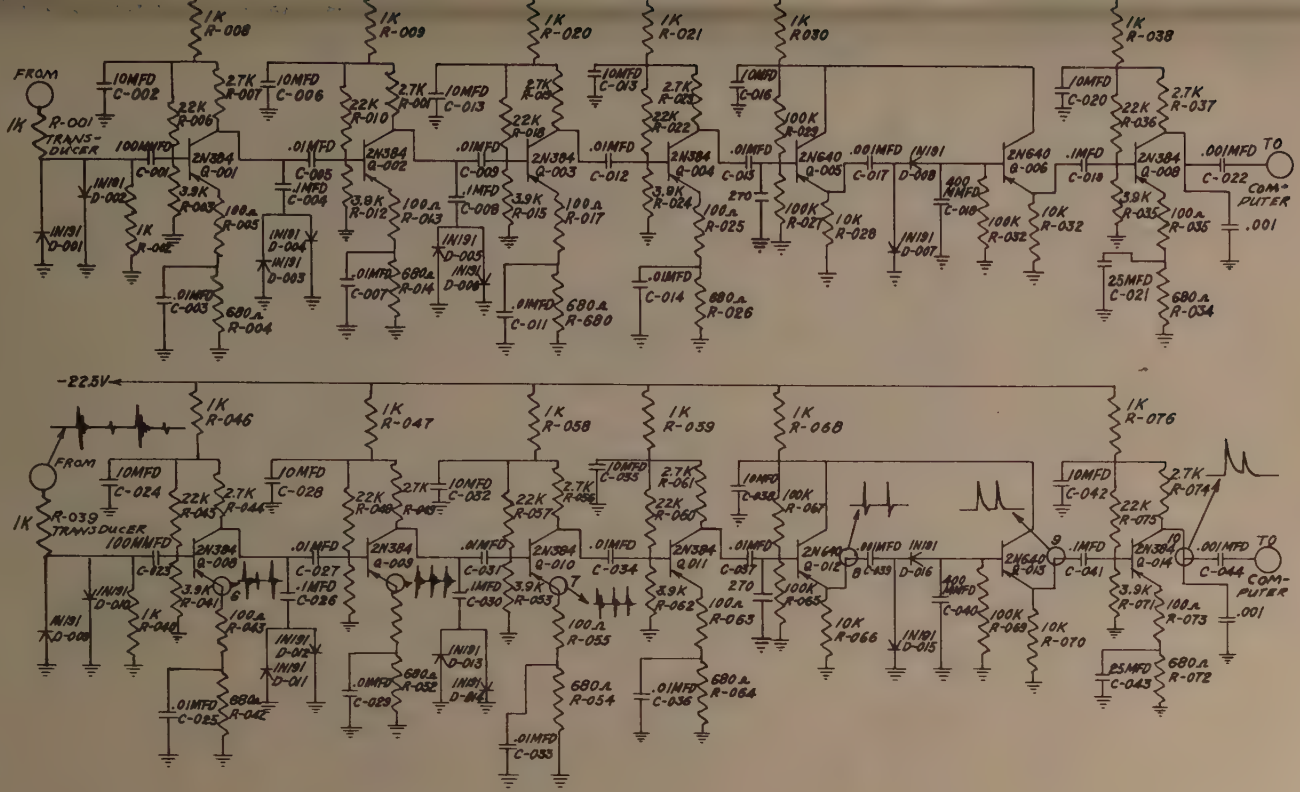

FIG. 5.—RECEIVER AND DETECTOR

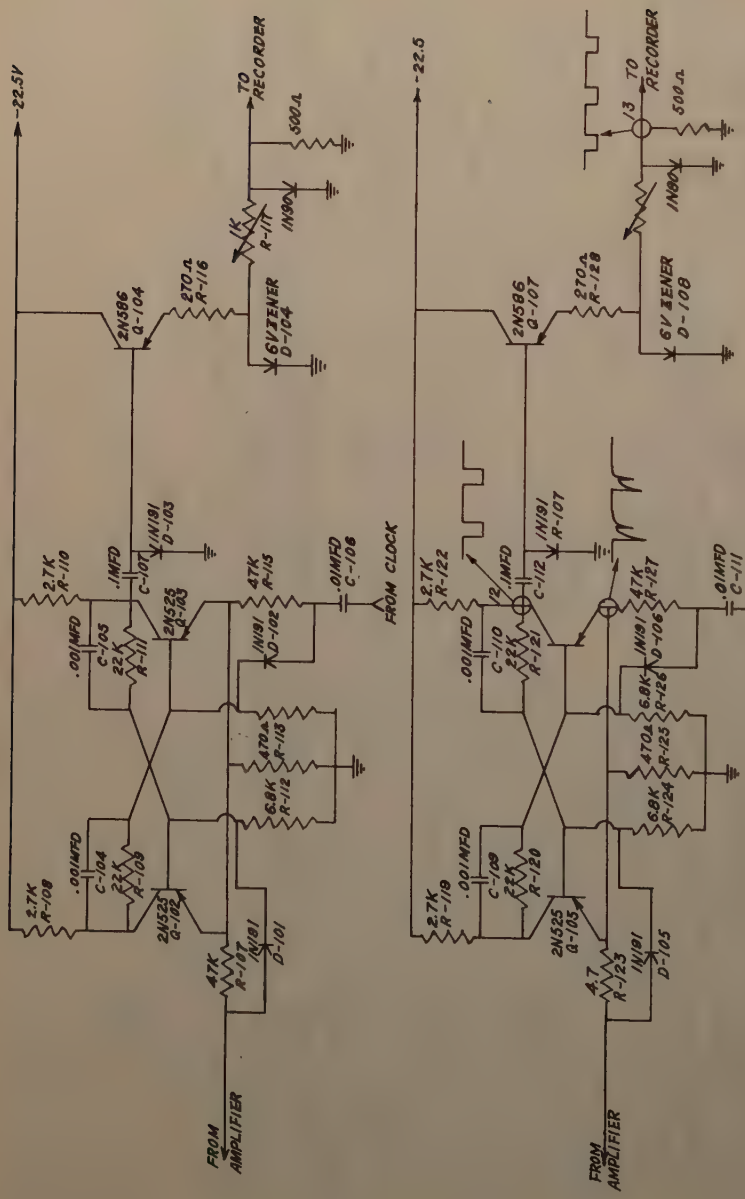

FIG. 6.—COMPUTER

ary echoes have sufficient intensity, they may cause vibration of the transucer, occurring after the succeeding transmitter pulse, but before the reurn of the principle echo. Secondary echoes are undesirable signals and may sually be eliminated by adjustment of the sensitivity control.

CALIBRATION

The dual channel stream monitor should be calibrated before each use. Because of the variations in sound wave velocity due to temperature, for most ccurate results the temperatures of the water during calibration and use hould be nearly the same. It is generally necessary to calibrate for only one istance, this distance being 1 ft, for both the 1-ft and 5-ft monitor scales.

TABLE 1.—STATIC CALIBRATION TESTS, STATIONARY TRANSDUCERS IN STATIC WATER OVER STATIONARY BED

Actual Depth	Monitored Depth				Reflecting Surface
	1 ft scale		5 ft scale		
	Channel A	Channel B	Channel A	Channel B	
(1)	(2)	(3)	(4)	(5)	(6)
0.17 ft	0.17	0.17+	0.21	0.20	Sand
0.32	0.31+	0.32	0.31	0.33	Sand
0.50	0.48	0.49+	0.49+	0.50	Sand
0.6[illegible]	0.66	0.65+	0.63	0.65	Sand
0.83	0.82+	0.82+	0.83	0.82+	Sand
1.0	0.99+	1.00	1.00-	1.00	Sand
1.0	----	----	1.00	0.98	Screen
1.5	----	----	1.49	1.48	Screen
2.0	----	----	1.99	2.00	Screen
2.5	----	----	2.50	2.50	Screen
3.0	----	----	3.00+	3.05	Screen
3.5	----	----	3.55-	3.55	Screen
4.00	----	----	4.05	4.05	Screen
4.5	----	----	4.55	4.50+	Screen
5.0	----	----	4.99	4.95	Screen

djustments for accurate recording of the distance can be made by the potentiometers R-902 and R-903 shown in Fig. 4. The R-902 potentiometer djusts the 5-ft range and R-903 the 1-ft range.

LABORATORY TEST RESULTS

The performance of the dual channel stream monitor under laboratory contions was tested in the hydraulics laboratory. Both fixed and alluvial bed umes were used.

Accuracy.—Tests were made to determine the accuracy of the instrument measuring depths. Sand beds and screen-covered metal surfaces were used targets. A screen was used over the metal surface to attenuate the sound aves and eliminate secondary echoes. Measurements of distances from the ansducer face to the reflecting surface were compared with the monitored esults. A tabulation of the results is shown in Table 1.

A reproduction of the particular chart used in the recorder is shown in Fig. 7. With the 1-ft monitor scale, the chart can be read to 0.02 ft and estimated to the nearest 0.01 ft. On the 5-ft scale the chart can be read to 0.10 ft and estimated to the nearest 0.05 ft. The recorder accuracy is specified by the manufacturer as $\pm$ 2% of full scale reading. The maximum deviation between actual depth and monitor reading in Table 1 was 0.05 ft on the 5-ft scale and 0.02 ft on the 1-ft scale. This is consistent with the accuracy of reading the chart scales.

Transducer Directivity.—Directivity tests of the transducers of the dual channel stream monitor were conducted qualitatively to determine the hydrophone sensitivity of the transducers and to determine the maximum reflecting

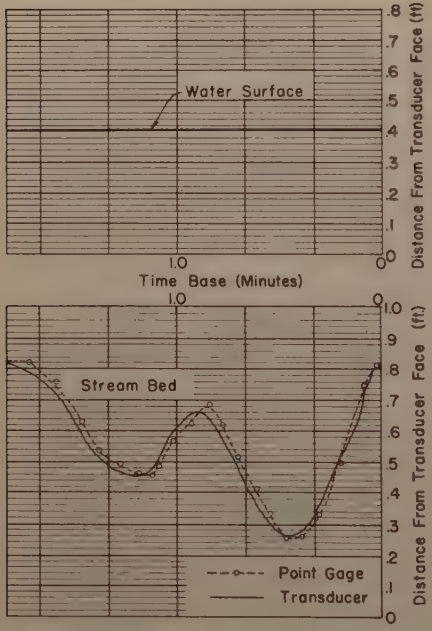

FIG. 7.—TRANSDUCER TRAVERSE OVER RIGID DUNES

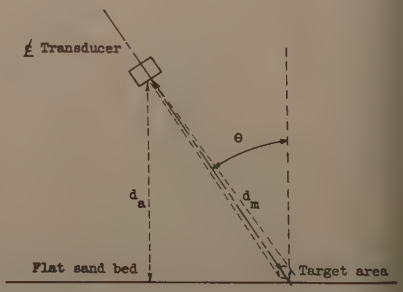

FIG. 8.—DEFINITION SKETCH

angle between the transducer face and the plane of the target. These tests were conducted over a stationary horizontal sand bed in static water. A sketch of the transducer arrangement is shown in Fig. 8 in which θ is the angle of inclination, d_a denotes the shortest distance from centerline of transducer to sand bed, and d_m refers to the distance along the centerline of transducer to sand bed. The results are given in Table 2. These results show that the interrogated target area is small as no difference in actual and monitored distance was evident.

Maximum allowable angle of the transducer was approximately 32°. Beyond this angle the echoes did not return to the transducer in sufficient intensity to actuate the transducer.

Traverses of Transducers over Stationary Sand Beds.—
Static Water.—Stationary sand dunes were constructed in a 2-ft flume with .3 mm diameter sand bonded together with a lean mixture of portland cement 1 part in 15). The dune profile was determined by point gage measurements. Comparison of the monitored results and measurements are shown on Fig. 7.

A significant feature noted was the apparent time lag between the recorded istance and the actual event. This time lag, which was also evident in some f the other tests, was primarily due to the response time of the recorder. The recorder chart speed during the traverse was 3 in. per min, and traverse elocity was approximately 18 in. per min. Another probable reason for the pparent time lag was the slight tilting of the transducer to a trailing position ith respect to the carriage because of the flexibility of the search tube assembly under dynamic conditions. However, this effect, in most instances, ill be small and can be controlled by making the probe unit more rigid. Considering the time lag, the maximum deviation between monitor reading and oint gage was about 0.02 ft, which is again consistent with the accuracy of the ecorder.

TABLE 2.—RESULTS OF TRANSDUCER ANGLE TESTS

θ	d_a	d_m	Monitor Reading
5°	0.69	0.69	0.69
10.50°	0.71	0.72	0.72
15.33°	0.71	0.74	0.74
20°	0.71	0.76	0.76
24.67°	0.72	0.80	0.80
32.33°	0.70	0.82	0.82

Moving Water.—The monitor was also tested under dynamic conditions in ne same flume and over the same rigid dunes as the static water tests. A ypical result of these tests is shown in Fig. 9. The time lag effect was also pparent in these tests, and the amount of lag was approximately the same as or the static tests. Accuracy of the monitored reading was about 0.02 ft. low velocity for this test was approximately 6 fps with considerable surface isturbance developed immediately downstream from each dune. An increase gain was necessary to monitor the water surface profile.

Stationary Operation in a Movable Bed Channel.—In this series of tests the ial channel stream monitor was subjected to operational tests in a movable ed, 2-ft tilting flume. Average sand size of the alluvial bed was 0.3 mm in iameter and flow velocity was varied from 0.75 fps to 4 fps. The instrument as tested in the ripple and dune regimes. Fig. 10 is a reproduction of the ed profile during a 3-hr test. A portion of the recorded results showing both ater surface and stream bed is shown in Fig. 11. The test results indicate at reflections from both stream bed and water surface are satisfactory under namic conditions in an alluvial flume.

Transducer Traverse Over Movable Sand Bed.—Fig. 12 is a reproduction test results of two traverses made in a downstream direction over a mov- g sand bed within a 5-min interval. Event markers (marks on the side of e chart) denote fixed 10-ft intervals on the flume; chart speed was 3 in. per

min and flow velocity was approximately 4 fps. The progress of the same dunes during the 5-min interval can be easily traced from the chart. It is also possible to determine, from the records, the height, shape, and spacing of the dunes.

Stationary Transducer in a Movable Sand Bed Channel with Standing Water Waves.—Fig. 13 is a reproduction of a typical chart record made under flow conditions exhibiting standing water waves in a sand bed flume. The tests were conducted in an 8-ft wide tilting flume. Flow velocity was approximately 7 fps, and average depth was about 0.5 ft. The average sand size of the bed was 1 mm in diameter. Chart speed was 3 in. per min. Tracing the water surface profile even with standing waves was satisfactory, although there was some disturbance caused by the air bubbles entrained on the cascade of the breaking waves. Because of the large sand size, and high concentration of suspended sediment near the bed, the monitored signal was scattered and often recorded an erroneous depth to the stream bed as indicated by the sharp peaks on the chart record. The minimum depth which the instrument would register

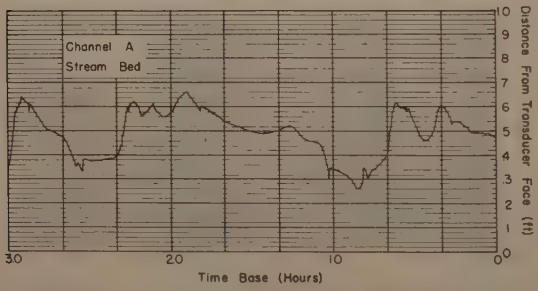

FIG. 10.—MOVING SAND DUNES STATIONARY TRANSDUCER POSITION V = 4.0 fps

was about 0.12 ft. Other tests indicated the maximum reliable operating depth to be about 0.20 ft.

Battery Life Test.—Determination of the useful life of the battery pack included in the dual channel stream monitor was made under continuous operation. It is normally expected that AC power will be used in the laboratory; however, for field use and specialized laboratory use, it may be convenient to utilize battery power. A new battery pack, consisting of two 90-v dry cells and one 45-v center tapped dry cell was installed immediately before the test was started. A 15 min warm-up period was allowed for the equipment during which final adjustments were made. After 14 hr of continuous operation, it was necessary to readjust the gain controls slightly because of some loss in power. After readjustment, normal operation continued for an additional 15 hr before further adjustments were necessary. Beyond the twnety-ninth hour of continuous operation, constant adjustment of the gain controls was found to be necessary to maintain satisfactory operating conditions. After approximately 33 hr of operation, the power provided by the battery pack was considered insufficient to maintain a satisfactory reliable performance.

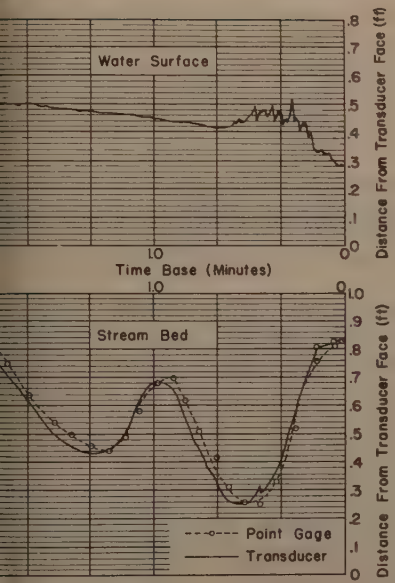

FIG. 9.—TRANSDUCER TRAVERSE OVER RIGID DUNES V = 6 fps

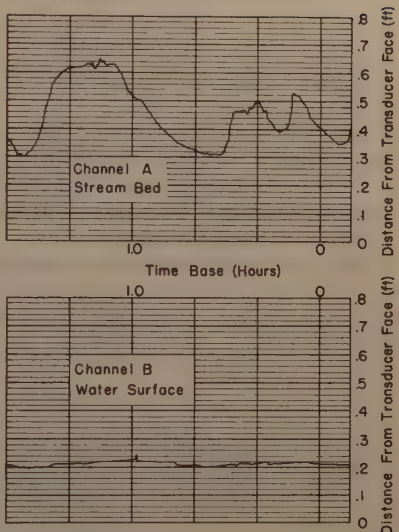

FIG. 11.—MOVING DUNES STATIONARY TRANSDUCERS V = 4 fps

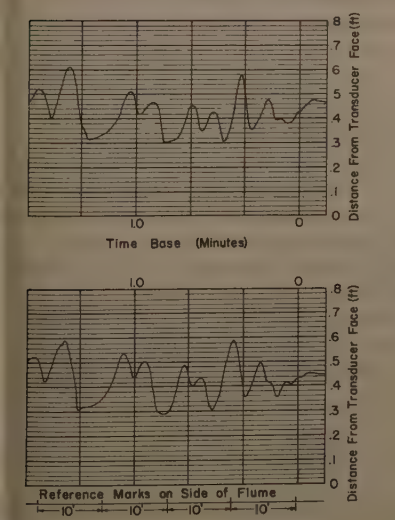

FIG. 12.—TRANSDUCER TRAVERSE IN DOWNSTREAM DIRECTION OVER MOVING DUNES. V = 4 fps. UPPER RECORD MADE 5 MIN. BEFORE LOWER

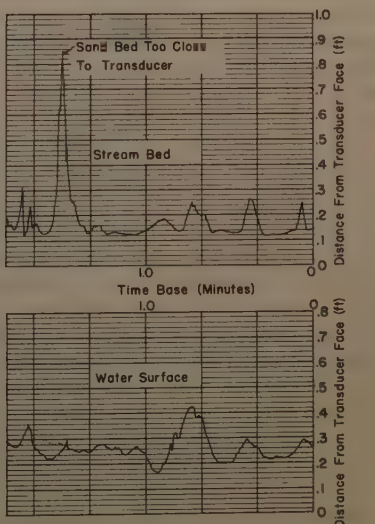

FIG. 13.—STATIONARY TRANSDUCERS. STANDING WAVES V = 7 fps

Operation in Deep Channels.—There is a possibility of utilizing the dual channel stream monitor in the field if the transducers can be satisfactorily positioned in the stream channel. One possibility is to mount the transducers in the body of a standard current meter weight. It is expected that the lead weight can be manipulated from a cableway in the same manner as current meters.

A lead weight assembly was constructed and tested in the laboratory. Satisfactory results of depth recordings were obtained. The transducers were mounted in the lead weight with one transducer placed flush with the top and the other flush with the bottom. The transducer cables were imbedded in the weight and rendered watertight.

Other Laboratory Tests.—The effect of flow velocity on monitored accuracy was tested with a fixed position of the transducer. The velocity of flow was varied from 1 fps to 12 fps. No noticeable deviation in depth reading was noted.

CONCLUSIONS

The dual channel stream monitor is a satisfactory instrument to monitor water surface and stream bed profiles in an alluvial channel for purposes of hydraulic research. When operated with a rectilinear recording milliameter, the total instrument complex has an accuracy of $\pm$ 0.05 ft on the 5-ft scale and $\pm$ 0.02 ft on the 1-ft scale. The accuracy of the instrument complex is restricted by the accuracy of the recorder. It is possible to improve the accuracy by using either a more sensitive recorder or an oscilloscope.

Depth measured by the instrument to a reflecting surface is the distance from the transducer face along the cylindrical axis of the transducer. The cylindrical axis coincides with the acoustic axis. The maximum angle of inclination of the plane of the target with respect to the transducer is approximately 32°. The transducers have a high directivity index in sound wave reception so that effectively small areas of the target are interrogated.

The instrument operates satisfactorily with transducers mounted in a stationary position or by traverse in the stream channel. A finite time lag from true position of the dunes relative to the channel is apparent where rapid change in depth occurs. This is caused by the response time of the recorder. In most hydraulic research the relative positions, sizes, and shapes of the sand dune formations are more important than the actual positions of the dunes with respect to laboratory flume. Surface turbulence and air entrainment associated with high velocity and layers of dense sediment concentration can cause some difficulty in monitoring true distances. Under normal stream flow velocities excellent results are achieved by the dual channel stream monitor.

ACKNOWLEDGMENTS

The development of the dual channel stream monitor was achieved by Colorado State University, Fort Collins, Colo. through financial sponsorship of the Watershed Research Branch of the Agricultural Research Service and the technical perception of Automation Industries, Inc. Valuable assistance and guidance in concept was also received from E. V. Richardson, A.M. ASCE, and D. B. Simons, M. ASCE, of the United States Geological Survey.

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CALCULATION OF POTENTIAL FLOWS WITH FREE STREAMLINES

By Garrett Birkhoff[1]

SYNOPSIS

A survey is given of methods for computing three classes of time-independent incompressible potential flows with free streamlines: (1) plane ows having free boundaries and curved fixed boundaries without gravity;) plane flows having free boundaries and straight fixed boundaries with ravity; and (3) axially symmetric flows having free boundaries without gravity.

FIXED BOUNDARIES

Because almost all flows with free boundaries also have fixed boundaries, is paper will begin with a brief analysis of methods for computing potential ows having fixed boundaries only. This problem is much simpler and relies closely related methods. Four classes of methods are available for solving is problem.

Difference Methods.—The Laplace equation can be approximated by a well-own 5-point difference approximation at each interior point of a rectangular t. The resulting system of linear algebraic equations can then be solved by veral techniques: by Southwell's method of relaxation using desk machines; the Young-Frankel method of Systematic Overrelaxation using high-speed odern computing machines, or by Implicit Alternating Direction methods.

Note.—Discussion open until April 1, 1962. To extend the closing date one month, a itten request must be filed with the Executive Secretary, ASCE. This paper is part of copyrighted Journal of the Hydraulics Division, Proceedings of the American Society Civil Engineers, Vol. 87, No. HY 6, November, 1961.

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Of these methods, it is recommended that the non-specialist first use the method of Systematic Overrelaxation.[2]

Numerical Conformal Mapping.—The determination of any plane potential flow past given fixed boundaries is equivalent to a problem in conformal mapping. There are numerous special techniques for computing conformal transformations between given regions.[3] The most effective methods (due to Gerschgerin and Theodorsen) reduce the problem to the numerical solution of singular integral equations in functions of one independent variable. A definitive survey of these methods will be presented elsewhere.[4,5]

Integral Equation Methods.—Other integral equation methods, taking as unknown function the distribution of sources, dipoles, or vorticity on the boundary that produces a given flow, can be applied to axially symmetric flows with fixed boundaries. But as their efficacy for hydraulic problems has not been carefully tested, they are not recommended at the present time (1961).

Graphical and Electrolytic Tank Methods.—Finally, there are practical methods for determining potential flow nets by graphical conformal mapping and by the electrolytic tank analogy. But as these methods involve only elementary mathematical ideas, and have limited accuracy; nothing will be said about them here.

TRIAL FREE STREAMLINES

Given any effective method for computing potential flows with given fixed boundaries and points of flow separation, one can try to find potential flows having unknown free streamlines as follows: Try a plausible free boundary shape, and compute the associated velocity distribution on the supposed "free" streamline. Using the Bernoulli equation, find the deviations from the desired condition of constant pressure. Then guess a new free streamline profile, pulling the boundary away from the flow where the pressure is low and pushing it into the flow where the pressure is high.

This method was used effectively by Southwell and G. Vaisey,[6] in conjunction with relaxation methods, to determine flows of all three types (1) to (3) described in the Synopsis.[7] However, as a systematic method, it suffers from various defects: The shape of the free boundary is sensitive to small variations in the velocity, and it is hard to achieve accuracy near points of flow separation. Finally, it is obviously quite tedious.

Young and his co-workers have tried to make the method more systematic by using Systematic Overrelaxation on the NORC at Dahlgren instead of hand

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2 "Iterative Methods for Solving Partial Differential Equations of Elliptic Type," by D. M. Young, Jr., Transactions, Amer. Math. Soc., Vol. 76, 1954, p. 92.

3 "Numerical Methods in Conformal Mapping," by G. Birkhoff, D. M. Young, and E. H. Zarantonello, Proceedings, Fourth Symposium of Applied Math., McGraw-Hill Book Co., New York, 1953, p. 117.

4 "Numerical Conformal Mapping," by Dieter Gaier, to be published in 1962 or 1963 by Springer.

5 "Construction and Applications of Conformal Maps," U. S. Natl. Bur. of Standards of Applied Math., Publication 18, U. S. Govt. Printing Office, 1952.

6 "Fluid Motions Characterized by Free Streamlines," by R. Southwell and G. Vaisey, Philosophical Transactions, Royal Soc. of London, Series A 240, 1948, p. 117.

7 "Applied Mechanics Reviews 4634," by D. Dumitrescu, V. Ionescu, and K. Yanke, Review of Mechanical Applications, Vol. 4, 1959-1960, p. 249.

elaxation. Their work is reviewed elsewhere.[8] The determination of flows ith free streamlines by the "trial free streamline" method, in conjunction ith an electrolytic tank will not be covered herein.[9]

REFLECTION PRINCIPLE

Instead, methods which rely on deeper function theoretic methods and seem etter adapted to obtaining accurate results will be discussed.

These may be considered as extensions of the technique used by Helmholtz nd Kirchhoff to compute two-dimensional jets from orifices in plates and avities behind plates. These authors recognized elementary conformal trans-rmations connecting the semicircular hodographs of the preceding flows ith the domain occupied by their complex potential $W = U + iV$, and then deter-ined the complex position $z = x + i\,y$ by the integral relations $z = \int \zeta^{-1}\, dW$, in hich $\zeta = dW/dz$ is the complex conjugate $dx/dt - i\,dy/dt$ of the flow velocity.

Using less elementary special conformal transformations, many other flows ith free streamlines of hydraulic interest have been obtained by this method.[10] ut the method is only applicable to polygonal fixed walls, and usually involves ne or more parameters, that also occur in the mathematical formulas. Hence, mputations must be performed over a range of values of these parameters nd interpolated. It is doubtful whether this method will be fruitful in the future.

A deeper study of the method in question reveals it as a special case of e Schwarz Reflection Principle[11] and suggests that one should try to make rther progress by extending the range of effective application of this princi-e, rather than by considering many-parameter families of elaborate special nctions.

CURVED BOUNDARIES, NO GRAVITY

By utilizing the Schwarz Reflection Principle, Levi-Civita and Villat were le to reduce free boundary problems for plane potential flows past curved rriers, without gravity, to the solution of an associated nonlinear singular tegral equation, of the form

$$\lambda(\sigma) = M\,\nu(\sigma)\,K(J[\lambda])\,e^{-D[\lambda]}, \qquad (1)$$

which M is a constant, $\nu(\sigma)$ a known function (that may involve parameters), $J[\lambda] = \sum a_k \cos(k\sigma)$, and $D[\lambda] = \sum a_k \sin(k\sigma)$. The problem is to solve Eq. 1 r the unknown (odd) function $\lambda(\sigma) = \sum k\,a_k \sin(k\sigma)$.

E. H. Zarantonello and the writer have developed an effective method for lving Eq. 1 by iteration or "averaged iteration," using trigonometric inter-lation with constant mesh-spacing. This method has been fully described sewhere.[3,12] The method works well, except for obstacles whose curvature ries by an order of magnitude. It would be desirable to extend it to such

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8 "Second ONR Symposium on Naval Hydrodynamics," by Ralph D. Cooper, Editor, oceedings, U. S. Govt. Printing Office, 1960.

9 "Jets, Wakes, and Cavities," by G. Birkhoff and E. H. Zarantonello, Academic ess, 1958, Chapter IX, § 10.

10 Ibid., Chapters II, III, and V.

11 Ibid., Chapter III, § 2; Chapter IV § 2.

12 Ibid., Chapter IX, §§ 8–9.

obstacles; perhaps this could be accomplished by replacing Eq. 1 by an alternative integral equation of Villat,[13] or by replacing trigonometric interpolation by a smoother interpolation formula, or by using unequal mesh-spacing with mesh-points concentrated in segments of large curvature. It would also be desirable to extend the method to jets from two-dimensional nozzles.

STRAIGHT WALLS WITH GRAVITY

The integral Eq. 1 is derived by reflecting the flow of interest in the free boundary and taking out the singularity (which appears in the factor $\nu(\sigma)$). Free boundary problems can be reduced for plane potential flows past straight walls, with gravity, to the solution of a nonlinear singular integral equation of similar form, by reflecting the flow in the fixed boundary. (In the case referred to elsewhere,[14] both can be done.)

This reduction has been described elsewhere in general terms,[15] and a numerical application is made to a bubble rising in a tube.[16] The results of

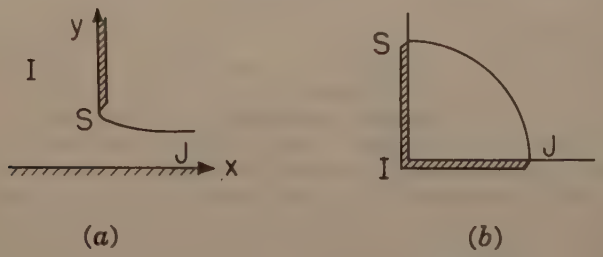

FIG. 1.—FLOW UNDER A VERTICAL SLUICE GATE

this computation have been confirmed by P. Garabedian,[17] as well as (approximately) by experiment. Carl de Boor[18] has determined approximately the flow under a vertical sluice gate as shown in Fig. 1, by this method.

If gravity is neglected, the exact solution is just half the jet from an orifice in a plane wall (slot), as determined by Helmholtz in 1868. If gravity is large in the sense that the flow is subcritical, then it seems probable that there is a family of flows.[19] However, if the flow is supercritical, it can be presumed that there is only one flow and it can be computed mathematically as follows.

Lay the x-axis along the "floor" under the flow, and the y-axis along the wall of the sluice gate, as in Fig. 1(a). Then map the flow conformally onto the

13 Ibid., p. 136, formula (15).
14 Ibid., § 3.
15 Ibid., Chapter VIII, § 11.
16 "Rising Plane Bubbles," by G. Birkhoff and D. Carter, Journal of Mathematics and Mechanics, Vol. 6, 1957, p. 769.
17 "On Steady-State Bubbles Generated by Taylor Instability," by P. Garabedian, Proceedings, Royal Soc. of London, Series A 241, 1957, p. 423.
18 "Flow Under a Sluice Gate," by Carl de Boor, Harvard Univ., Contract Nonr 1866(34), Mimeographed Report, March, 1961.
19 "The Flow of Water Under a Sluice Gate," by A. M. Binnie, Quarterly Journal of Mechanical and Applied Mathematics, London, Vol. 5, 1952, p. 395.

irst quadrant of the unit circle in an auxiliary t-plane, as in Fig. 1(b), so that he floor and sluice gates again lie on the real and imaginary axes, thus putting he free surface on the quarter-circumference $t = e^{i\sigma}$, $0 < \sigma < \pi/2$. The com-plex potential will then satisfy $W = R \ln [2t/(1-t^2)]$ for some value of $R > 0$ which depends on the units and dimensions involved.

Now define $\Omega(t) = \tau(t) + i\theta(t)$ by the equation $\zeta = t \exp \Omega$, or $\Omega = \ln(\zeta/t)$. The differentiated free boundary condition on $t = e^{i\sigma}$ may then be shown to be equivalent to the nonlinear singular integral equation for $\lambda(\sigma) = \tau'(\sigma)$.

$$\lambda(\sigma) = \gamma \cot(\sigma)\, e^{-3J[\lambda]} \sin(\sigma + D[\lambda]),\ 0 \leqq \sigma \leqq \pi/2 \quad \ldots\ldots \quad (2)$$

n which $J[\lambda]$ and $D[\lambda]$ have the same meaning as in Eq. 1, and γ is a dimen-sionless parameter which depends monotonically on the reciprocal Froude umber

$$\frac{1}{F} = \frac{g\, y(S)}{v_J^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

he sluice gate problem is thus reduced to the solution of Eq. 2 over a range f value of the parameter γ.

The approximate numerical solution of Eq. 2, performed by de Boor, gives he coefficient of contraction C_c as a function of the reciprocal Froude number, s tabulated in the following:

$1/F$	0.000	0.0785	0.157	0.236	0.314
C_c	0.611	0.581	0.559	0.529	0.474

he singularity at $\sigma = 0$, in which $\cot \sigma = \infty$ is multiplied by $\sin [\sigma + D[\lambda]] = 0$, reatly increases the truncation error.

DIMENSIONAL PERTURBATION

Computations have been made of axially symmetric flows with free stream-nes, having straight fixed boundaries, without gravity. An excellent analysis f this work has been given.[20,21]

The axially symmetric jet from a circular orifice was computed by Trefftz 1916, using trial free streamlines in conjunction with integral equation ethods. He estimated the coefficient of contraction to be $C_c = 0.61 \pm 0.01$. his value was confirmed by Southwell and Vaisey,[22] using trial free stream-nes in conjunction with relaxation methods.

A more sophisticated method of attacking axially symmetric problems is at of dimensional perturbation, due to Garabedian,[23] who considers the fferential equation and boundary conditions as analytic functions of a contin-ously varying dimension-number. It is remarkable that Garabedian[24] obtains e different value $C_c = 0.58$ by this method (which also gives the cavity drag

20 "Jets and Cavities," by D. Gilbang, Handbuch der Physik, Vol. IX, 1960, pp. 311-445.
21 "Jets, Wakes, and Cavities," by G. Birkhoff and E. H. Zarantonello, Academic ress, 1958, Chapter X.
22 "Fluid Motions Characterized by Free Streamlines," by R. Southwell and G. Vaisey, ilosophical Transactions, Royal Soc. of London, Series A 240, 1948, p. 117.
23 "The Mathematical Theory of Three-Dimensional Cavities and Jets," by P. Gara-dian Bulletin, Amer. Math. Soc., Vol. 62, 1956, p. 219.
24 "Calculation of Axially Symmetric Cavities and Jets," by P. Garabedian, Pacific urnal of Math., Vol. 6, 1956, p. 611.

coefficient of a disc as 0.83). Though this discrepancy is perhaps not very important for the engineer, it brings out the need for further scientific research into the accuracy of free streamline computations by the method of "trial free streamlines," before great reliance can be placed on the results.

CONCLUSIONS

In principle, it seems clear that many plane potential flows with free boundaries can be rigorously computed with great accuracy, through the iterative solution of nonlinear integral equations on high-speed computing machines. This has already been done in a number of cases. To apply similar methods effectively to other problems would seem a fruitful area of work for the civil engineer interested in fundamental theoretical research.

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STUDY OF SCOUR AROUND SPUR-DIKES

By R. J. Garde,[1] M. ASCE, K. Subramanya,[2] and K. D. Nambudripad[3]

SYNOPSIS

The results of an investigation about the maximum scour depth occurng at a spur-dike are reported herein. This investigation was conducted a 2 ft wide by 25 ft long non-recirculating flume. The effect of flow, spurke and sediment characteristics on the maximum scour depth was studied verifying the dimensional analysis with the help of experimental data. It as found that the Froude number of the normal channel, the opening ratio, e angle of inclination of the spur-dike, and the average drag coefficient of e sediment particle adequately represent the influence of flow, spur-dike, d sediment characteristics on the maximum scour depth.

INTRODUCTION

The problem of scour around any obstruction placed in an alluvial chanl is of great importance to hydraulic engineers. In practice, a channel is ten obstructed by one means or other, such as training dikes, bridge piers, utments, spur-dikes and so on; to be able to design a safe and economic ructure, it is important to have a clear picture of scour phenomenon ound these obstructions. In order to study the variables governing the pth of scour around obstructions such as spur-dikes, investigations were

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conducted at the Hydraulics Laboratory of the University of Roorkee, Roorkee, India. The salient features of this investigation are summarized herein.

A spur-dike is a structure constructed at an angle to the flow direction, extending from the bank of an alluvial channel into the main flow. It serves one or more of the following functions: (1) Training of the stream flow; (2) protection of the stream bank from erosion; (3) improvement of depth for navigation. As the water flows around the spur-dike, the flow pattern is changed due to the reduction of the width of channel, and as a result of this, the shear distribution around the spur-dike is modified. This leads to scouring action until equilibrium is established between the various forces influencing the scouring action.

Some work has been done about various aspects of the hydraulic characteristics of spur-dikes. Lacey[4] gives a formula for maximum scour depth at a spur-dike. Based mainly on the field observation, Claude C. Inglis,[5] F. ASCE, compared the maximum scour depths with those obtained from Lacey's equation. He recommended certain coefficients to be applied to Lacey's regime depth to find the maximum scour depth and also analyzed the influence of size of material on the maximum scour depth. Andru and J. Blench,[6] F. ASCE, after analyzing various laboratory and field data on scour at obstructions, concluded that the depth of local scour depends on discharge intensity and the size of bed material. Mushtaq Ahmad[7] conducted investigations on the behavior of spur-dikes and drew some valuable conclusions regarding the effect of various parameters on the maximum scour depth.

More laboratory investigations have been conducted on problems related to scour at bridge piers. Emmett M. Laursen,[8,9,10] M. ASCE, has concluded that the depth of scour for a given geometry of obstruction is dependent on the width of obstruction and the depth of flow when there is appreciable sediment supply. Hsin-Kuan Liu, M. ASCE, and M. M. Skinner[11] have given some qualitative results regarding scour at abutments. Carl F. Izzard, F. ASCE, and Joseph Bradley, F. ASCE,[12] reported the preliminary findings of an investigation of scour at bridge abutments.

The purpose of this investigation was the study of the characteristics of scour at a spur-dike in general and the maximum depth of scour in particu-

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4 "Design of Weirs on Permeable Foundations," by A. N. Khosla, Publication No. 12, Central Bd. of Irrig., India, 1936.

5 "The Behaviour and Control of Rivers and Canals with the Aid of Models," by C. C. Inglis, Research Publication No. 13, Part II, CWINRS, Poona, India.

6 "Regime Behaviour of Canals and Rivers," by T. Blench, Butterworth Scientific Publication, London, 1957.

7 "Experiments on Design and Behaviour of Spur-dikes," by M. Ahmad, Proceedings, I.A.H.R., Minnesota, 1953.

8 "Progress Report of Model Studies of Scour Around Bridge Piers and Abutments," by E. M. Laursen, Highway Research Bd., Research Report No. 13-B, 1951.

9 "A Generalized Model Study of Scour Around Bridge Piers and Abutments," by E. M. Laursen, Proceedings, I.A.H.R., Minnesota, 1953.

10 "Scour at Bridge Crossings," by E. M. Laursen, Proceedings, ASCE, Vol. 85, No. 86, No. 2, February, 1960.

11 "Laboratory Observations of Scour at Bridge Abutments," by H. K. Liu, and M. M. Skinner, Highway Research BD., Bulletin No. 242, 1960.

12 "Field Verification of Model Tests on Flow Through Highway Bridges and Culverts," by C. F. Izzard and J. N. Bradley, Proceedings, 7th Hydr. Conf. Iowa, 1957.

lar. Attention was focussed mainly on the influence of sediment, spur-dike, and flow characteristics on the maximum scour depth.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in the Appendix.

PROBLEM AND LIMITATION

The problem posed for the investigation was to study variation of maximum scour depth under the following limitations:

1. A single idealized spur-dike placed at right angles to the flow was considered.
2. The opening ratios $\frac{\text{width of contracted section}}{\text{width of the flume}}$ used were 0.900, 0.835, 0.667, and 0.530.
3. Bed materials of median diameters 0.20 mm, 0.45 mm, 1.00 mm and 2.25 mm and of average specific gravity 2.70 were used to investigate the effect of sediment characteristics.
4. The discharge varied between 0.07 cfs to 0.45 cfs per ft width of the flume.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

The experiments were conducted in a horizontal glass walled flume 2 ft wide by 25 ft long. The water was supplied to the system from a constant head tank (Fig. 1). The water then passed through a 12-ft long entrance box which was provided with a series of baffles. These baffles distributed the flow uniformly over the entire width of the flume and also helped in destroying the excess energy of the flow. The water then passed over a sand bed of 2-in. thickness, discharged over a radial tail-gate, and returned to the sump. A Venturimeter located in the supply line was used to measure the discharge flowing through the system. In order that the discharge measurements be accurate, the Venturimeter was calibrated volumetrically prior to beginning of this investigation.

The water and the bed surface readings were taken with the help of a pointgage mounted on a wooden frame. This frame could slide on the horizontal side walls of the flume. The temperature of the water was recorded during the experiment with a thermometer.

The spur-dike models used were vertical steel plates placed at right angles to the flow. Their widths were such as to give the opening ratios of 0.900, 0.835, 0.667 and 0.530.

Before the beginning of each run the sand bed was screeded to approximate a predetermined slope. The desired discharge was then allowed to flow in the flume by slowly opening the valve. The tail-gate was then so adjusted that the water surface slope and the bed surface slope were reasonably the same. On some occasions the bed had to be rescreeded to make both slopes equal. The water was allowed to flow in this condition for a period varying between 1 hr and 3 hr so that the flow could attain equilibrium by adjustment of the bed slope. The water-surface and bed-surface readings were

taken at 1 ft intervals. The water-surface slope, the bed-surface slope, and the depth of flow were determined from these readings.

When the flow was in equilibrium the spur-dike model was introduced and bed readings at the nose were taken at various time intervals. The run was continued until the bed reading at the nose changed so slowly with time that it was difficult to record a change. This time, after introduction of the model, varied from 3 hr to 5 hr. After recording the maximum scour depth, the flow was slowly decreased so that the scour pattern was not disturbed. Bed readings were then taken around the spur-dike and used to prepare contour maps.

After completion of experiments with one size of a sand, the sand in the flume was replaced by another sand size and a new series of experiments were run. Four different sizes of sands, 0.29 mm, 0.45 mm, 1.00 mm, and 2.25 mm median diameters with average specific gravity of 2.70 were used. Fig. 3 shows the size distribution curves of these sands.

The flume being a non-recirculating type, clear water flowed over the upstream portion of the sand bed from which it picked up some sediment. This scoured material supplied the necessary sediment in the downstream portion of the flume where the spur-dike model was introduced. In a few runs, in which the sediment transport rate was high, sediment was fed manually at the upstream end. The use of a recirculating system would have been better for this investigation, however, it was thought that much valuable information could be obtained with the simple non-recirculating system that was available to the writers.

COMPUTATION OF HYDRAULIC PARAMETERS

The discharge for each run was obtained from the Venturi meter readings. The depth of flow was taken as the average of depths of flow measured at every foot along the length of the flume. Mean velocity of flow, V, was obtained from the relationship

$$V = \left(\frac{Q}{BD}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which Q is the discharge, B denotes width of the flume and D is the average depth of flow.

The opening ratio, α, defined as the ratio of the width of the flume at the contracted section (Fig. 2) to the full width of flume is used hereafter as the measure of the relative opening $\left(\alpha = \frac{B - b}{B}\right.$, in which b is the width of the spur-dike$\left.\right)$.

DEVELOPMENT OF SCOUR

According to Liu and Skinner[11] the scour depth in the scour hole increases indefinitely in the absence of sediment supply to the scour hole from the upstream. It was found in this investigation that for all practical purposes the maximum scour depth was attained after 3 hr to 5 hr. After this the lowering of the bed was too slow to record with the available equipment. This can be seen from Fig 4 which shows a plot of the scour depth, d_t, as a function of time for various sand sizes. All the curves seem to approach

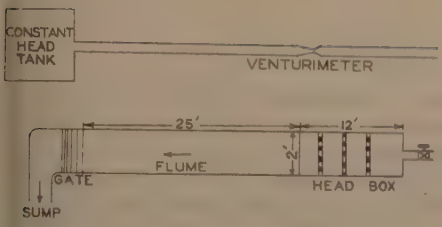


FIG. 1.—EXPERIMENTAL SET UP

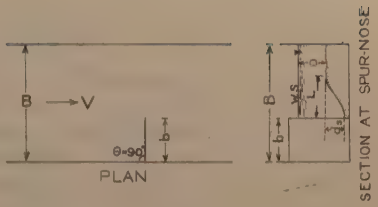
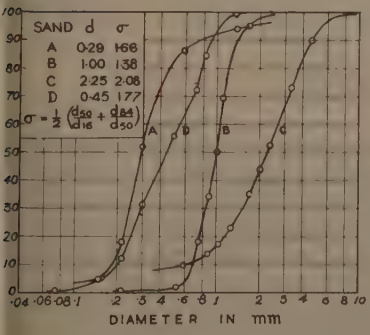


FIG. 2.—DEFINITION SKETCH



G. 3.—SIZE DISTRIBUTION CURVES

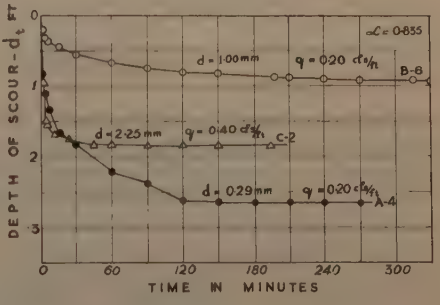
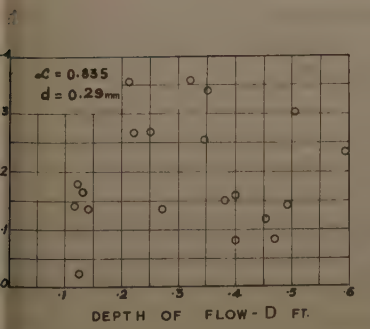


FIG. 4.—DEVELOPMENT OF SCOUR



IG. 5.—VARIATION OF d_s WITH D

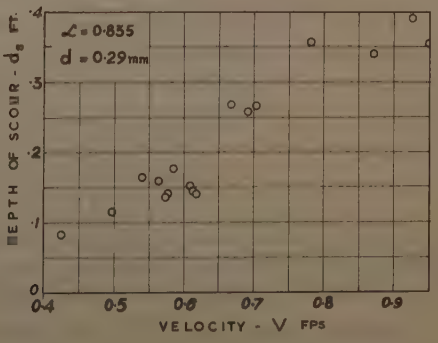


FIG. 6.—VARIATION OF d_s WITH V

a limit. This limiting scour depth is termed the maximum scour depth and is used in the subsequent computations.

GEOMETRY OF SCOUR HOLE

In almost all cases, the maximum scour depth occurred at the nose of the spur-dike. The scour hole upstream of the spur-dike was conical in shape, whereas downstream, it was elongated and had a shallower slope. A deposition bar was formed adjacent to the spur-dike on the downstream side in all cases. It was further found that the width of the scour hole L (Fig. 2) in front of the spur-dike had no consistent correlation with the maximum scour depth, d_s. The value of $\frac{L}{d_s}$ varied from 1.80 to 5.00 and had no systematic correlation either with the opening ratio α, or the size of the bed material d. This is at variance with the constant value of $\frac{L}{d_s} = 2.75$ used by Laursen[10] in the calcualtion of scour around abutments.

MAXIMUM SCOUR DEPTH

Various ideas and expressions have been proposed by investigators for the calculation of the maximum scour depth caused by the introduction of obstruction in an alluvial channel. It was thought necessary to verify these relationships with respect to the data collected by the writers before developing an independent approach to the problem of determination of maximum scour depth.

According to Laursen[9], scour around bridge piers is a function of depth of flow and is independent of mean velocity and size of bed material for a given opening ratio. Fig 5 shows a plot of d_s against D for $\alpha = 0.835$. Considerable scatter of experimental data indicates that such a relationship as proposed by Laursen does not exist for the spur-dikes. On the other hand the velocity of flow (Fig. 6) seems to be an important factor in determining the maximum scour depth.

Laursen[10] has recently considered a bridge crossing as a fore-shortened long contraction. Under the condition of an appreciable sediment supply, he has obtained the following relationship for scour at an encroaching abutment.

$$\frac{d_s}{D} = f\left[\frac{b}{D}, \frac{V_*}{w}\right] \quad \ldots\ldots\ldots\ldots \quad (2$$

in which V_* is the shear velocity, b is the width of obstruction, and w is the settling velocity of the sediment. Shear velocity is defined as the square root of average shear stress on the bed divided by the mass density of fluid. For small variations in $\left(\frac{V_*}{w}\right)$ Eq. 2 reduces to

$$\frac{d_s}{D} = f\left[\frac{b}{D}\right] \quad \ldots\ldots\ldots\ldots \quad (3$$

pplication of this relationship to the present investigation was studied by nsidering the spur-dike as the limiting case of an abutment. Fig. 7 shows plot of $\left(\frac{d_s}{D}\right)$ against $\left(\frac{b}{D}\right)$ for all the runs in which scour did not reach the her wall of the flume. It can be seen that there is a wide scatter of points. herefore, the writers wonder whether the functional relationship $\frac{d_s}{D} = f\left(\frac{b}{D}\right)$ adequate to express the maximum scour depth as a function of flow, fluid, diment, and spur-dike characteristics.

Using Lacey's regime formula, the maximum scour depth relative to water rface is expressed as

$$(D + d_s) = 0.47 \left(\frac{Q}{f_1}\right)^{1/3}, \quad \ldots\ldots\ldots \quad (4)$$

N. Khosla,[4] F. ASCE, from Lacey's formula has related the scour depth the discharge intensity by the relation

$$(D + d_s) = 0.90 \left(\frac{q^{2/3}}{f_1^{1/3}}\right) \ldots\ldots\ldots \quad (5)$$

which Q is discharge, f_1 is Lacey's silt factor, which is a function of grain ze, and q denotes the discharge per foot width of main channel. This for- ula is intended for normal regime channels, and application to the calcula- n of scour at different modifications of channel like bridge piers, bends, d so on, requires the use of a coefficient[4,5], which ranges from 1.0 to 3.5. Blench[6] refers to the work of Andru in which data from various sources e plotted as $(D + d_s)\ F_b^{1/3}$ against q. Here F_b is the bed factor. The equa- n of the best fit line is given as

$$(D + d_s)\ F_b^{1/3} = 1.35\ q^{0.74} \ldots\ldots\ldots \quad (6)$$

C. F. Izzard, F. ASCE, and J. N. Bradley,[12] F. ASCE, suggest an empiri- equation for scour at bridge abutments and river spurs as

$$(D + d_s) = 1.40\ q_1^{2/3} \ldots\ldots\ldots \quad (7)$$

which q_1 is the discharge intensity at the contraction q/α. This is based a plot of various data from models and prototypes. Eqs. 4 through 7 are pe expressed in feet per second.

Ahmad[7] has suggested the formula for $(D + d_s)$ as

$$(D + d_s) = K\ q_1^{2/3} \ldots\ldots\ldots \quad (8)$$

which K is a constant depending on flow concentration and angle of incli- ion of spur-dike.

Eqs. 4 through 8 indicate that $(D + d_s) = f(q)$, but they do not show what e sediment size and opening ratio may have on scour depth. Fig. 8 shows lot of $(D + d_s)$ against q for a constant value of d = 0.29 mm and for the

four opening ratios used in the present investigation. The plot indicates a promising relation between q and $(D + d_s)$ for a constant value of α and d

It need not be stressed further that a significant correlation exists between q and $(D + d_s)$. However, it is the writers' contention that hidden in this relationship must be a combination of various dimensionless terms which log-

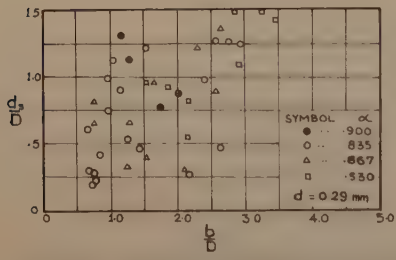


FIG. 7.–VARIATION $\frac{d_s}{D}$ WITH $\frac{b}{D}$ AND α

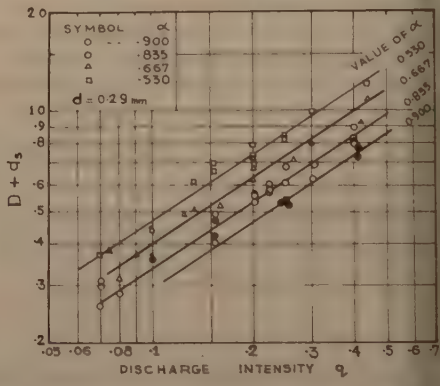


FIG. 8.–VARIATION OF $(D + d_s)$ WITH q AND α

ically influence the phenomenon of scour. In order to determine these logica dimensionless parameters and to study their role in the phenomenon of scour dimensionless analysis is carried out.

DIMENSIONAL ANALYSIS

The variables entering the problem can be grouped into the following categories:

1. Variables describing geometry of channel and of the spur-dike,
   - The width of the channel — B
   - Width of the spur-dike — b
   - Angle of spur-dike — θ
   - (Assume rectangular channel)
2. Variables describing the flow,
   - Mean velocity — V
   - Depth of flow — D
   - Maximum scour depth — d_s
3. Variables describing the fluid,
   - Mass density of water — ρ
   - Difference in specific weight between water and air — $\Delta\gamma$
   - Dynamic viscosity — μ

4. Variables describing the sediment,

Median size	d
Standard deviation	σ

is defined as

$$\sigma = \frac{1}{2}\left[\frac{d_{84}}{d_{50}} + \frac{d_{50}}{d_{16}}\right] \quad \ldots\ldots \quad (9)$$

Difference in specific weight between sediment and water (The sediment is assumed spherical)	$\Delta\gamma_s$

the maximum scour depth relative to water surface is taken as the de-
ident variable, then

$$\left(D + d_s\right) = f\left(B, b, \theta, V, D, \rho, \Delta\gamma, \mu, d, \sigma, \Delta\gamma_s\right) \ldots \quad (10)$$

wever, because w, the settling velocity of the sediment, is a function of d, μ, and $\Delta\gamma_s$, one can substitute w in place of any of the previous four vari-
es. Substituting w for μ, Eq. 10 becomes

$$\left(D + d_s\right) = f\left(B, b, \theta, V, D, \rho, \Delta\gamma, w, d, \sigma, \Delta\gamma_s\right) \ldots \quad (11)$$

ecting V, D, and ρ as the repeating variables yields

$$\left(\frac{D + d_s}{D}\right) = f\left(\frac{B}{D}, \frac{b}{D}, \theta, \frac{V}{\sqrt{\frac{\Delta\gamma}{\rho}D}}, \frac{V}{w}, \frac{d}{D}, \sigma, \sqrt{\frac{V}{\frac{\Delta\gamma_s}{\rho}D}}\right) \ldots (12)$$

us, $\left(\frac{D+d_s}{D}\right)$ seems to depend on eight dimensionless terms. With certain
nipulations, permissible within the theory of dimensional analysis, Eq. 12
be written as

$$\left(\frac{D + d_s}{D}\right) = f\left(\frac{B}{D}, \frac{B - b}{B}, \theta, \frac{V}{\sqrt{gD}}, C_D, \frac{d}{D}, \frac{\gamma_s}{\gamma}, \sigma\right), \quad \ldots \quad (13)$$

which γ_s and γ are specific weights of sediment and water respectively, g
gravitational acceleration, and C_D is the average drag coefficient of the
iment which is defined as

$$C_D = \frac{4}{3}\frac{\Delta\gamma_s d}{w^2\rho} \quad \ldots\ldots\ldots \quad (14)$$

uming further that $\frac{B}{D}$, $\frac{d}{D}$, and σ are of secondary importance, that the
ct of $\frac{\gamma_s}{\gamma}$ is included in C_D, and substituting $\frac{B-b}{B} = \alpha$, Eq. 13 reduces

$$\left(\frac{D + d_s}{D}\right) = f\left(\alpha, \theta, \frac{V}{\sqrt{gD}}, C_D\right) \quad \ldots\ldots \quad (15)$$

In the present investigation θ is held constant at 90°. Hence,

$$\left(\frac{D + d_s}{D}\right) = f\left(\alpha, \frac{V}{\sqrt{gD}}, C_D\right) \quad \ldots\ldots \quad (16$$

IMPORTANCE OF FROUDE NUMBER

Dimensional analysis has shown that the Froude number $F = \frac{V}{\sqrt{gD}}$ is significant parameter for scour around obstructions. This is justified on tw points:

First, in the case of flow in open channels with obstructions, variation i Froude number gives the extent of the influence of gravitational action on flo phenomena. Different characteristics of flow through a constriction in a open channel are the same for the same value of Froude number. Thus, th water surface configuration and the flow pattern around a spur-dike will b identical for the same values of F and α. Because it is the flow pattern aroun the spur-dike which changes the shear distribution and causes scour, both and α are important parameters governing the maximum scour depth.

Second, in the case of an unobstructed channel with an alluvial bed, the firs author[13] has shown that once the ripples grow into dunes, the height and wav length of the dunes is a function of the Froude number and dimensionless func tion of average shear stress on the bed. Recent investigations at Imperia College, London,[14] have also indicated that resistance to flow in alluvial chan nels with rippled beds depends on the Froude number. The writers are n aware of any literature on scour around spur-dikes in which this dependenc of scour depth on F is stressed.

VERIFICATION OF THE ROLE OF F AND α

As a first phase of the investigation, experiments were conducted usin sediment of median size 0.29 mm and varying the F and the opening rati Eq. 16 becomes

$$\left(\frac{D + d_s}{D}\right) = f(F, \alpha) \quad \ldots\ldots\ldots\ldots \quad (1$$

Fig. 9 shows the variation of $\left(\frac{D + d_s}{D}\right)$ with Froude number, with α as the thi variable. The correlation seems to be quite satisfactory. The slope of ea line is $\frac{2}{3}$. Therefore, Eq. 17 can be expressed as

$$\left(\frac{D + d_s}{D}\right) = C F^{2/3} \quad \ldots\ldots\ldots\ldots \quad (1$$

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13 "Characteristics of Bed Forms and Regimes of Flow in Alluvial Channels," by J. Garde and M. L. Albertson, Colorado State Univ., Fort Collins, Colo., 1959.

14 "Transport of Bed Load in Channels with Special Reference to Gradient and Form by Bharat Singh, thesis presented to the Imperial College, in London, England, in Janua 1960, in partial fulfilment of the requirements for the degree of Doctor of Philosop

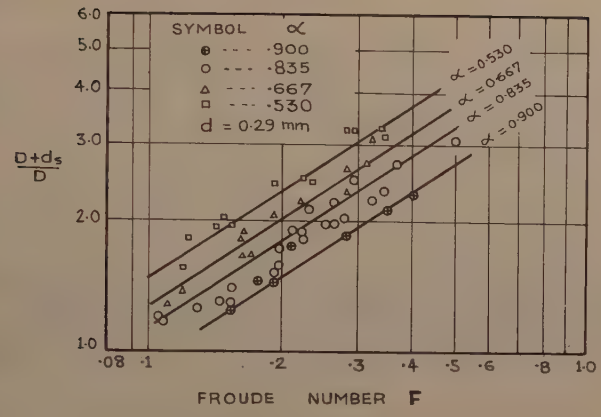

FIG. 9.–VARIATION OF $\frac{D + d_s}{D}$ WITH F AND α

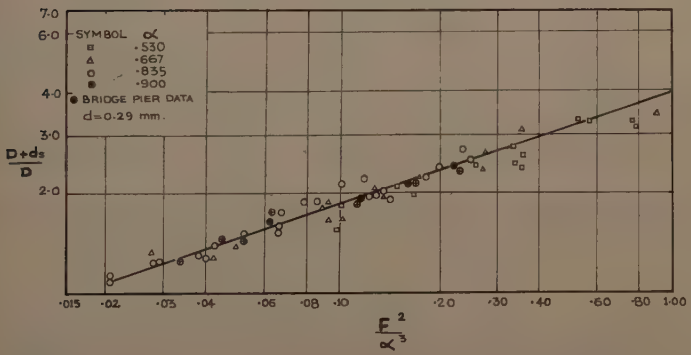

FIG. 10.–VARIATION OF $\frac{D + d_s}{D}$ WITH $\frac{F^2}{\alpha^3}$

which C is a constant equal to $f(\alpha)$. Furthermore, it is found that,

$$C = K \frac{1}{\alpha} \quad \ldots\ldots\ldots\ldots \quad (19)$$

being another constant. Hence,

$$\left(\frac{D + d_s}{D}\right) = \frac{K}{\alpha} F^{2/3} \quad \ldots\ldots\ldots \quad (20)$$

One would, therefore, expect all data to fall on a single curve if $\left(\frac{D + d_s}{D}\right)$ is plotted against $\left(\frac{F^2}{\alpha^3}\right)$. Fig. 10 shows such a plot, and it can be seen that the correlation is fairly good. This indicates the validity of the dimensional analysis and the significant parameters obtained therefrom. The variation of the three parameters can be expressed by the equation

$$\left(\frac{D + d_s}{D}\right) = 4.0 \frac{1}{\alpha} F^{2/3} \quad \ldots\ldots\ldots \quad (21)$$

On Fig. 10 are plotted results from four experiments in which the obstruction was kept in the middle of the flume to act like an idealized pier. These data also show the same tendency as the rest of the spur-dike data indicating that the preceding analysis could be adapted to the problem of scour around bridge piers also.

EFFECT OF SEDIMENT SIZE

Laursen[9] has stated that the maximum scour depth is independent of the sediment size. Izzard and Bradley[12] feel that the sediment size and gradation affects only the rate of development of scour hole and not the maximum scour depth. The formulas suggested by Lacey[4] and Blench[6] include a factor to account for the effect of the bed material size on the maximum scour depth. Inglis[5] and Mushtaq Ahmad[7] are of the opinion that the maximum scour depth is affected by change in the sediment size. According to I. Iwagaki,[15] the size of the material is important in the study of scour, and C_D is a significant parameter that takes into consideration the size and the specific weight of the sediment.

The writers believe that the size of the sediment has an influence both on the rate of scour and the maximum scour depth and that Eq. 14

$$C_D = \frac{4}{3} \frac{\Delta \gamma_s d}{w^2 \rho} \quad \ldots\ldots\ldots\ldots \quad (14)$$

is a suitable parameter whose variation indicates the role of sediment characteristics on the nature of scour.

In order to study the effect of sediment size on the maximum scour depth, experiments were conducted with a constant opening ratio $\alpha = 0.835$ while varying Froude number and sediment size. Four sand sizes in the range 0.29 mm to 2.25 mm were used, such that the average C_D varied between 0.46 and 3.22. Fig. 11 shows variation of $\left(\frac{D + d_s}{D}\right)$ with F for various sediment sizes. For a constant value of C_D, the dimensionless scour depth varies according to

$$\left(\frac{D + d_s}{D}\right) = C F^n \quad \ldots\ldots\ldots\ldots \quad (22)$$

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15 "Analytical Study of the Mechanics of Scour for Three Dimensional Jet," by Iwagaki, G. L. Smith, and M. L. Albertson, presented at the August, 1958 ASCE Hydraulics Conf. at Atlanta, Ga.

nd because $C = K \frac{1}{\alpha}$

$$\left(\frac{D + d_s}{D}\right) = K \frac{1}{\alpha} F^n \quad \ldots\ldots\ldots\ldots \quad (32)$$

igs. 11 and 12 show that the value of n, which assumed a value of $\frac{2}{3}$ for the .29 mm sediment, varies with the size of sediment or in other words with verage C_D. Within the limits of sediment size in which these experiments ere performed, it seems that the value of n reaches a maximum (Fig. 12) f 0.90 for $C_D = 0.84$; that is for the sediment size of approximately 1.00 mm. or sediment sizes smaller or larger than this, n decreases. Similar is the ariation of coefficient K with C_D. The value of K gradually increases with

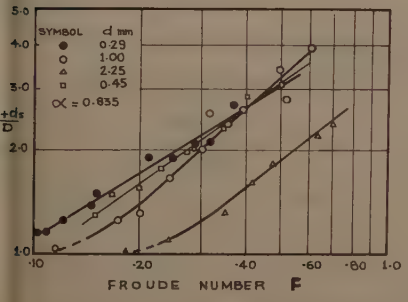

FIG. 11.–VARIATION OF $\frac{D + d_s}{D}$ WITH F AND d

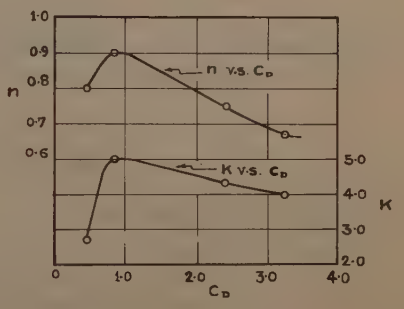

FIG. 12.–VARIATION OF n AND K WITH C_D

eduction in the value of C_D (Fig. 12) until it reaches a value of 5.00 at C_D = 84 (d = 1.00 mm) and decreases rapidly with further decreases in C_D. The alue of K varied between 5.00 and 2.75 for the conditions of the experiment.

EFFECT OF THE ANGLE OF INCLINATION OF THE SPUR-DIKE

Preliminary investigation about the effect of the inclination of the spur-ke, θ, on the maximum scour depth is being conducted. It has been found at other conditions remaining the same, (that is, for constant value of F, α d C_D) the maximum scour depth has the greatest magnitude for the spur-ke inclination of 90°. For all other inclinations upstream or downstream, e scour depth is smaller.

The previous information has been presented not for its completeness bu only to indicate the role of angle of inclination of spur-dike on the maximum scour depth.

CONCLUSIONS

Under the limitations imposed on this investigation, the following conclusions can be drawn:

1. The maximum scour depth is affected significantly by the size of the sediment.
2. The use of the Froude number for the normal channel $\left(F = \frac{V}{\sqrt{g D}}\right)$ adequately considers the effect of flow characteristics on the maximum scour depth.
3. The opening ratio α represents a significant dimensionless term characterising the spur-dike geometry, while C_D seems to represent adequately the role of sediment size on the maximum scour depth.
4. The dimensionless scour depth $\left(\frac{D + d_s}{D}\right)$ is given by the Eq. 23,

$$\left(\frac{D + d_s}{D}\right) = K \frac{1}{\alpha} F^n \quad \ldots\ldots\ldots\ldots \quad (23$$

in which K and n are functions of C_D.

ACKNOWLEDGMENTS

The preliminary draft of this paper was reviewed by R. S. Chaturvedi Head of Civil Engineering Department, University of Roorkee, and also b I. Iwagaki, University of Kyoto, Japan. The writers appreciate this assistance.

APPENDIX.—NOTATION

The following symbols, adopted for use in this paper, conform essentiall with "American Standard Letter Symbols for Hydraulics" (ASA Z10.2-1942) prepared by a committee of the American Standards Association with Societ representation and approved by the Association in 1942:

B = width of the spur-dike;

b = width of the flume;

C_d = drag coefficient of the sediment;

D = Average depth of flow;

d = median size of the sediment (also denoted by d_{50});

d_s = maximum scour depth;

d_t = scour depth at any time t;

d_{16}, d_{50}, d_{84} = sediment size finer than the given percent;

F = Froude number $\left(= \frac{V}{\sqrt{g D}}\right)$;

f_b = bed factor of Blench;

$f(\ )$ = any functional relationship;

f_s = Lacey's silt factor;

g = gravitational acceleration;

K = width of the scour hole;

k = exponent of Froude number;

Q = discharge in volume;

q = discharge in volume per unit width of flume;

q_c = discharge in volume per unit width at contracted section;

V = mean velocity of flow;

V_* = shear velocity;

V_s = settling velocity of sediment;

β = opening ration $\left(= \frac{B - b}{B}\right)$;

γ = specific weight of water;

γ_s = specific weight of sediment;

γ_s = difference of specific weight between sediment and water;

γ' = difference in specific weight between water and air;

μ = dynamic viscosity of water;

ρ = mass density of water;

σ = standard deviation; and

θ = angle of inclination of spur-dike with the direction of flow.

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QUEUING THEORY AND SIMULATION IN RESERVOIR DESIGN

By Myron B. Fiering,[1] A. M. ASCE

SYNOPSIS

The theory of queues, or waiting lines, and Monte Carlo techniques are applied to the problem of selecting the optimal design of a single multi-purpose reservoir. The proposed uses for the reservoir are irrigation, hydroelectric power generation, and flood control. Simulation of the reservoir system response to hydrologic inflows is performed on a large-scale electronic digital computer. The technique of the application is demonstrated here by describing a typical numerical solution for the design of a multi-purpose reservoir to be built across a stream for which the inflow distribution (and appropriate parameters) are known. The solution is generalized so that the effects of serial correlation among the inflow data may be evaluated. It will be assumed throughout that annual data alone are available for the study, but it will be made clear that the technique can easily be expanded to include seasonal or monthly data as well. Statistical theory, especially the theory of bivariate regression, plays a large role in this study. The extremely subtle (and largely unanswered) questions concerning the reliability of the population parameters themselves will not be examined.

INTRODUCTION

The criterion for the selection of a single proposed design from among several alternative schemes will be its optimality, and in the context of this paper,

Note.—Discussion open until April 1, 1962. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 6, November, 1961.

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"optimal" will have special meaning. A design will be considered to be optimal when the average (or expected value) of gross benefits, derived from marketing the outputs of the system, exceeds the cost of the system by an amount greater than for any alternative design. A unit increase, for example, $1.00, in net benefits, would justify the doubling, trebling, and so on, of the capital outlay. In other words, no budgetary restriction is assumed to exist. In addition, the change in storage requirements and derived benefits, induced by the existence of serial correlation among the inflows, will be investigated.

Walter B. Langbein,[2] F. ASCE offers a queuing theory solution to the problem of specifying the reservoir capacity required to maintain a minimum regulated draft, defined by the quantity b. Thus, if the inflows be normally and independently distributed, and if the operating rule for the reservoir be written so that the draft in any period, D, is a linear function of the storage, S, Langbein gives

$$S = \frac{(\overline{X} - m)^2 - (\overline{X} - b)^2}{(\overline{X} - b)} \quad \ldots\ldots\ldots\ldots \quad (1)$$

or

$$S = t\left(\frac{\sigma^2 - \sigma_d^2}{\sigma_d}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

in which: S is the storage required, $\overline{X}$ is the mean inflow, b, k are the coefficients in the linear expression relating storage and draft, namely, $D = b + kS$, σ^2 denotes the variance of the inflow probability distribution, σ_d^2 refers to the variance of the draft probability distribution, and t is the standardized normal deviate, with m, the minimum probable natural inflow, given by $m = \overline{X} - t\sigma$. The effect of serial correlation among the inflow data is investigated in Langbein's paper, and by introducing a correlation correction factor, C, he derives the generalized solution for the storage requirement:

$$S = \frac{C^2(\overline{X} - m)^2 - (\overline{X} - b)^2}{\overline{X} - b} \quad \ldots\ldots\ldots\ldots \quad (3)$$

or

$$S = t\left(\frac{C^2\sigma^2 - \sigma_d^2}{\sigma_d}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

in which: C, the correction factor, is given $\sqrt{1 + \frac{2r}{1 + k - a}}$ and is never less than unity, r denotes the serial correlation coefficient between successive inflows and a is defined by virtue of the assumption that the correlation coefficient between inflow data spaced n-items apart, or of lag n, is $a^n r$. Clearly, the effect of positive serial correlation is to increase the storage requirement.

Langbein provides additional solutions, dealing with non-normal inflow distributions and non-linear operating rules, but the details are not relevant here

2 "Queuing Theory and Water Storage," by Walter B. Langbein, Proc. Paper No. 1811 ASCE, Vol. 84, No. HY 5, October, 1958.

Two points might be noted, however, with regard to Langbein's technique. Firstly, economic criteria, though they may be implicit in the selection of the deviate t (which specifies a probability level), are not explicitly included. This is perfectly plausible because Langbein's solution does not purport to delineate the capacity of turbines, irrigation canals, and the like, but is, rather, an application of queuing theory to the specification of reservoir capacity only. Secondly, the importance of the draft probability distribution should be emphasized. Eqs. 2 and 4 contain the variance of this distribution, and, as the newly proposed method is developed in the following sections, the vital role played by this function will be manifest.

J. Gani and P. A. P. Moran[3] have given the outline of a solution which utilizes queuing theory and Monte Carlo analysis, but the distribution of inflows, restricted to the negative binomial and Poisson types, are a bit unrealistic. Furthermore, economic considerations are not included in their analysis. Pertinent comments on the length of Monte Carlo synthesis are given, and these will be considered directly.

Harold A. Thomas, Jr., F. ASCE, gives[4] an elegant queuing theory solution for selecting the optimal scale of development of and optimal operating policy for a single multi-purpose reservoir. In his solution, it is assumed that the sequence of inflows entering the reservoir has a particularly simple probability distribution, and, moreover, that it demonstrates no serial correlation from interval to interval. Although his assumptions tend to simplify the computational and analytical difficulties that are encountered in the queuing theory technique, a vast amount of numerical work must, nonetheless, be performed to compute the draft distribution and economic schedules, and in fact, without the use of a high quality desk calculator, this application of queuing theory would be inordinately difficult to use.

In this paper the solution devised by Thomas for a particular inflow distribution is generalized to include many types of inflow probability distributions, with any magnitude of serial correlation (0 to $\pm$ 1). This entire general solution is coded for the IBM 709 data processing system so that the numerical tedium is eliminated.

SIMULATION STUDY

The use of queuing theory in the reservoir design process requires a nonambiguous, quantitative statement regarding each of several parameters or functions:

1. The inflow probability distribution;
2. The magnitude of the serial correlation that obtains between inflows for successive years;
3. The reservoir capacity (or, more properly, the active storage available for regulation);

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3 "The Solution of Dam Equations by Monte Carlo Methods," by J. Gani and P. A. P. Moran, Australian Journal of Applied Science, Vol. 6, No. 3, September, 1955, p. 267.

4 "Queuing Theory of Streamflow Regulation Applied to the Cost-Benefit Analysis of Multipurpose Reservoir," by Harold A. Thomas, Jr., Private Communication to the Harvard Water Resources Program, Harvard Univ., Cambridge, Mass., November 20, 1959.

4. The operating policy for determining the outflow (that is, the release) in each period;
5. The functions relating cost to the various reservoir sizes and operating policies;
6. The benefit functions, specifying the market value of that fraction of the release allocated to each of the several available uses; and
7. The payoff function, taken here to be the average or expected value of net benefit, which is to be maximized subject to the several constraints imposed by the analysis and by good sense.

It should be noted that all numerical values assigned to parameters in the following sections are representative only of the model which is assumed here for illustrative purposes. In actual solutions, different probability distributions and parameters would be used. The units of storage will be taken to be millions of acre feet (maf), those of draft shall be millions of acre feet per year (maf per yr), and those of cost and benefit shall be millions of dollars.

The key point in the solution is the generation of the draft probability distribution, and the major portion of this paper deals with its evaluation. Its use in the economic analysis is reserved until the final portions of the solution. A moment's consideration will show that the draft distribution function depends, in turn, on items 1 through 4 previously noted, and these are considered in sequence. There is no question but that the draft distribution for a real system depends on many other factors, such as season, population trends, type of agricultural and industrial development, and accumulated water shortages (or surpluses), to name a few. However, in this simplified model, which does not distinguish between seasonal draft patterns, the annual draft is a function only of the water available in each year.

Inflow Probability Distribution.—It is assumed that the inflows to the model reservoir system conform to a truncated normal distribution with known mean and standard deviation. (Those readers not familiar with elementary statistical theory might consult any one of many fine statistics textbooks.[5]) The inflows enter the reservoir at a uniform rate throughout the year. Fig. 1 shows a graph of this distribution, with the mean flow equal to 2 units (million-acre-feet per year), the standard deviation equal to 1.414, and the upper and lower limits set at 4.0 and 0.0, respectively. The reason for truncation of the distribution may be clarified by noting that the normal distribution extends asymptotically to plus-and-minus infinity, but there is no significance to negative inflows. The writer has shown,[6] for the Clearwater River basin in Idaho, that truncation does not seriously affect the reliability of conclusions drawn from data so distributed. It is presumed that a similar demonstration could be made for this model and, therefore, the truncation is introduced. Furthermore, as will subsequently be explained more fully, the nature of the benefit functions chosen in this example cause the benefits to be relatively insensitive to the exact magnitude (but not frequency) of high flows. Thus truncation at the upper portion of the inflow distribution introduces no serious errors into the analysis. However, it must be noted that other design problems, involving different

5 "Introduction to Mathematical Statistics," by Paul Hoel, John Wiley and Sons, New York, 1954.

6 "Statistical Analysis of Streamflow Data," by Myron B. Fiering, thesis presented to Harvard Univ., in Cambridge, Mass., in June, 1960, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

recipitation. Conversely, if the level of ground water is elevated during a ear, normal or deficient precipitation will tend to run off into the streams in n unusually high proportion. Thus, years of low flow will tend to be clustered ogether, as will years of high flow. The mathematical statement of this con- ition is that there exists a positive serial correlation coefficient of lag one be- ween the annual flows of the river basin. This coefficient, generally denoted y "r," has an absolute value in the range $0 \leq r \leq 1$, in which zero represents o correlation (or no relationship) and unity represents perfect correlation (or functional relationship) between successive inflows.

Statistical theory permits the evaluation of parameters in a generalized lin- ar equation such that the inflow I_i, is written as a function of $I_{i-1}, I_{i-2}, \ldots,$ $_{-n}$. The set of coefficients in this equation is derived from the theory of auto- egression, and presents no theoretical difficulty. Again, for simplicity, an utoregressive scheme of lag one is assumed, in which I_i depends only on I_{i-1}. n the Thomas and Moran models, serial correlation is assumed to be zero; hus, the inflows in those models are randomly (independently) distributed.

Reservoir Size.—The active storage capacity of the reservoir is denoted by m, and the solution for the model considers five possible values of S_m (0, 1, 3, and 4 million-acre-feet) in which the smallest, $S_m = 0$, denotes no con- truction and the largest, $S_m = 4$, denotes the largest feasible structure for the ite of the model river basin. The notation for active storage in the reservoir t the end of the i^{th} interval is S_i.

Operating Rule.—It is assumed that drafts are released at a uniform rate roughout the year and, as will be explained, that the magnitude of the draft or any single year is a single-valued function of the sum of the initial storage olume plus the current inflow rate. The draft in the i^{th} period is denoted by i.

The continuity principle as applied to annual flows may be written in terms f the parameters of the model (or queue).

$$\left.\begin{aligned} D_i &= I_i + S_{i-1} - S_i \\ 0 &\leq S_i \leq S_m \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (5)$$

In the terminology of queuing theory, I_i represents the arrival rate, S_i rep- esents the length of the queue, and D_i is the service function. This queuing elation is indicated by Fig. 3, in which the reservoir is depicted as a container ith a petcock at the lower end. Further consideration of Eq. 5 places bounds n the draft, D_i, as follows:

1. D_i must be equal to or less than $I_i + S_{i-1}$, because this sum, defined as i, represents the total amount of water available for distribution, and
2. D_i must be equal to or greater than zero or $I_i + S_{i-1} - S_m$, whichever greater, because the storage cannot exceed S_m.

he shaded portion of Fig. 4, lying between the lines identified as $S_i = 0$ and S_i S_m, delineates the zone of permissible drafts specified in 1 and 2.

It remains to select an operating curve, necessarily contained within the aded portion of Fig. 4, such that D_i may readily be computed as a function of i. The operating rule is not derived analytically from the foregoing material, t is, rather, a simplified, empirical specification of the computational scheme be followed so as to define the release in any period. It is not a proposal for

benefit functions and inflow data, might not warrant truncation at an upper bound. If this be the case, it is an easy matter to alter the equations that are subsequently developed so as to allow the inflows to range from zero to infinity.

The selection of the normal distribution as being representative of the annual inflows to the model reservoir is well justified by the result of hydrologic studies and experience in many river basins. It might be argued that some transform of the data, say a logarithmic transform, more nearly reflects the distribution of inflows, and indeed the United States Geological Survey, Dept. of Interior (USGS), often makes this type of transform in hydrologic computations. Such a procedure would have, in this study, the distinct advantage of eliminating the possibility of negative inflows, because no matter how small the logarithm might be, the antilogarithm, or flow value, is invariably positive. This transform, if justified by the data, is easily built into the equations. For simplicity in this example, however, we deal here with no data transform.

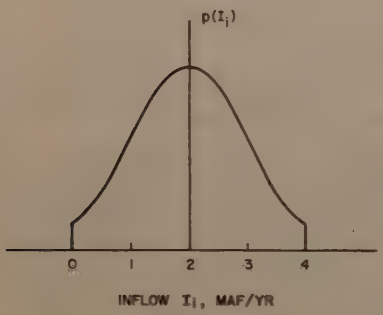

FIG. 1.—TRUNCATED NORMAL INFLOW DISTRIBUTION, MEAN FLOW = 2 maf per yr STANDARD DEVIATION = 1.4 maf per yr

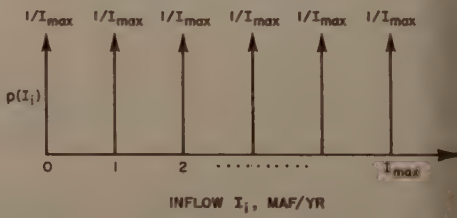

FIG. 2.—RECTANGULAR INFLOW DISTRIBUTION

The inflow probability distribution assumed in Thomas' solution is rectangular and is based on the distribution of discrete points in the range from zero to the maximum inflow. If I_i represents the inflow in the i^{th} time interval, and if I_m is the maximum inflow which obtains in the model, then the probability of any inflow is $1/I_m$, as shown in Fig. 2.

Serial Correlation.—The magnitude of the serial correlation coefficient of lag one is a measure of the closeness with which the annual inflow in the $(i+1)^{st}$ period can be estimated from the magnitude of flow in the i^{th} period. The underlying hydrologic cause for the existence of correlation, or persistence between successive annual inflows, regardless of the presence of significant correlation between annual precipitation data, can be found by considering the soil strata and vegetative cover in the drainage basin to act as a large reservoir. Thus, if the level of ground water is drawn down due to drought conditions in a particular year, normal or excessive precipitation in the following year will tend first to replenish the ground water supply and will not reach the well-defined stream courses in amounts commensurate with the antecedent

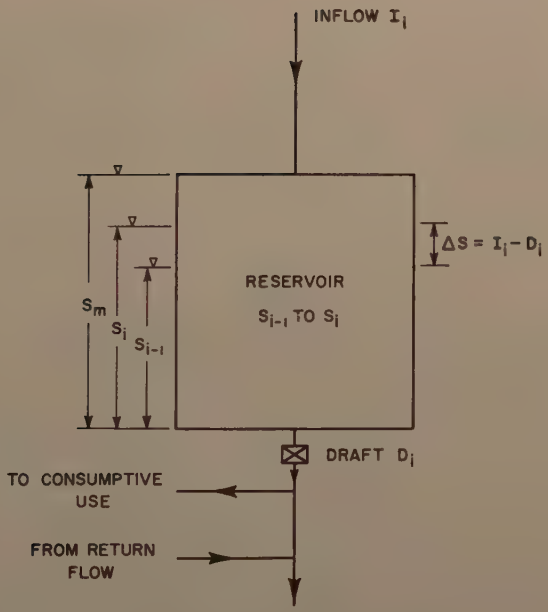

FIG. 3.—RESERVOIR SCHEME

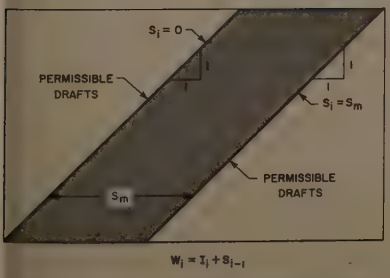

FIG. 4.—ZONE OF PERMISSIBLE DRAFTS

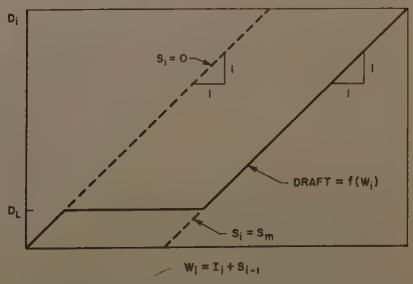

FIG. 5.—RESERVOIR OPERATING RULE

any real reservoir operation. The rule is characterized by a parameter D_L, the normal draft level, as indicated in Fig. 5. For $W_i < D_L$, the reservoir is emptied. For $W_i \geq D_L + S_m$, the reservoir is kept full. For intermediate values of W_i, the draft is set at D_L. A little algebra summarizes the three cases:

1. For $S_{i-1} + I_i < D_L$,

$$D_i = S_{i-1} + I_i \text{ . . . . . (leave reservoir empty) . . . . . (6a)}$$

2. For $D_L < S_{i-1} + I_i < D_L + S_m$,

$$D_i = D_L \text{ . . . . . (constant draft) . . . . . (6b)}$$

3. For $I_i + S_{i-1} \geq D_L + S_m$,

$$D_i = S_{i-1} + I_i - S_m \text{ . . . . (leave reservoir full) . . . . . (6c)}$$

The limiting reservoir operating rules, $D_L = 0$ and $D_L = I_m$ are not considered, because in each of these cases the outflow distribution is identical to that of the inflow and the problem reduces to $S_m = 0$ (no reservoir). This may be proven by noting that in the first case, $D_L = 0$ the reservoir once filled remains permanently filled, and in the second case, $D_L = I_m$ the reservoir once empty remains permanently empty. For the model river basin, it is necessary to consider Eqs. 6a, 6b, and 6c for $S_m = 0, 1, 2, 3, 4$, and $D_L = 1, 2, 3$ in such a fashion that every combination of S_m and D_L is investigated.

Draft Probability Distribution.—To this juncture the solution proposed here is identical to the Thomas solution in all respects save one, the single departure being the characteristics of the inflow distribution. For the computation of the draft probability, the methods diverge completely, only to rejoin at the conclusion for the economic analysis. The Thomas and Langbein solutions, based on a uniform, bounded inflow probability distribution, generates the draft probability distribution (corresponding to a particular selection of S_m and D_L) by way of computing the matrix of transfer probabilities of reservoir storage. Fig. 6 shows a typical matrix in which the elements represent the probability that the reservoir, containing a specified volume S_{i-1} at the start of an interval, will contain another particular volume S_i at the end of that interval. Note that the row sums of the matrix elements are unity, because the reservoir must contain at least one of the several permissible volumes at the end of the interval in question ($S_i = 0$ is a permissible condition). Using this matrix, and the known probability distribution of the inflows, the draft probability distribution is derived.

When the same techniques are applied to evaluating the matrix of transfer probabilities for the model assumed in this paper, serious analytical and computational difficulties arise due to the effects of serial correlation among the inflows. To overcome this, and render the solution suitable for a wide range of actual problems, a simulation technique is devised for use on a large capacity high-speed electronic digital computer. Although the coding is written for the IBM Type 709 computer, it should be possible to use the flowchart, or logical computer solution, contained in Fig. 9 to write a machine code suitable for whatever equipment might be available.

Simulation Technique (General).—The logical basis of the computer program for this solution is trivial. There is no question but that solutions of this

pe are not very elegant . . . in fact, it would not be unwarranted to classify em as being "brute force" solutions. However, once programmed, they are tremely simple to use and interpret, and if this factor be a criterion of their gineering value, many more types of problems will be attacked in this fashion computers become more widely available and familiar in civil engineering actice.

The essence of the technique is the routing of a long hydrologic flow record rough a proposed reservoir. Drafts are made in accordance with the pre-ribed operating rule, and a tally sheet is maintained so that entries may be ade to accord with the frequency of the occurrence of particular magnitudes the draft. For example, if the draft is rounded to the nearest million-acre-et, a mark is made in the column appropriate to that draft magnitude. When

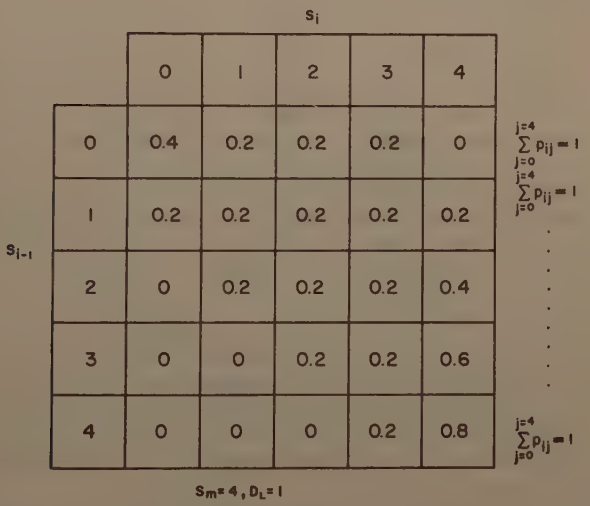

FIG. 6.—A TRANSFER PROBABILITY MATRIX FOR RECTANGULARLY DISTRIBUTED RANDOM INFLOWS, WITH $D_L = 1$ AND $S_m = 4$

set of inflows is exhausted, the frequency of each group is computed by mming the tally marks, and the probability of each group is computed by di-ing its frequency by the total number of years of record. It is clear that the ger the gaging record the more reliable will be the draft probabilities com-ed in this fashion, because a long record implies an opportunity for the re-tion of random sampling errors. Because long and complete stream gaging ords are infrequently encountered, some method is needed whereby a syn-tic record can be obtained. The technique of generating an appropriate syn-tic-hydrologic record will be examined.

Synthetic Hydrologic Records.—Engineers, planners, and hydrologists have g been interested in the use of synthetic hydrologic records as an aid in res-oir design studies. At the outset, it would be well to define clearly what is ant by synthetic hydrology in the hope that an accurate definition will make

the entire subject a little more palatable for those who have a suspicious distrust of statistical methods applied to hydraulic engineering practice. Synthetic hydrology is a series of simulated streamflows that did not actually occur but that, based on certain statistical considerations, could have been the real record. If the statistical model used to represent the flow regime is suitable in all respects, it would be impossible to distinguish between real and synthetic records by means of the usual statistical tests of significance.

A simple explanation and justification of synthetic hydrology may be derived using the example of a pack of playing cards as the source of a 52-yr record of streamflow data. Numerical values are assigned to face cards--the jack is weighted eleven, the queen twelve, and the king thirteen. After shuffling, all the cards are drawn in succession, thereby simulating a 52-yr streamflow record. Assume a design problem of determining the size of a storage structure which is to provide a constant annual draft equal to the mean inflow, 7.0 units.

A set of 52 values drawn without replacement from a deck can be considered as a set of inflows to a reservoir. Because the draft is constant for all periods (that is, equal to the mean inflow), the increment of required storage in each period is the deviation of the period's inflow from the mean, and the cumulative sum of deviations is the storage in each period. This is the traditional mass curve analysis, in which the range of the cumulative deviation defines the required reservoir size. Each time the cards are shuffled and played out again the required storage is apt to be different. As a test, 20 such games were played, resulting in a mean storage requirement of 31.15 with extreme values of 22 and 45. These results indicate the wide range of reservoir design values inherent in this simple model. Because each of the 20 records has an equal chance of occurring, it is evident that the trace of any single sequence in this example does not provide a definitive basis for design. The first step in obtaining a more reliable design requires the consideration of the probability distribution of future inflows based on a statistical analysis of the existing sequence. Because a single trace does not indicate the full potentialities inherent in the entire population of traces, the distribution of future inflows cannot be deduced from the single trace alone.

It might be thought that a suitable way to vary the sequence and thereby provide a different storage requirement is merely to cut the cards without reshuffling and start the game again. A little consideration shows that for any cyclical hydrograph of flow versus time, with draft equal to the mean inflow, the storage requirement remains unchanged by such a cut. Another way of expressing this is to state that for any cyclic function of fixed period, the definite integral over one period is independent of the location of the limits of integration.

There are two fundamental difficulties with the card game insofar as its applicability to real data is concerned. First, note that when a card is drawn the probability of drawing another card of that rank on any successive try is reduced. Ideally, each card should be replaced following the draw and the deck thoroughly shuffled before drawing another. Second, the deck represents a rectangular distribution of inflow magnitudes, and, because this type of distribution rarely occurs in nature, it is essential to change the components of the deck so as to conform to real situations.

The concept of a super-deck is introduced to treat these difficulties. Among its important properties are (1) the fixed probability of drawing any rank on any draw, regardless of what has been drawn before, and (2) the realistic dis-

ibution of inflows. By playing successive games with super-decks, data are tained so as to provide a more rational basis for fixing the storage require-ent. It is not important that the super-deck be a physical deck of cards, be-use a readily available substitute possesses all the pertinent properties. A ble of random sampling numbers serves as an acceptable substitute because e entries are independently distributed and may be made to conform to any sired distribution. For example, many statistics textbooks include such bles for both the normal and rectangular distributions.

Synthetic hydrology helps to overcome the paucity of possible patterns of gh and low flows by providing a large number of new sets of data that could tain on the stream in question. The model utilized here specifies that the nthetic streamflow in any period consists of two additive components:

1. A trend-line component consisting of a linear function of the flow during e previous time period, and
2. a random component, the magnitude of which varies from period to pe-od.

Statistical Considerations.—The method of least squares will be briefly ex-ined so that the equations for generating synthetic hydrology for the model ll be meaningful. Let I_i represent the numeric value corresponding to the easured inflow during the i^{th} period, and let there be n such measured values. is desired to estimate I_{i+1} using a linear regression model of the form

$$\hat{I}_{i+1} = \bar{I}'' + b\left(I_i - \bar{I}'\right) \quad \ldots\ldots\ldots\ldots \quad (7)$$

which $\hat{I}_{i+1}$ is the best estimate of the trend-line component of I_{i+1}, $\bar{I}'$ de-tes the mean flow over the period $1 \le i \le n-1$, $\sum_{i=1}^{n-1} I_i \Big/ (n-1)$, $\bar{I}''$ is the mean w over the period $2 \le i \le n$, $\sum_{i=2}^{n} I_i \Big/ (n-1)$, and b, the regression coefficient,

$$b = \frac{\sum_{1}^{n-1} I_i I_{i+1} - n\bar{I}'\bar{I}''}{\sum_{1}^{n-1} I_i^2 - n(\bar{I}')^2} \quad \ldots\ldots\ldots\ldots \quad (8)$$

The closeness of fit between the variables I_i and I_{i+1} is measured by the fficient of correlation,

$$r = \frac{\sum_{i=1}^{n-1} I_i I_{i+1} - n\bar{I}'\bar{I}''}{\left[\sum_{1}^{n-1} I_i^2 - n(\bar{I}')^2\right]^{1/2}\left[\sum_{1}^{n-1} I_{i+1}^2 - n(\bar{I}'')^2\right]^{1/2}} \quad \ldots\ldots \quad (9)$$

which is identical to the serial correlation coefficient of lag one previously described. Define the sample variances

$$\left.\begin{aligned}(s')^2 &= \sum_{i=1}^{n-1} \frac{(I_i - \bar{I}')^2}{n-1} \\ (s'')^2 &= \sum_{2}^{n} \frac{(I_i - \bar{I}'')^2}{n-1}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (10)$$

and note that

$$b = \frac{r\, s''}{s'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

If the length of record is moderate, then $\bar{I}'$ and $\bar{I}''$ are approximately equal and may be written as $\bar{I}$, the mean inflow. Similarly, s' and s'' may be written as s. Thus, Eq. 7 may be rewritten

$$\hat{I}_{i+1} = \bar{I}\,(1 - r) + rI_i \quad \ldots\ldots\ldots\ldots (12)$$

To I_{i+1} it is necessary to add a random component which is a function of the variance s^2. The standard error of estimate, defined by

$$\text{standard error} = s\left(1 - r^2\right)^{1/2} \quad \ldots\ldots\ldots\ldots (13)$$

is a useful function because it may be shown to be a measure of the unexplained portion of the total variance, s^2.

Thus, for r = 1, it is clear that $s(1 - r^2)^{1/2} = 0$, and all the variance, $(s'')^2$ is accounted for by the regression of I_{i+1} on I_i. On the other hand, for r = 0 $s(1 - r^2)^{1/2} = s$ and none of the variance is accounted for by virtue of the linear relation between I_{i+1} and I_i. For values of r intermediate between 0 and 1, the observed values of I_{i+1} are distributed about the estimated value, $\hat{I}_{i+1}$ with the closeness of grouping related to r^2.

Another way to visualize the standard error is to consider a band of width $s(1 - r^2)^{1/2}$ on each side of the trend line defined by Eq. 12, within which observations will fall with a specified frequency, and that this width is a function of the random influences or inaccuracies in the estimating procedure. Many of these notions are further clarified by Fig. 7, which shows the results of regression analysis applied to two variables, X and Y.

For a given I_i, the values of I_{i+1} will be normally distributed about $\hat{I}_{i+1}$ due to the assumption of normality among the inflows. Eq. 12 may now be rewritten to include an additional term;

$$\hat{I}_{i+1} = \bar{I}\,(1 - r) + rI_i + ts\left(1 - r^2\right)^{1/2} \quad \ldots\ldots\ldots (14)$$

in which t is a random normal and independently distributed variate with mean equal to zero and variance equal to unity. This has the effect of adding to the previous estimate of I_{i+1} a positive or negative component whose magnitude exceeds the band width of one standard error 32% of the time and the band width

f two standard errors 5% of the time. The nature of the distribution of the t-alues is such that some parameters, such as the mean, computed from a sam-le of many estimates of the type in Eq. 14 will not differ significantly from ıose of the type in Eq. 12.

Eq. 14 satisfies the two requirements previously set forth, namely, that the stimated value of the streamflow shall contain a trend-line component, iden-fied by $\bar{I}(1-r) + rI_i$, and a random component, identified by $s(1-r^2)^{1/2}$. If ıe parameters s, r, and $\bar{I}$ are measured from the record, if suitable starting alues are assigned to the storage S_i and the inflow I_i, and if a sufficiently long able of normal random sample numbers is available, it is a simple, albeit te-ious, task to generate a long sequence of inflow values $I_1, I_2, I_3 \ldots I_N$ by re-eated use of Eq. 14.

Gani and Moran have developed criteria for estimating the length of syn-ıetic records required to reduce random errors to tolerable limits. Accept-

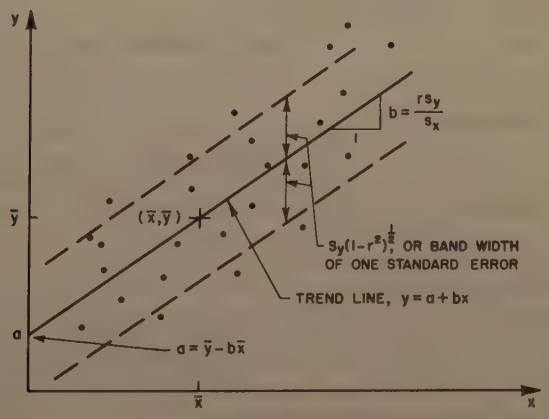

FIG. 7.—GENERAL LINEAR REGRESSION MODEL
$y = a + bx$

ble results were obtained, in a particular study, with records of 1000 yr. As test, a few trial runs of this model study were extended to 1000 yr but very ttle difference (that is, in the first decimal place), was observed between nefits for 500-yr and 1000-yr sequences. To conserve computer time, par-cularly because many trials were required, 500 yr was chosen as the mini-um acceptable length. The upper limit on the magnitude of N is that of hu-an endurance and, thus, it becomes clear that a digital computer might prof-ibly be used to perform these several tasks:

1. Generate or store a set of random sampling numbers,
2. Store preassigned values of N, $\bar{I}$, s, r, I_1, and s_1,
3. Produce, with the parameters of task 2, synthetic hydrology by repeated clic evaluation of Eq. 14,
4. Evaluate the draft D_i in accordance with the given operating rule and lues of S_m and D_L,

5. Perform the necessary rounding and bookkeeping on D_i so as to compute the draft probabilities, for an integral (quantized) range of draft, and

6. Provide for the capacity to change any parameters mentioned in tasks 2 and 4, either individually or jointly, so as to rerun the analysis from step 3.

The generating scheme contained in tasks 1 through 4 is known as a random walk, and provides one of the fundamental tools for the mathematical and statistical discipline known as queuing theory.

Summary of the Technique.—A data processing system has been programmed by the writer to perform the several tasks previously itemized. Details of the program and the format of the output statements will subsequently be examined, but it will suffice to say that if computing facilities are available there is no step among the six listed that should cause insurmountable theoretical or practical difficulties. Using the following parameters

N, length of synthetic record,	500 (years)
r, serial correlation coefficient,	0.2
$\bar{I}$, mean inflow,	2.0 (million-acre-feet per year)
s^2, variance of the inflow,	2.0 (maf per yr)^2
S_1, initial reservoir state,	2.0 (maf)
I_1, initial inflow value,	2.0 (maf per yr)
I_m, maximum inflow,	4.0 (maf per yr)
S_m, maximum reservoir size,	4.0 (maf)
D_L, normal draft level,	1.0 (maf per yr)

the draft probabilities are computed in accordance with the outlined scheme and an example of the computation is given in Table 1. It should be noted that

TABLE 1.—DRAFT PROBABILITIES, $S_m = 4$, $D_L = 1$, $r = 0.2$

Draft (Rounded)	0	1	2	3	4
Frequency	0	239	115	87	59
Probability	0.000	0.478	0.230	0.174	0.118

the complete study for each value of r requires 14 additional tables similar to Table 1, because there are 5 possible values of S_m (0, 1, 2, 3, 4) and 3 possible values of D_L (1, 2, 3). Each combination yields a table of draft probabilities and it is the task of the economic analysis to select that combination which is optimal. Appendix I, Tables 4 through 9, contains the complete set of 15 draft probability tables for each value of r in this model basin. The values of draft D_i, are rounded to the nearest integer so that a meaningful discrete distribution can be constructed. It might be argued that a real design problem involves only one value of the serial correlation coefficient, namely that which is computed from the available record, and that the references to a range of values of r is extraneous. This reasoning is irrefutable, but it will be recalled that

is study is designed to investigate, among other phenomena, the effect of se-al correlation on the optimal design. Therefore, a range of values of r will considered, each being used with precisely the same inflow mean and vari-ce; in actual practice only a handful of values of r would be tried so as to acket the range within which it is likely that the population value of the cor-lation coefficient (not necessarily equal to the estimate computed from the mple), will lie.

The major contribution offered by this study is the rapid and effortless eans for computing a large number of tables of the type of Table 1. By any her method, evaluation of these draft probabilities would be intractable or, best, so tedious that useful and reproducible results would likely be very re. The ease with which a range of parameter values can be investigated ables the designer to develop a degree of insight into the sensitivity of the stem response to changes in the parameter values. All this would be im-ssible with hand computation methods.

Benefit Functions.—It will be recalled that the model river basin provides ree benefits; irrigation, hydroelectric power, and flood control. The com-ents relating to the utility functions for the first two are identical, and they ll be grouped together for examination purposes.

It is assumed that a water-use facility will be built to a scale measured by parameter D_S. With irrigation, for example, D_S might be proportional to the tal number of acres of land prepared for crops together with the cost of suit-le structures for conveying and distributing the reservoir output. For hy-oelectric power, D_S might denote generator capacity. Usually it is convenient state D_S in terms of the same units used for draft.

If the scale of development D_S is known, it is assumed that a particular lue of the gross benefit (utility) can be computed as a single-valued function the annual draft. In this model, the annual benefit does not depend on ante-dent draft nor on draft forecasts. Figs. 8 and 9 are graphs of utility func-ns whose general shape is characteristic of the irrigation and power utility ıctions assumed in this study. It is seen that the utility function is a four-rameter function, these four parameters being shown in Figs. 8 and 9 for the se of irrigation and hydropower, respectively. With reference to Figs. 8 and it should be noted that three of the four parameters are slopes and the four-D_S, is an index of location. Thus, D_S and AG1 (or PW1) jointly determine location of the discontinuous change of slope which characterizes the benefit ıctions, and AG2 and AG3 (or PW2 and PW3) define the slopes below and ove the scale of development, D_S. If U_1 is the utility due to irrigation, and that due to hydroelectric power, it is possible to write

$$U_1 = -(AG2 - AG1) \cdot (D_S) + AG2 \cdot (D), \text{ for } D \leq D_S \quad \ldots . (15a)$$

$$U_1 = -(AG3 - AG1) \cdot (D_S) + AG3 \cdot (D), \text{ for } D > D_S \quad \ldots . (15b)$$

$$U_2 = -(PW2 - PW1) \cdot (D_S) + PW2 \cdot (D), \text{ for } D \leq D_S \quad \ldots . (15c)$$

$$U_2 = -(PW3 - PW1) \cdot (D_S) + PW3 \cdot (D), \text{ for } D > D_S \quad \ldots . (15d)$$

From the graph it may be noted that D_S is merely a parameter of the utility ction, in exactly the same fashion that the several slopes AG and PW are

parameters of this function. D_S should not be confused with D_L, the norma draft level, which is a parameter of the operating rule. It may turn out tha the optimal solution prescribes $D_S = D_L$, but this will not generally be the case In this simple model, D_S is assumed to be the same for irrigation and hydro power generation. In more elaborate models, it would be appropriate to defin D_{S1} and D_{S2}, where the indices 1 and 2 refer to the different uses. In this ex ample

AG1 = 1.0

AG2 = 2.0

AG3 = 0.2

PW1 = 2.0

PW2 = 3.0

PW3 = 0.3

A vast amount of study could be used, and the talents of many specialist enlisted, to define adequately the shape of the curves that are used to represen

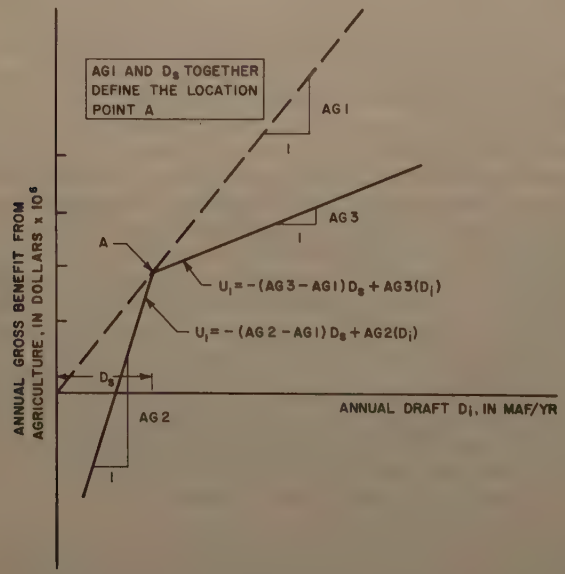

FIG. 8.—TYPICAL BENEFIT FUNCTION FOR AGRICULTURAL USES

benefit functions. It is not the purpose of this paper to study the exact natur of these curves, nor to investigate the social, legal, and political aspects tha must enter into a more thorough analysis. It can be agreed, however, that th general shape of many benefit functions tends to be convex, indicating the in fluence of the oft-quoted law of diminishing returns. Moreover, the solutio

the computer is so rapid and easily altered that several benefit functions, epresenting a range of parameters, can conveniently be investigated. This rocedure is recommended, because one can often gain insight into the funda- ental nature of a problem by examining a range of solutions. Admittedly, is is an imperfect substitute for a complete understanding of the nature of e problem and its solution, but it is helpful in many applications of engineer- g design studies.

In this simple model, flood control benefits are measured by a linear equa- on that defines the reduction in flood probability attributable to the structure. or example, let a flow $D_i = 3.5$ be sufficiently large to cause flood damage in e model system, and let the parameter FL2 be the probability that $D \geq 3.5$ rior to the construction of the reservoir. The probability is that $D \geq 3.5$ after nstruction can be read from the draft probability distribution, so that $[FL2 - p(D \geq 3.5)]$ is the reduction of flood probability attributable to the reservoir.

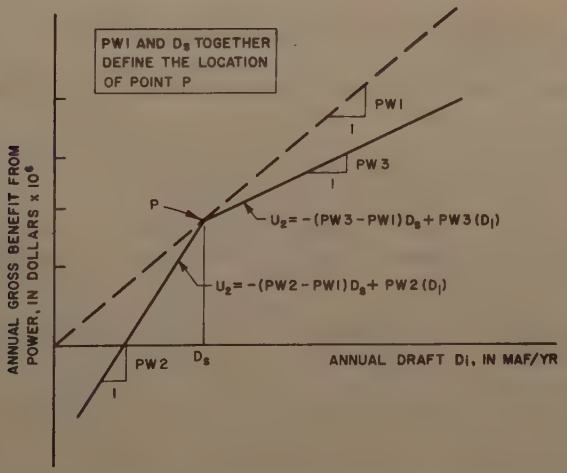

FIG. 9.—TYPICAL BENEFIT FUNCTION FOR HYDRO-ELECTRIC POWER

(x) is a generally accepted notation that indicates the probability of an event) Because it is postulated that benefits are proportional to this reduction,

$$U_3 = FL1\,[FL2 - p(D \geq 3.5)] \quad \ldots\ldots\ldots\ldots \quad (16)$$

which FL1 is a constant of proportionality. It is manifest that this function a gross oversimplification, but most hydrologists and engineers will agree t some function can be designed so as to approximate flood benefits, at least thin a certain range of flood flows. It might be necessary to specify several uations, each applicable within a particular range of draft, in order to clearly stinguish the pertinent factors. But this is a matter for further study, and ll not be treated here.

Expected Annual Gross Benefits.—It has previously been demonstrated, for a known value of r, that the proposed model gives rise to 15 draft probability distributions, one of which has been tabulated in the text as an example. For each such table, the parameter D_S can assume any value in its range $0 \leq D_S \leq I_m$. Thus, if we select some particular value of D_S, the expected value of the annual gross benefits from irrigation and hydropower may be computed. Expected value is a statistical term that denotes theoretical average or mean value. The computation proceeds as shown in Table 2, with $S_m = 4$, $D_L = 1$, $D_S = 1$, and $r = 0.2$.

It is clear that $p(U_i) = p(D_i)$ because, for D_L fixed, Eqs. 15a, 15b, 15c and 15d define U_i as a single-valued function of D_i only. Thus, the probability distribution of U_i can be derived directly from that of D_i by means of a point-by-point transform between the two variables. The bottom two rows of Table 2, which sum to give average annual gross benefits of \$1.187 and \$2.279 x 10^6 for irrigation and power, respectively, are representative of a standard method of

TABLE 2.—GROSS BENEFITS FROM IRRIGATION AND POWER, $S_m = 4$, $D_L = 1$, $D_s = 1$, $r = 0.2$

D_i	0	1	2	3	4	
$p(D_i)$	0.000	0.478	0.230	0.174	0.118	from Table 1
$p(U_i)$	0.000	0.478	0.230	0.174	0.118	
U_{i_1} (irr)	-1.000	1.000	1.200	1.400	1.600	from Eqs. (15)
U_{i_2} (pow)	-1.000	2.000	2.300	2.600	2.900	from Eqs. (15)
$p(U_i) \times U_{i_1}$ (irr)	0.000	0.478	0.276	0.244	0.189	$\sum_{i=0}^{I_m} p(U_i) \times (U_{i_1}) = 1.187$
$p(U_i) \times U_{i_2}$ (pow)	0.000	0.956	0.529	0.452	0.342	$\sum_{i=0}^{I_m} p(U_i) \times (U_{i_2}) = 2.279$

computing average values. For example, if a number of persons be divided into several subgroups by weight G_i, and the relative frequency (or probability) of each subgroup is known, an estimate of the average weight of the entire group might be made by summing $G_i \times p(G_i)$ in which the index i ranges over all subgroups. This is exactly analogous to the method of computing average annual gross benefits given in Table 2.

For benefits due to flood control, a relation of the type in Eq. 16 is predicated. The parameter FL2, defined as the probability of a damaging flood on the unregulated river, is evaluated by consideration of the inflow probability distribution. In the given basin example, the tables of the normal distribution are consulted to evaluate the probability of a flow equal to or greater than 3.5 maf per yr. With a mean inflow of 2 maf per yr and a standard distribution of 1.414 maf per yr, the computation is, apart from a small truncation error

$$t' = \frac{3.5 - \bar{I}}{s} = \frac{3.5 - 2.0}{1.414} = 1.06 \quad \ldots\ldots\ldots\ldots \quad (17$$

hich gives FL2 = 0.145. Here, t' is a normalized variate, one whose magni-
ıde is the number of standard deviations that separate the variate value from
ıe mean, and the corresponding FL2 comes from the tables. This computa-
on is equivalent to computing the area under the truncated right-hand tail of
ıe normal distribution in Fig. 2. With other inflow distributions, the compu-
tion would be performed differently, but the rationale of the technique is suf-
ciently straightforward. For $D_L = 1$ and $S_m = 4$, Table 1 gives the probability
$D \geq 3.5$ as 0.118. It can happen that the probability of $D \geq 3.5$ exceeds the
robability without regulation, namely FL2 = 0.145; thus, the flood benefits
ould be negative. The magnitude of FL1 is determined from damage studies
the area, and is taken here to be 55.20. The flood benefit in this model study
55.2 (0.145 - 0.118) or $\$1.490 \times 10^6$.

$\bar{U}$ is the average annual gross benefit for all uses, and in this simple model
taken to be the algebraic sum of the three benefits considered independently.
ocial benefits, such as wildlife, fish and game preservation, and recreational
ses of the reservoir are not considered in this model.

The incredibly difficult and only partially clarified aspects of water law and
ater rights, so vital to the water distribution schemes in our western states,
re constantly being reinterpreted and adjudicated in the courts. Appropriative
ersus riparian systems, waters that flow along interstate boundaries, exemp-
fied by the decision of the United States Supreme Court Special Master in the
se of the lower Colorado River, . . . all these instances and many more exert
normous influence on the nature of the mathematical models which are to be
nstructed. Moreover, it is to be hoped that hydrologic insight and statistical
ference, in their turn, will provide the guides and basic criteria within which
e socio-legal framework is to be constructed. Such questions demand further
udy, and much patience and goodwill, to reach adequate and just resolution.

The value of $\bar{U}$ is simply 1.187 + 2.279 + 1.490, or $\$4.956 \times 10^6$, but this
m must be discounted to its present value in order to justify the present ex-
nditure of funds that will generate the average annual return over the eco-
mic life of the reservoir. For this computation, it is necessary to assume
economic life span for the reservoir and to specify an interest rate. Using
yr and 2-1/2%, respectively, the discounted value can be written

$$\text{Present Value of Benefits} = \bar{U} \sum_{j=1}^{50} \frac{1}{(1 + 0.025)^j} = 28.362\ (4.956)$$

$$= \$140.57 \times 10^6 \qquad \ldots\ldots\ldots\ldots\ldots\ (18)$$

It should be noted that this entire computation of benefits, starting from
ble 2, must be repeated a large number of times because each of the three
rameters pertinent to its numerical evaluation can vary independently of the
ers. Thus, in this model, there are 75 combinations to be evaluated:

D_L can be 1, 2, 3	(3 increments)
S_m can be 0, 1, 2, 3, 4	(5 increments)
D_S can be 0, 1, 2, 3, 4	(5 increments)

H 5 x 5 x 3 is 75. Clearly this type of computation is ideally suited for a dig-
l computing machine that processes, each iteration at great speed, each time
ng exactly the same set of instructions as before.

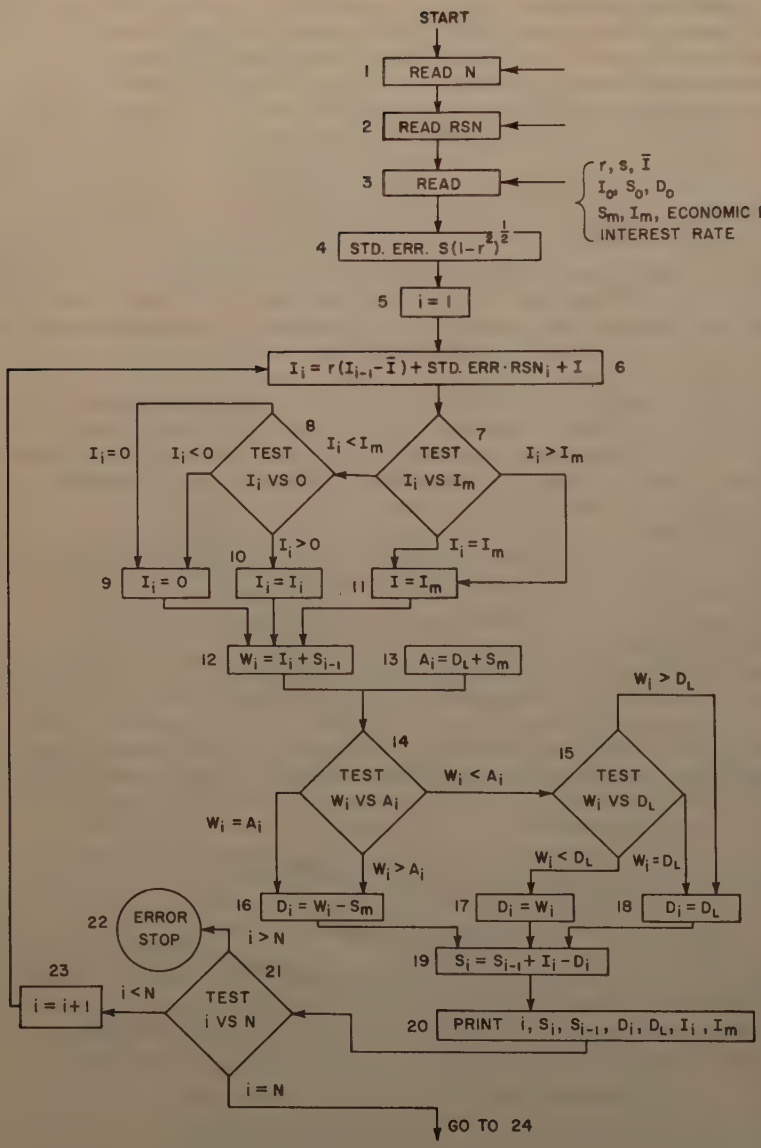

FIG. 10.—FLOW CHART FOR PROGRAM

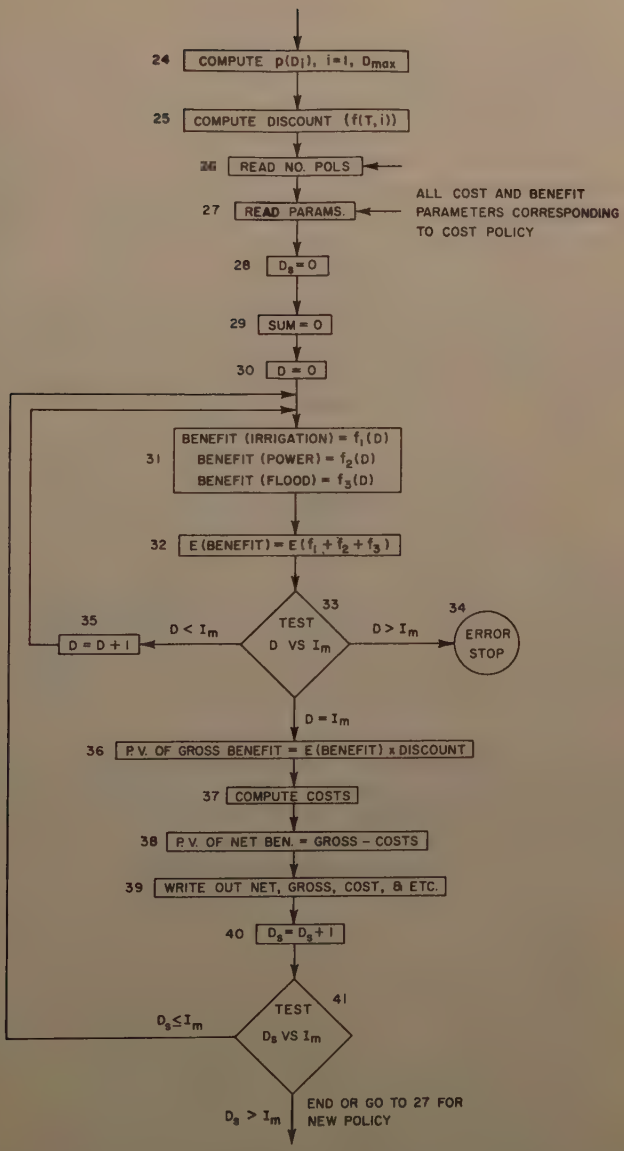

FIG. 10.—FLOW CHART FOR PROGRAM *CONTINUED*

Costs.—Costs are divided into two categories—reservoir cost and costs al located to the specific uses. In this simple model the reservoir cost is take as a function of S_m, the maximum storage capacity, alone. In other, more elab orate models, the reservoir cost may be a function of both the scale of develop ment S_m and the operating policy, specified by D_L. Likewise, it is assume that the costs allocated to irrigation and hydroelectric power are functions onl of their scale of development, D_s. The cost of flood control is not specificall allocated in this model, but rather is assumed to be inherent in the reservoi cost. It is assumed that all monies are invested at time T = 0, and that sub sequent costs of operation, maintenance, and replacement are accounted for i the utility functions for annual benefit. Table 3 summarizes the several co functions, with the entries being in dollars x 10^6. For $S_m = 4$ and $D_s = 1$, th total cost is 51.05 + 5.67 + 8.51 = \$65.23 x 10^6.

Net Benefits.—It is possible to compute the present value of the expected n benefit, which is merely the present value of gross benefit less the total cos 140.6 - 65.23 = \$75.34 x 10^6, indicating a substantial profit for this combinatio of sizes and D_L. Tables 10 through 15 of Appendix II contain the value of th net benefits for each of the possibilities, and it is a simple matter to scan thes

TABLE 3.—RESERVOIR, IRRIGATION, AND POWER COSTS VERSUS SCALE OF DEVELOPMENT, IN DOLLARS x 10^6

Size, S_m or D_s	0	1	2	3	4
Reservoir Cost	8.51	11.34	17.02	28.36	51.05
Irrigation Cost	0.00	5.67	11.34	17.02	22.69
Power Cost	0.00	8.51	17.02	25.53	34.03

tables and select the single combination of parameters which is expected to re turn the greatest excess of benefits over costs. Because this satisfies the de inition of optimal design set forth in the first portion of the paper, the initi portion of the design problem is solved. For this model study, taking r = 0. the optimal design has the following characteristics:

S_m	= 3 maf
D_L	= 3 maf per yr
D_s	= 2 maf per yr
Total Cost	= \$56.74 x 10^6
Gross Annual Benefits	= \$329.97 x 10^6
Net Annual Benefits	= \$273.23 x 10^6

The relatively large returns associated with this study are due, to a gre extent, to the effects of flood control. The probability of a flood in the unre ulated system is 0.145, or almost 1 yr in every 7 yr. Thus, reservoirs th provide great storage capacity are apt to induce large flood control benefit

Computer Studies.—The IBM 709 Data Processing System at the Weste Data Processing Center, University of California, Los Angeles, Calif., was u

ed in this study. The FORTRAN II Language was used in writing the source ogram, from which the computer compiled a machine code and performed bsequent execution of the program. Fig. 10 is a flow chart, or logical dia- am, for the program. The segments of the program in which computations e performed, or decisions are made, are numbered on the flow chart, and the t indicates several details of the computation that might not be described equately by the rather cryptic style of the flow chart itself. The time re-

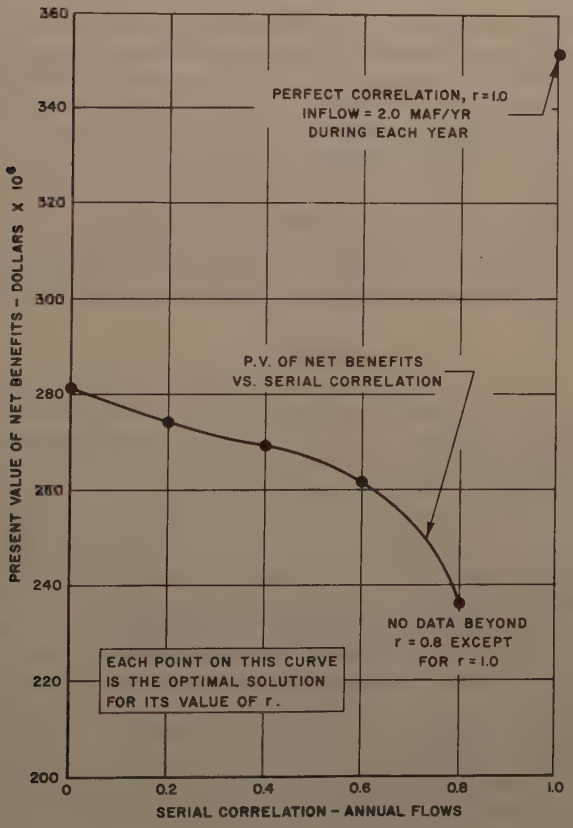

FIG. 11.—SERIAL CORRELATION VS PRESENT VALUE OF NET BENEFITS

red for the generation and economic analysis of 15 synthetic records, each yr in length, is 6.3 min on the IBM 709. A substantial portion of this time, ne order of 5 min, is consumed by the recording of output data.

ffect of Serial Correlation.—In addition to the study in the previous sec- s, for which r, the serial correlation coefficient, is assumed to be 0.2, a ge of values of r is considered so that the effect of serial correlation along

data possessing identical values of the first two statistical moments may evaluated. Fig. 11 shows the net benefits that obtain for optimal solutions different values of the correlation coefficient, from which it is deduced th net benefits tend to decrease with increasing correlation.

At perfect correlation, or r = 1.0, the following anomaly occurs: The stan ard error, containing a multiplicative factor $(1 - r^2)$, goes to zero. Likewis from Eq. 11, note that (1 - r) is zero and, therefore,

$$\hat{I}_{i+1} = I_i$$

However, this restriction does not allow the synthetic record to fulfill t assumption that the standard deviation of annual flows, s, equals 1.414, becau all values of the record are equal, and their variance is zero. The values benefits, then, that accrue when the serial correlation is unity, should be di regarded because the only possible configuration of inflows that would mai tain the first two moments as defined would be a monotonically increasing decreasing function. This implies a long-term trend, which is not within t scope of the proposed solution and should be discounted.

Because inordinately high values of the annual correlation coefficient a rarely obtained in hydrologic practice, poor delineation of the benefit functi in the area of r = 1.0 is not damaging to the proposed analysis.

CONCLUSIONS

With regard to the use of queuing theory to aid in the selection of optim reservoir design, a method is proposed whereby a simple model is coded f machine simulation studies. Draft probabilities and economic benefits a computed and compared when long synthetic records are routed through a pr posed design. A rather restrictive set of assumptions is applied to the exam solved here, and it is anticipated that real problems, if coded for analysis accordance with the method described, will not conform to all the assumptio and simplifications made for the model. Steps 2, 3, and 6 of the flow chart Fig. 10 allow a good deal of flexibility in that many different types of rand sampling digits may be provided or computed for the simulation techniq (step 2), and many parameters and transforms may be built into the synthe represented by step 6. For example, if a log transform were desired, it wo be necessary only to insert a step (between 6 and 7) which computes an logarithms, and the analysis would continue unchanged in all other respec A competent FORTRAN programmer, using the flow chart in Fig. 10, sho have no difficulty with this insertion or with any other reasonable variation the basic program.

Synthetic seasonal or monthly flows may be generated by a straightforwa expansion of the basic computer logic. Different values of the parameters the inflow distribution will be needed in the computation, corresponding to cyclic order of seasons or months. A different operation rule may be invo for each period, thereby simulating a real system very closely.

A study of synthetic hydrology, generated for a wide range of assu serial correlation coefficients, shows that increasing serial correlation te to depress that value of net benefits corresponding to the optimal selectio

reservoir and appurtenant structures, but an abrupt rise in benefits occurs en $r \rightarrow 1.0$.

ACKNOWLEDGMENTS

The writer wishes to express his appreciation to the staff of the Western Da- Processing Center, University of California, Los Angeles, Calif., for the op- rtunity to perform these studies on the fine equipment available there. John rton, visiting the University of California while on leave from his faculty sition at the University of New South Wales, Australia, and Walter Langbein, the USGS, read the manuscript and made many useful comments. Finally, writer wishes to express his gratitude to Harold A. Thomas, Jr., of Har- rd University, Cambridge, Mass., whose original solution, so often quoted l referred to throughout this paper, provides a sound basis for the further estigation encompassed by this study.

APPENDIX I.—DRAFT PROBABILITY TABLES (TABLES 4 - 9)

ABLE 4.—DRAFT PROBABILITIES FOR S_m = 0, 1, 2, 3, 4 and D_L = 1, 2, 3, r = 0.0

S_m (maf)	D_L (maf per yr)	Draft (maf per yr)[a]				
		0	1	2	3	4
0	1	0.156	0.240	0.242	0.210	0.152
0	2	0.156	0.240	0.242	0.210	0.152
0	3	0.156	0.240	0.242	0.210	0.152
1	1	0.034	0.408	0.250	0.186	0.122
1	2	0.084	0.188	0.484	0.174	0.070
1	3	0.126	0.218	0.246	0.374	0.036
2	1	0.004	0.458	0.238	0.180	0.120
2	2	0.072	0.120	0.646	0.104	0.058
2	3	0.122	0.216	0.238	0.416	0.008
3	1	–	0.466	0.236	0.178	0.120
3	2	0.052	0.100	0.734	0.068	0.046
3	3	0.122	0.216	0.234	0.426	0.002
4	1	–	0.466	0.238	0.176	0.120
4	2	0.042	0.080	0.790	0.052	0.036
4	3	0.122	0.214	0.236	0.428	–

[a] Drafts are rounded to nearest integer.

TABLE 5.—DRAFT PROBABILITIES FOR S_m = 0, 1, 2, 3, 4 AND D_L = 1, 2, 3, r = 0.

(S_m (maf)	D_L (maf per yr)	Draft (maf per yr)[a]				
		0	1	2	3	4
0	1	0.150	0.246	0.260	0.190	0.154
0	2	0.150	0.246	0.260	0.190	0.154
0	3	0.150	0.246	0.260	0.190	0.154
1	1	0.044	0.402	0.242	0.188	0.124
1	2	0.100	0.178	0.456	0.190	0.076
1	3	0.132	0.220	0.258	0.342	0.048
2	1	0.012	0.454	0.238	0.178	0.118
2	2	0.082	0.128	0.604	0.120	0.066
2	3	0.132	0.210	0.248	0.398	0.012
3	1	0.002	0.470	0.236	0.174	0.118
3	2	0.066	0.102	0.688	0.094	0.050
3	3	0.132	0.208	0.242	0.414	0.004
4	1	–	0.478	0.230	0.174	0.118
4	2	0.056	0.082	0.748	0.072	0.042
4	3	0.132	0.206	0.242	0.418	0.002

[a] Drafts are rounded to nearest integer.

TABLE 6.—DRAFT PROBABILITIES FOR S_m = 0, 1, 2, 3, 4 AND D_L = 1, 2, 3, r = 0

S_m (maf)	D_L (maf per yr)	Draft (maf per yr)[a]				
		0	1	2	3	4
0	1	0.144	0.254	0.266	0.190	0.146
0	2	0.144	0.254	0.266	0.190	0.146
0	3	0.144	0.254	0.266	0.190	0.146
1	1	0.052	0.404	0.224	0.194	0.126
1	2	0.108	0.180	0.432	0.194	0.086
1	3	0.142	0.208	0.266	0.340	0.044
2	1	0.018	0.454	0.220	0.186	0.122
2	2	0.096	0.128	0.570	0.136	0.070
2	3	0.142	0.196	0.248	0.398	0.016
3	1	0.006	0.478	0.216	0.178	0.122
3	2	0.076	0.118	0.644	0.104	0.058
3	3	0.142	0.190	0.252	0.410	0.006
4	1	0.002	0.488	0.210	0.178	0.122
4	2	0.066	0.096	0.702	0.086	0.050
4	3	0.142	0.188	0.252	0.416	0.002

[a] Drafts are rounded to nearest integer.

ABLE 7.—DRAFT PROBABILITIES FOR S_m = 0, 1, 2, 3, 4 AND D_L = 1, 2, 3, r = 0.6

S_m	D_L	Draft (maf per yr)[a]				
(maf)	(maf per yr)	0	1	2	3	4
0	1	0.134	0.262	0.252	0.234	0.118
0	2	0.134	0.262	0.252	0.234	0.118
0	3	0.134	0.262	0.252	0.234	0.118
1	1	0.066	0.376	0.232	0.212	0.114
1	2	0.118	0.190	0.420	0.174	0.098
1	3	0.134	0.240	0.218	0.354	0.054
2	1	0.032	0.434	0.222	0.198	0.114
2	2	0.104	0.162	0.518	0.130	0.086
2	3	0.132	0.232	0.204	0.406	0.026
3	1	0.014	0.464	0.214	0.194	0.114
3	2	0.088	0.138	0.590	0.116	0.068
3	3	0.132	0.232	0.188	0.438	0.010
4	1	0.008	0.478	0.210	0.190	0.114
4	2	0.078	0.122	0.646	0.090	0.064
4	3	0.130	0.230	0.192	0.444	0.004

Drafts are rounded to nearest integer.

BLE 8.—DRAFT PROBABILITIES FOR S_m = 0, 1, 2, 3, 4 AND D_L = 1, 2, 3, r = 0.8

S_m	D_L	Draft (maf per yr)[a]				
maf)	(maf per yr)	0	1	2	3	4
0	1	0.128	0.248	0.278	0.232	0.114
0	2	0.128	0.248	0.278	0.232	0.114
0	3	0.128	0.248	0.278	0.232	0.114
1	1	0.072	0.352	0.234	0.228	0.114
1	2	0.120	0.198	0.396	0.184	0.102
1	3	0.128	0.242	0.242	0.318	0.070
2	1	0.048	0.394	0.226	0.218	0.114
2	2	0.114	0.116	0.466	0.160	0.094
2	3	0.128	0.234	0.234	0.362	0.042
3	1	0.034	0.420	0.214	0.220	0.112
3	2	0.104	0.158	0.504	0.150	0.084
3	3	0.128	0.232	0.224	0.390	0.026
4	1	0.024	0.432	0.218	0.216	0.110
4	2	0.102	0.144	0.534	0.142	0.078
4	3	0.128	0.232	0.218	0.404	0.018

Drafts are rounded to nearest integer.

TABLE 9.—DRAFT PROBABILITIES FOR S_m = 0, 1, 2, 3, 4 and D_L = 1, 2, 3, r = 1.

S_m (maf)	D_L (maf per yr)	DRAFT (maf per yr)[a]				
		0	1	2	3	4
0	1	-	-	0.998	-	0.002
0	2	-	-	0.998	-	0.002
0	3	-	-	0.998	-	0.002
1	1	-	-	0.998	0.002	-
1	2	-	-	0.998	0.002	-
1	3	-	-	0.996	0.004	-
2	1	-	-	1.000	-	-
2	2	-	-	1.000	-	-
2	3	-	-	0.996	0.004	-
3	1	-	0.002	0.998	-	-
3	2	-	-	1.000	-	-
3	3	-	-	0.996	0.004	-
4	1	-	0.004	0.996	-	-
4	2	-	-	1.000	-	-
4	3	-	-	0.996	0.004	-

[a] Drafts are rounded to nearest integer.

APPENDIX II.—VALUES OF NET BENEFITS (TABLES 10 - 15)

TABLE 10.—VALUE OF NET BENEFITS, IN DOLLARS x 10^6, r = 0.0 (As of 1961)

D_s	S_m				
	0	1	2	3	4
			D_L = 1		
0	8.35	52.38	49.83	38.44	15.74
1	45.17	104.77	106.05	95.17	72.47
2	51.35	105.08	103.81	92.42	69.72
3	26.64	73.47	71.18	59.54	36.58
4	-24.85	18.15	15.60	3.96	-19.00
			D_L = 2		
0	8.35	133.85	146.93	154.35	147.40
1	45.17	179.85	194.46	204.44	198.76
2	51.35	201.86	226.68	241.77	239.92
3	26.64	162.09	176.44	185.40	180.23
4	-24.85	100.13	112.95	120.38	113.93
			D_L = 3		
0	8.35	187.33	225.43	223.44	203.90
1	45.17	227.98	266.59	264.59	245.05
2	51.35	240.80	280.17	278.18	258.90
3	26.64	222.21	263.37	261.89	242.61
4	-24.85	155.91	193.50	191.25	171.71

TABLE 11.—VALUE OF NET BENEFITS, IN DOLLARS x 10^6, r = 0.2 (As of 1961)

D_s	S_m				
	0	1	2	3	4
			$D_L = 1$		
0	5.08	49.13	52.71	41.35	18.62
1	42.66	100.24	107.90	97.82	75.34
2	48.85	100.05	105.15	94.30	71.06
3	21.84	68.95	72.01	60.65	37.42
4	-29.40	13.88	16.18	4.82	-18.42
			$D_L = 2$		
0	5.08	124.54	134.46	148.15	138.03
1	42.66	168.50	180.72	196.45	187.61
2	48.85	189.75	210.64	231.73	226.72
3	21.84	152.78	163.47	179.20	170.36
4	-29.40	91.59	100.99	114.68	104.82
			$D_L = 3$		
0	5.08	168.23	218.83	220.02	200.51
1	42.66	208.11	258.71	259.90	240.39
2	48.85	219.91	271.78	273.23	253.98
3	21.84	198.77	253.20	255.67	236.67
4	-29.40	134.00	183.84	185.28	166.03

TABLE 12.—VALUE OF NET BENEFITS, IN DOLLARS x 10^6, r = 0.4 (As of 1961)

D_s	S_m				
	0	1	2	3	4
			$D_L = 1$		
0	17.44	45.89	46.50	35.03	12.33
1	55.78	95.98	100.93	90.99	68.80
2	61.71	94.50	97.41	85.94	62.99
3	33.68	64.43	65.81	53.31	30.36
4	-18.58	9.62	10.48	-2.01	-24.96
			$D_L = 2$		
0	17.44	108.97	128.14	135.48	125.45
1	55.78	151.91	172.61	182.51	173.75
2	61.71	171.88	200.75	214.47	209.80
3	33.68	136.70	156.13	164.23	156.24
4	-18.58	76.78	94.16	100.74	91.73
			$D_L = 3$		
0	17.44	174.24	212.60	216.86	200.46
1	55.78	212.84	251.20	255.47	239.06
2	61.71	224.90	264.78	269.82	253.67
3	33.68	202.99	246.71	252.00	236.10
4	-18.58	137.71	177.86	181.87	165.46

TABLE 13.—VALUE OF NET BENEFITS, IN DOLLARS x 10^6, r = 0.6 (As of 1961)

D_s	S_m				
	0	1	2	3	4
			$D_L = 1$		
0	61.27	64.59	58.85	47.52	24.77
1	100.90	112.89	111.50	102.46	80.47
2	107.08	113.21	108.75	98.18	75.17
3	81.09	83.90	77.65	66.58	43.06
4	25.26	27.56	21.31	10.23	-13.29
			$D_L = 2$		
0	61.27	89.81	102.75	119.65	103.28
1	100.90	131.48	146.20	165.15	150.05
2	107.08	148.89	168.98	193.03	181.25
3	81.09	112.69	125.63	145.60	129.99
4	25.26	54.31	65.71	83.38	67.26
			$D_L = 3$		
0	61.27	158.84	197.11	210.80	197.52
1	100.90	198.46	236.99	250.68	237.66
2	107.08	207.45	247.26	260.95	248.44
3	81.09	188.61	231.46	247.21	234.70
4	25.26	124.61	163.90	177.59	164.32

TABLE 14.—VALUE OF NET BENEFITS, IN DOLLARS x 10^6, r = 0.8 (As of 1961)

D_s	S_m				
	0	1	2	3	4
			$D_L = 1$		
0	67.76	64.99	59.25	51.02	31.48
1	108.15	112.53	109.85	103.41	85.15
2	116.89	115.14	110.16	102.19	83.67
3	90.13	87.87	81.62	73.65	54.37
4	33.79	31.53	25.28	17.05	-2.49
			$D_L = 2$		
0	67.76	83.63	90.54	94.80	81.50
1	108.15	125.04	132.71	138.26	125.21
2	116.89	141.18	153.70	161.54	150.53
3	90.13	106.77	115.20	120.49	107.70
4	33.79	48.89	56.31	60.32	46.76
			$D_L = 3$		
0	67.76	133.88	171.98	185.64	175.46
1	108.15	174.26	212.36	226.03	215.85
2	116.89	183.77	222.89	236.80	226.63
3	90.13	162.37	203.54	218.98	209.58
4	33.79	100.41	138.00	151.41	140.98

TABLE 15.—VALUE OF NET BENEFITS, IN DOLLARS x 10^6, r = 1.0 (As of 1961)

D_s	S_m				
	0	1	2	3	4
			$D_L = 1$		
0	243.79	244.06	238.35	226.97	204.27
1	300.52	300.79	295.08	283.69	260.99
2	357.24	357.51	351.81	340.16	317.21
3	286.58	286.85	280.89	269.25	246.29
4	215.94	215.96	210.00	198.35	175.40
			$D_L = 2$		
0	243.79	244.06	238.35	226.99	204.32
1	300.52	300.79	295.08	283.72	261.05
2	357.24	357.51	351.81	340.45	317.78
3	286.58	286.85	280.89	269.53	246.86
4	215.94	215.96	210.00	198.64	175.97
			$D_L = 3$		
0	243.79	244.09	238.41	227.05	204.38
1	300.52	300.82	295.14	283.78	261.11
2	357.24	357.54	351.86	340.50	317.83
3	286.58	287.14	281.46	270.10	247.43
4	215.94	216.24	210.56	199.20	176.53

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CYCLICAL VARIATIONS IN WORLD-WIDE HYDROLOGIC DATA

By Gordon R. Williams[1]

SYNOPSIS

The nature and causes of cyclical changes in hydrologic data have been investigated by engineers and scientists for many years. Correlations between hydrologic data and sunspots have been attempted with varying success. Astronomic and meteorologic research in recent years has revealed that the sunspot numbers are, if not a measure of solar activity, at least an index of such activity. In addition to the well-known 11-yr sunspot cycle that affects hydrologic data in an irregular manner, there is evidence of a quasi 100-yr cycle, that also appears to cause long term variations in hydrologic data. Both precipitation and river runoff records are analyzed. All data are expressed as ratios to the mean in order to obtain a dimensionless basis for comparison. Short-term cycles are revealed through use of 3-yr moving averages. Long-term variations from the mean are indicated by use of the differential mass curve or cumulative deviations from the mean. Similar long-term variations are evident in widely spaced records around the world, although the peaks and troughs are not exactly coincident in time. There is some indication of a progressive displacement in time from place to place.

INTRODUCTION

Engineers concerned with water resources development have known almost since precipitation and runoff records were first studied that annual supplies

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are not random occurrences. Sequences of wet and dry years form irregular cyclical patterns. In some records repeating patterns may be evident for several decades and then lose their form. A search for the cause of these variations has been pursued by astronomers, physicists, climatologists, and inquisitive engineers. It is important for engineers to study these effects in order to appraise the relatively short precipitation and runoff records that they are often compelled to use in the design of hydraulic engineering projects. In other words, do these records give a reliable indication of the water supplies that will be available during the many decades that a water resources project will operate? This is a vitally important question that must be removed from the realm of pure guessing.

SOLAR RADIATION AND CLIMATIC CHANGE

The circulation of the atmosphere is dependent on only two basic causes (1) unequal heating of the earth's surface by the sun and (2) rotation of the earth. As the rotation of the earth is constant, it is natural to turn to a study of the sun for the cause of variations in the climate in any locality. Carefu measurements of the radiant energy from the sun indicate that the annual totals received on the earth vary only 1% to 2% from year to year and, hence the total radiation figure is called the "solar constant." However, some investigators, particularly G. G. Abbott of the Smithsonian Institution, Washington, D. C. have impressive evidence[2] to indicate that even these small changes in total radiation are influential in causing irregular cycles in climatic and hydrologic data.

Because the sources of atmospheric moisture are evaporation and transpiration, because both these phenomena are dependent on energy received from the sun, and because total solar radiation is essentially constant, it might be reasoned that the total moisture content in the earth's atmosphere is constant. If the available moisture is constant, then all parts of the earth or even all parts of a large continent cannot be expected to have simultaneous increases or decreases in precipitation and runoff. However, the selected records that have been analyzed for this study indicate that enormous areas of the earth have had simultaneous deficiencies or excesses in water supply. If further study substantiates the initial findings, it must be concluded that the portion of total solar radiation that goes into heat of vaporization is not constant after all. It is reasonable to assume that for varying periods of years there are substantial changes in the component of the energy of the sun which is intercepted in outer layers of the atmosphere, or else varying amounts of the total energy are transformed into kinetic or electric energy.

SUNSPOTS, AN INDEX OF SOLAR ACTIVITY

Even though the total radiation emitted from the sun is almost constant there are other types of solar activity that are far from constant and may influence the amount or form of energy available at the earth's surface. In 161

2 "A Long-Range Forecast of United States Precipitation," by G. G. Abbott, Smithsonian Miscellaneous Collections, Smithsonian Inst., Washington, D.C., Vol. 139, No. March 23, 1960.

oth Galileo in Padua, Italy, and Johannes Fabricius in Wittenburg, Germany[3] oticed spots on the sun. It is now known that these spots are violent eruptions n the gaseous matter of the sun accompanied by emissions of invisible charged articles which in turn traverse the 93,000,000 miles of space between the un and the earth in one to four days. Magnetic effects of these eruptions isturb the earth's magnetic field in a short time, cause serious dislocations n radio transmission, produce brilliant auroral displays and apparently con- ribute to changes in atmospheric density.[4] Observations made during the past nternational Geophysical Year (July 1957-January 1958) reveal that the course f the jet stream sometimes changed at times of solar eruptions. As the posi- ion of the jet stream has a marked effect on surface weather, it is likely that ncreases and decreases in sunspot activity, if prolonged, cause changes in recipitation and resulting runoff on the earth's surface. It is not known wheth- r the influence of the solar eruptions on the jet stream results from the pres- nce of the solar particles in the atmosphere, or from magnetic effects,[5] or rom both. It is known, however, that sunspots change in polarity at the time of inima and that the periods of similar polarity last about 22 yrs. It is quite ertain that the sunspot numbers are only a crude index of the real causes of hort-term and long-term changes in climate. The sunspot number, usually nown as the Wolf or Zurich number, is an empirical total based on a simple ormula which considers both the number of observed groups as well as the otal number of individual spots. More reliable measures of solar influence, if ney could be obtained, might be the concentration of solar particles in the tmosphere, or some measure of magnetic effects yet to be devised. For want f a better index, the variations in hydrologic data shown in this paper will be ompared with short-term and long-term changes in the annual sunspot umbers.

It would seem logical in view of the studies by Abbott and others that con- iderable research would be available on the relation between the measured ariation in solar radiation and sunspot numbers. This does not seem to be the ase and the only work that has come to the attention of the writer is a paper ublished in 1945 by L. B. Aldrich[6] of the Smithsonian Institution, in which a omparison between variations in the solar constant and in sunspot numbers as made for the period 1923-1944. The results were not conclusive, but did how that solar radiation both increased and decreased during periods of in- reased sunspot numbers. Some of the greatest solar radiation values oc- urred during sunspot numbers from 10 to 30, and also between 60 and 85 see Fig. 1). This may explain in part why hydrologic records often show peaks uring periods of both maximum and minimum sunspot numbers.

SUNSPOT NUMBERS AND CLIMATIC CHANGES

Some investigations of the probable correlation between sunspot numbers nd climatic changes have attempted to define a regular pattern into which both

3 "The Sun," by Karl Kiepenheuer, The Univ. of Michigan Press, Ann Arbor, Mich., 959, p. 19.

4 "Effects of a Severe Solar Storm on the Orbit of Echo I," by Robert Jastrow and obert Bryant, Transactions, Amer. Geophysical Union, Vol. 42, No. 1, March, 1961, 120.

5 "Long-Period Variations in Geomagnetic Activity," by Edwin J. Chernosky, Trans- ctions, Amer. Geophysical Union, Vol. 36, No. 4, August, 1955, p. 591.

6 "The Solar Constant and Sunspot Numbers," by L. B. Aldrich, Smithsonian Miscel- neous Collections, Smithsonian Inst., Washington, D. C., Vol. 104, No. 12, July 2, 1945.

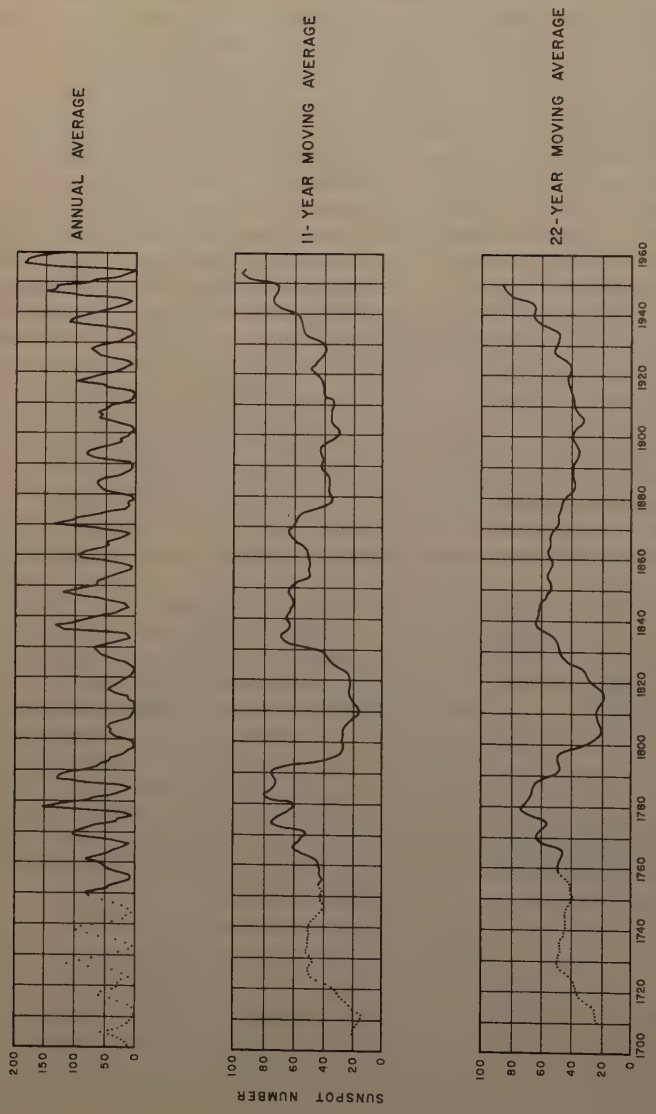

FIG. 1.—VARIATIONS IN MEANS OF ZURICH SUNSPOT NUMBER

ne sunspots and a climatic factor would fit. The similarities between sunspot ycles and climatic cycles sometimes persist for a few decades and then are o longer evident. This leads to the conclusion that the first correlation was nere chance and further studies are not pursued. Other studies have applied armonic analysis to climatic cycles and have been able to prove that climatic ariations can be reproduced by creating an irregular pattern which results rom summating a number of different regular cycles with different period-cities and amplitudes. Abbott's studies reveal[2] that the component cycles that e uses to reproduce weather cycles can be identified with variations in the olar constant. He states that there are 27 different periodic cycles affecting recipitation.

Other studies have utilized complex statistical methods to prove or dis-rove that precipitation, runoff, or other phenomena increase in periods of unspot maxima and decrease in periods of sunspot minima, but the results re usually inconclusive. There are several reasons for the failure of these ttempts to prove such a statistical correlation. One of the reasons, as will e shown in examples in this study, is that extreme sunspot activity over much f the earth has an adverse effect on moisture supplies. Willett was aware of his relationship in 1950 and stated:[7] "By and large, the major dry periods of lacial recession and cold wet periods of glacial advance, have coincided oughly with the longer periods of sunspot activity and inactivity, respectively."

Another, and very important reason why attempts to establish statistical orrelations between sunspots and variations in precipitation are almost com-lete failures, is that precipitation and runoff records often exhibit a cycle hose frequency is double the sunspot cycle, or show a peak once in 5 yr - 6 yr, nce with sunspot maxima and once with sunspot minima. The greatest peaks ften occur at times of sunspot minima. British scientists studying variations Lake Victoria in the equatorial region at the head of the Nile river found a erfect correlation between sunspots and the level of the lake in the period 896-1922; but subsequent to 1922, a double cycle appeared in the lake levels, ith a peak at both sunspot maxima and minima. The conclusion[8] of H. E. urst, that the earlier correlation was accidental is not borne out by analyses ade for this paper. To attempt to prove or to disprove by purely mathematical orrelations that variations in solar activity (of which sunspots are one index) ave no effect on climatic and hydrologic cycles is to deny what the eye can etect in graphs of data from all over the world. Changes in solar activity ertainly appear to have a complex relationship with climatic cycles, and much emains to be discovered about the true nature of the emissions from the sun d their interaction with the earth's atmosphere.

ADOPTED METHODS OF STATISTICAL ANALYSIS

The initial impetus for this paper came from two theses,[9,10] that the writer roposed and supervised while a member of the faculty at the Massachusetts

7 "Extrapolation of Sunspot - Climate Relationships," by H. C. Willett, Journal of eteorology, Vol. 8, No. 1, February, 1951, p. 1.

8 "The Nile," by H. E. Hurst, Constable and Co., London, 1952, p. 263.

9 "An Analysis of Trends in Stream Flow Records of Selected Rivers in the United tes," by J. R. Groves, thesis presented to the Massachusetts Inst. of Tech., in Cam-idge, Mass., in June, 1959, in partial fulfilment of the requirements for the degree of ster of Science.

10 "Changes in Precipitation and Solar Activity," by H. J. Parmelee, thesis presented the Massachusetts Inst. of Tech., in Cambridge, Mass., in June, 1960, in partial fulfil-nt of the requirements for the degree of Bachelor of Science.

Institute of Technology, Cambridge, Mass. These theses presented two differen approaches to a statistical correlation between hydrologic data and sunspots neither of which produced satisfying results, for the reasons previousl examined. The basic statistical compilations contained in the theses have bee invaluable in the preparation of this paper.

Even the limited number of records that were compiled for the two theses and that have been supplemented by additional analyses for this paper, sho conclusively that both short-term (5 yr to 15 yr) and long-term (20 yr to 60 yr cyclical variations in precipitation and runoff records occur in various part of the world. To show these variations, the annual values were first expresse as ratios to the mean, and then a 3-yr moving average was computed and com pared with a plot of annual sunspot numbers. The second method of analysi was to compute cumulative deviations from the mean, or the so-called differ ential mass curve. The cumulative deviations were plotted and compared wit an 11-yr moving average of the annual sunspot numbers. A moving average o the latter type eliminates the well-known 11-yr cycle in sunspots and reveal long-term trends. In the 250-yr period in which sunspot data have been ob served, it has been noted that there is a quasi 100-yr cycle between peaks an between troughs in both the 11-yr and 22-yr moving average. The full rang of variations is shown in Fig. 1, which is taken from a paper by E. J. Chernosk and M. P. Hagan.[11]

Before presenting the hydrologic data, it is of interest to examine furthe the known record of sunspot data in Fig. 1. Sunspot numbers vary continuall from day to day, but only the annual means have been utilized in this study The 11-yr cycle between annual peaks is evident on the top graph. It is als apparent that adjacent peak values vary and that there has also been som progressive variation in the magnitude of the peaks. Computations of 11-y and 22-yr moving averages reveal irregular long-term cycles as shown o the middle and bottom graphs of Fig. 1. The quasi 100-yr cycle is indicate by the minima about the years 1710, 1810, and 1900. The time intervals be tween the maxima in the long-term cycle are more irregular, but the period of greatest activity have fallen between the one-quarter and three-quarte points in each century. The world is passing through the greatest known perio of activity in the history of sunspot records.

Because of the method of compiling the hydrologic data, that is, the use o 3-yr moving averages of annual totals and cumulative deviations of the annu totals from the mean, it is probably immaterial whether precipitation o runoff records are used to illustrate cyclical trends, because the effect delays within the hydrologic cycle (that might affect individual years in th runoff records) will have been obscured. Runoff records for large drainag areas will tend to average localized effects, but over several years both pre cipitation and runoff records will define the regional trend in moisture suppl

ANALYSIS OF SELECTED HYDROLOGIC RECORDS

As these studies originated near Boston, Mass., it is appropriate to prese first the analysis of the precipitation record at that city extending back to 181 Prior to 1871 the record was unofficial and probably represents readings tak

11 "The Zurich Sunspot Numbers and Its Variations for 1700–1957," by E. J. Chernos and M. P. Hagan, Journal of Geophysical Research, Vol. 63, No. 4, December, 1958, p. 7

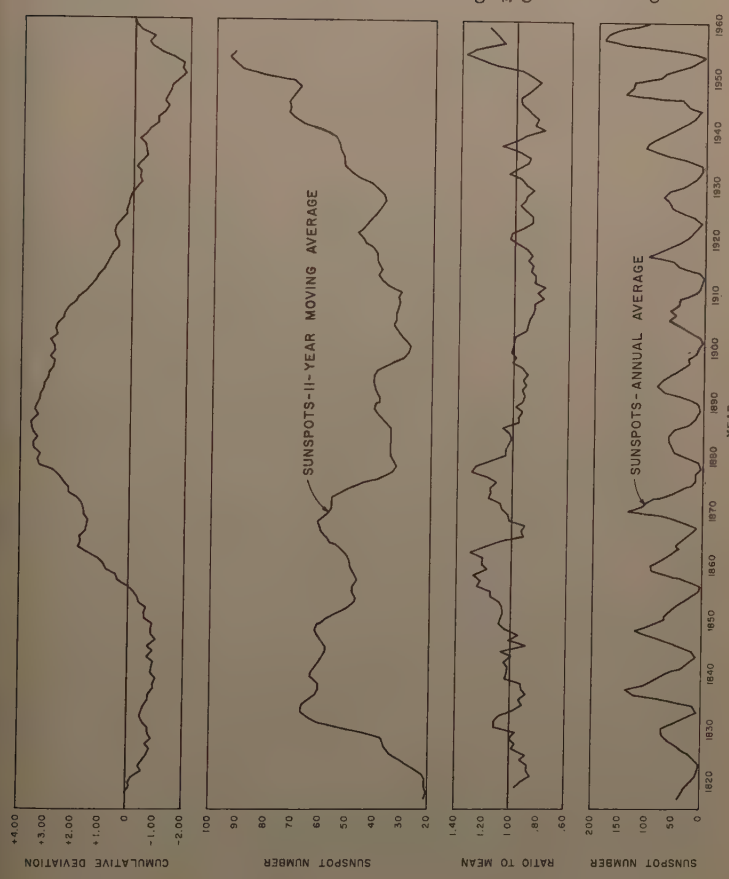

FIG. 2.—CYCLICAL VARIATIONS IN PRECIPITATION BOSTON, MASS.

some miles from the measurement point for the official record. The data ar presented in Fig. 2. Graph A is the plot of annual sunspot means from Fig. and is plotted adjacent to the 3-yr moving average of precipitation (Graph B) expressed as a ratio to the mean. Graph C is the 11-yr moving average of sun spots and is to be compared with cumulative deviations from mean precipita tion in Graph D. For those not accustomed to use of the differential mas curve (Graph D), it should be noted that positive slopes indicate periods o greater than average precipitation (or runoff), negative slopes represent peri ods of less than average precipitation, and a horizontal line represents period of average precipitation.

There is no apparent correlation between maxima and minima in Graph A and B of Fig. 2, but there is a definite tendency for peak periods of precipita tion to fall within periods of sunspot minima or at times minor maxima. A outstanding example of this characteristic is the period 1951-55. The 5-yr t 6-yr cycle is very evident from 1918-1946. A comparison of Graphs C and shows that over long periods of time Boston precipitation is affected adversel by increased solar activity. If the long-term sunspot cycle reversed in 195 it is reasonable to predict that the 50-yr downtrend beginning about 1900 ha been reversed.

Fig. 3 and 4 are the 3-yr moving averages and the cumulative deviation from the mean, respectively, for four selected Eastern rivers. The Potomac James, and Kanawha River Basins were chosen in order to detect the similar ities that might exist in contiguous basins. Although the Androscoggin Rive is 500 miles to 600 miles northeast of the other three, it shows rises and fall generally coincident in time with the other rivers for the short-term trends Fig. 4 shows that the runoffs of the three southern rivers have had a genera downward trend since 1920, or have decreased with increased sunspot activity The runoff of the northern river turned down in 1902, about 20 yr sooner, b has been normal or above normal on the average since 1927. Other record show that all across the northern United States there have been favorabl moisture supplies from 1940-1960.

In order to sample hydrologic trends for a large area in the northeaster sector of the United States, an analysis was made of the flow of the S Lawrence River at the outlet of Lake Ontario (drainage area, 300,000 sq miles The record was adjusted for diversions into and out of the basin. The 3-y moving average (see Fig. 5) shows long cycles of 8 yr to 15 yr or more betwee peaks. Runoff peaks have occurred at times of both sunspot maxima an minima. Undoubtedly, the enormous lake storage eliminates minor peaks inflow and changes the timing of all peaks. The entire relation to both shor term and long-term sunspot cycles is obscure probably because the area large enough to sample opposing climatic influences. The cumulative deviatio from the mean shows normal or below normal runoff since 1891, somewh similar to the Boston precipitation record, except that the yield from the Gre Lakes turned upward in 1942, whereas the Boston record did not change un 1950. A comparison between a point record and an area record may see illogical, but there is some evidence that long-term cycles govern for wi belts of latitude and longitude and that the sampling can be by either a poi or an area method.

A record that averages water supply conditions over 38% of the land ar of the United States is that of the Mississippi River at Vicksburg, Miss., whi

as been computed by the Mississippi River Commission back to the year 817. The 3-yr moving average (see Fig. 6) shows twenty-three hydrologic ycles of different amplitude in 143 yr with an average interval of about 6 yr.

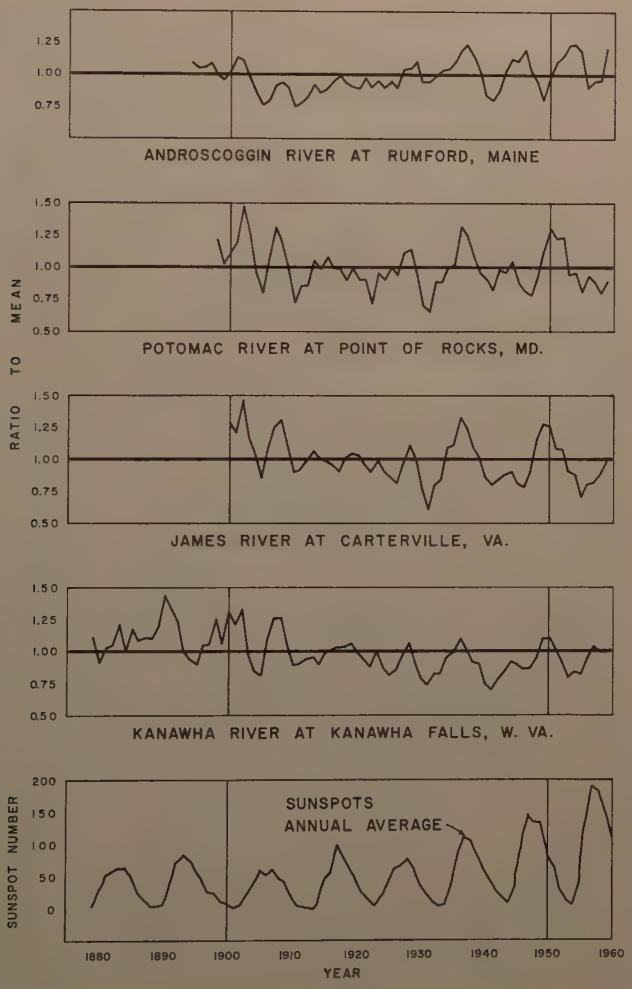

FIG. 3.—SHORT TERM VARIATIONS IN RUNOFF FOR SELECTED RIVERS IN EASTERN UNITED STATES 3-YR MOVING AVERAGES

oincidence in time with the annual sunspot cycle is not consistent, but peaks ave coincided with both sunspot maxima and minima, and a solar influence is eadily apparent. The short-term cycles are of sufficient duration and magni-

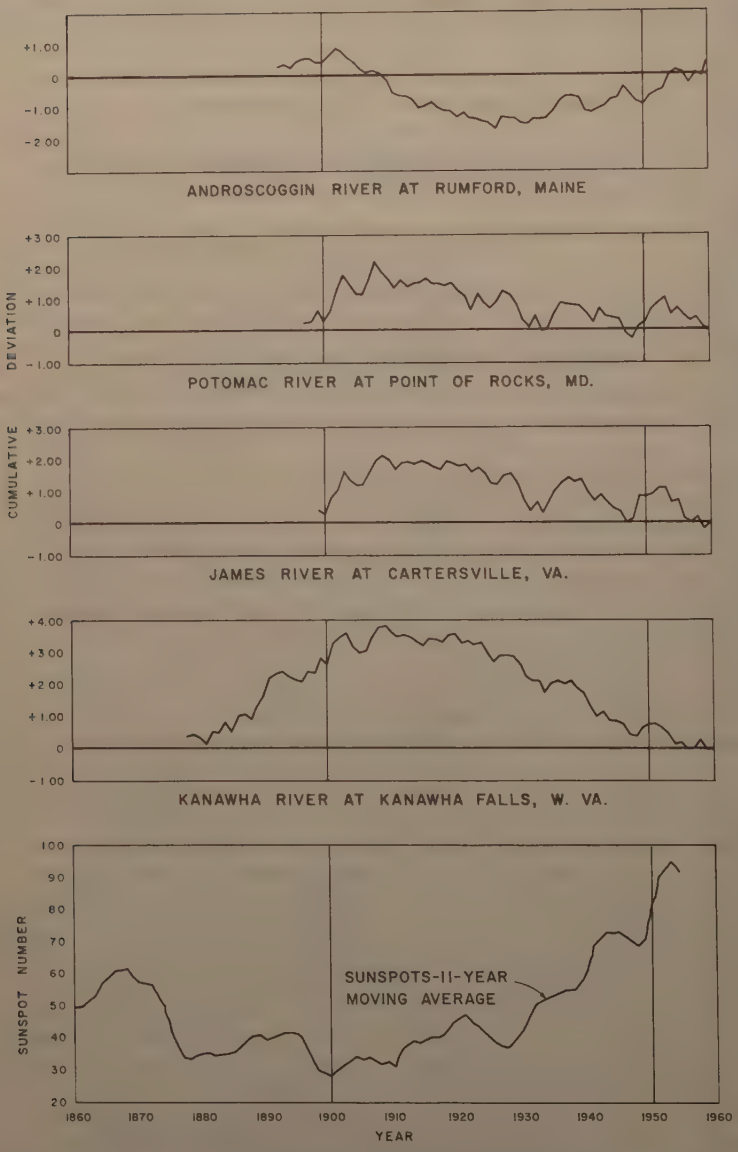

FIG. 4.—LONG TERM VARIATIONS IN RUNOFF FOR SELECTED RIVERS IN EASTERN UNITED STATES CUMULATIVE DEVIATIONS FROM THE MEAN

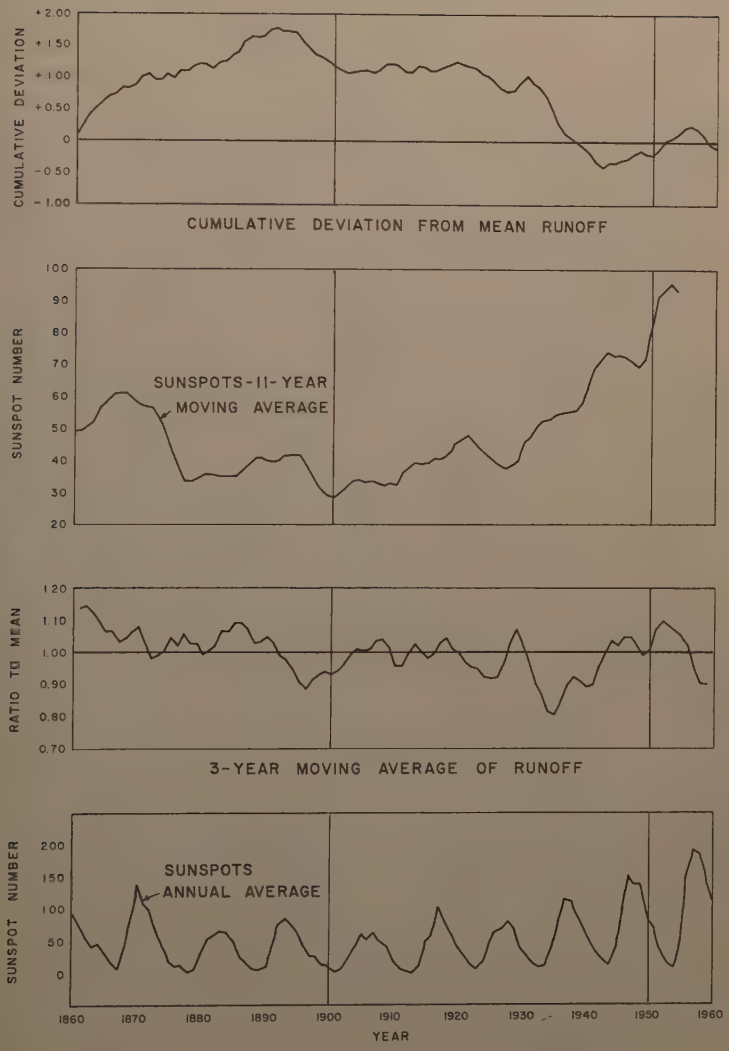

FIG. 5.—CYCLICAL VARIATIONS IN RUNOFF ST. LAWRENCE RIVER AT OUTLET OF LAKE ONTARIO

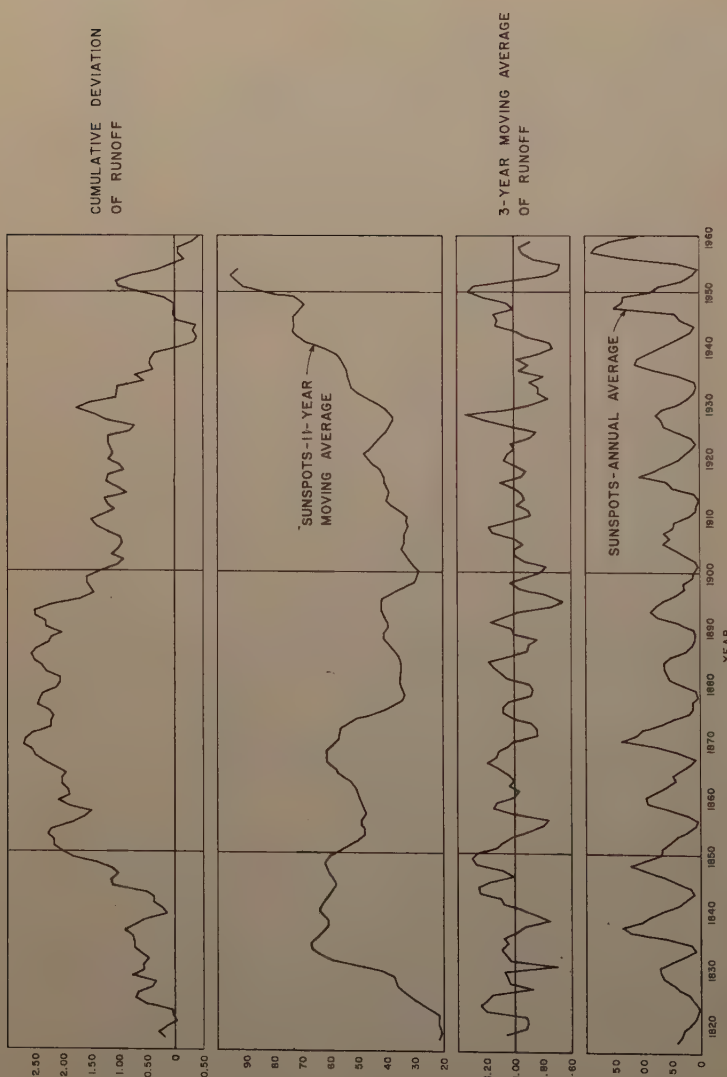

FIG. 6.—CYCLICAL VARIATIONS IN RUNOFF MISSISSIPPI RIVER AT VICKSBURG MISS.

ude to cause appreciable waves in the differential mass curve. The effect of he long-term sunspot cycle is not clear, probably because of counter-balancing egional cycles in the enormous drainage area. The drought from 1929-1942 tands out as the period of greatest deficiency in 143 yr. The break in 1944 n the long period of deficiency beginning with the widespread drought of 1930 s due to above normal contributions from northern latitudes, as evidenced by he runoff of the St. Lawrence River (Fig. 5).

In order not to convey the impression that the drainage basins of the entire Jnited States are simultaneously drying up, differential mass curves have een plotted for seven river basins in the western half of the United States see Fig. 7). The basins vary widely in location and size but the curves show definite progression of long-term wet and dry cycles across the country rom north to south and from west to east. The long decline in water supply eginning with the turn of the century has been first detected along the northern order from Maine (Fig. 4) through the Great Lakes to Washington. The down-rend seems to have broken first in the northeast in 1927, in the Great Lakes n 1942, and in the northwest United States in 1945. The uptrend of the Missis-ippi River at St. Louis, Mo. in 1942 (shown also at Vicksburg, Miss., in Fig. 6) eflects increased supplies in the general region of the Great Lakes. The umber of records is too small to predict the trend, but it appears that in the outhwestern and south central United States a return to normal or above water upplies can be anticipated, perhaps because of an expected decline in solar ctivity. It is interesting to note that the drought of the 1930's, which affected ost of the United States, except the extreme northeast and the southwest, be-an with an upturn in the long-term sunspot cycle.

The deficiencies in water supplies since the beginning of the twentieth entury have been world-wide, but there are records that exhibit a contrary rend. The 92-yr runoff record from the 102,000 sq-mile drainage area of he upper Indus River at Attock in West Pakistan (see Fig. 8) averaged above ormal from 1904-1945, then was deficient for 12 yr, and finally returned to n above normal cycle in 1955. This record was computed in 1960 from recent ischarge measurements and a gage-height record collected at one of the few table sections in the river. The record is confirmed by 38 yr of daily dis-harge measurements downstream. The drainage area is probably nearer the un than any large basin in the world as the bordering mountains rise 20,000 ft 25,000 ft above sea level. Most of the total runoff takes place from May to eptember and results from monsoon rainfalls in the lower valley, from melt-ng of the previous winter's snowfall, as well as melting from snow accumula-ons from other years and from glaciers. Annual variations in runoff may esult as much from advances and retreats of the snow-line and glaciers as om concurrent precipitation. Regardless of conjectures regarding the ources of annual runoff, it can be seen from Fig. 8 that the short-term peaks runoff have, in general, coincided with troughs in the annual sunspot cycle, ut the long-term trend in water-supply has followed the long-term trend in e sunspot numbers, in contrast to the inverse relationship that is more pre-lent in the other records investigated.

A comparison of records of either river runoff or precipitation that have own a resistance to the widespread downward trend in this century is pre-nted in Fig. 9. The trend of precipitation of Adelaide, Australia, is opposite that of Sidney, Australia, on the other side of the continent (see Fig. 10).

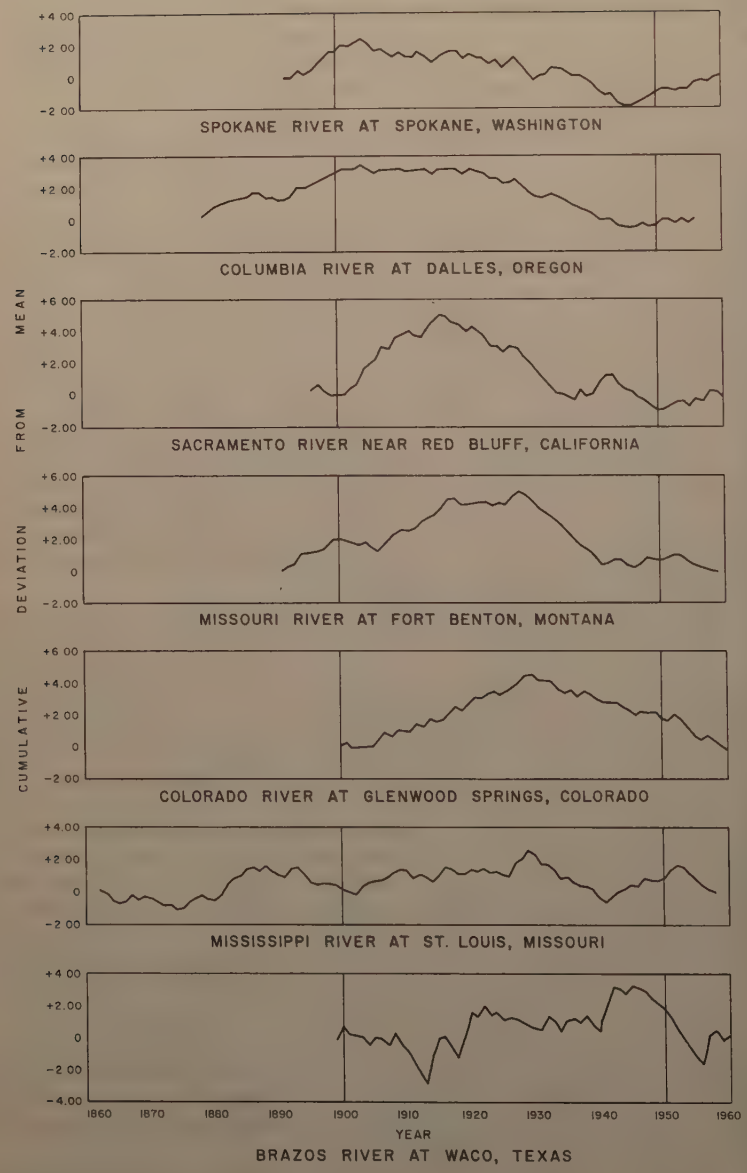

FIG. 7.—LONG TERM VARIATIONS IN RUNOFF FOR SELECTED RIVERS IN WESTERN UNITED STATES

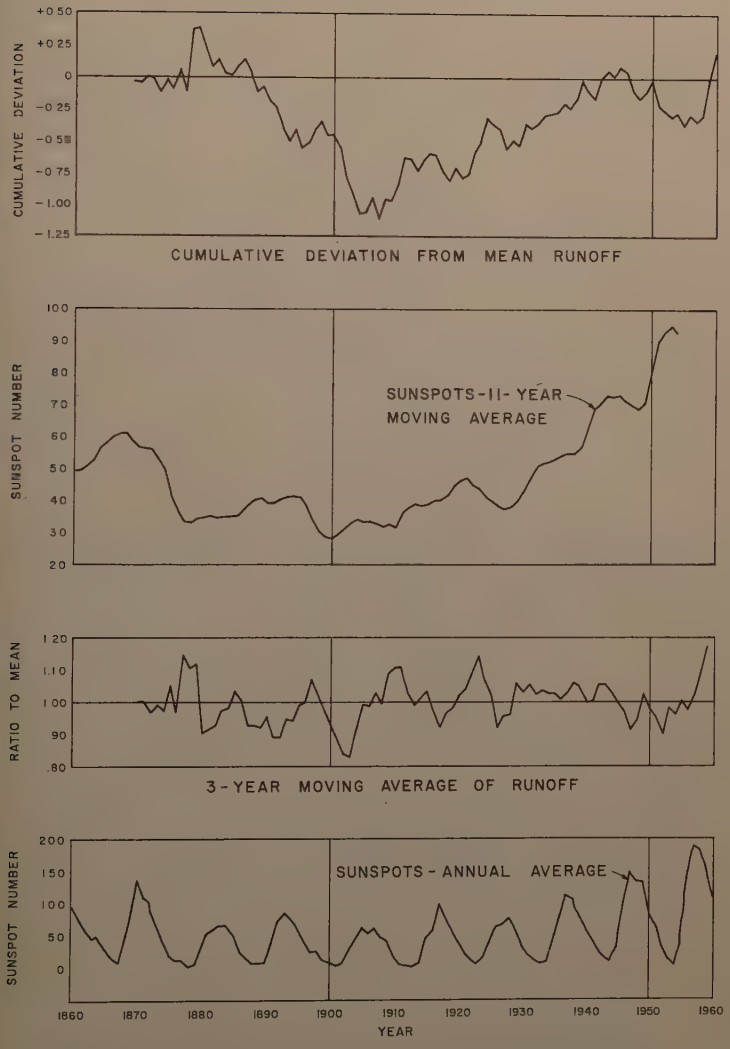

FIG. 8.—CYCLICAL VARIATIONS IN RUNOFF INDUS RIVER AT ATTOCK, WEST PAKISTAN

Each of the records is probably typical of a belt of longitude and latitude considerable extent, but it must remain for future studies to confirm thi

The selected records in Fig. 10 are plotted to show that thousands of mil separate places with trends that are similar, except for relatively small di

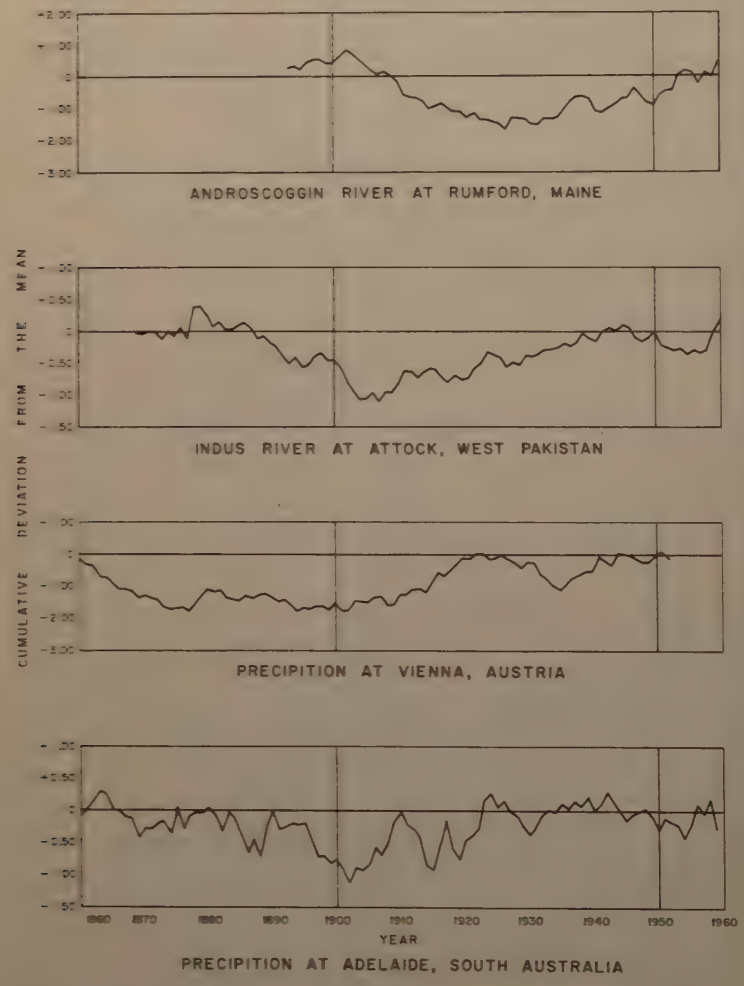

FIG. 9.—LONG TERM CYCLICAL VARIATIONS FOR SELECTED RIVERS AND PRECIPITATION STATIONS AROUND THE WORLD

placements in time. If it were not for the opposing trends in Fig. 9 and a in Fig. 7, it would be possible to arrive at the erroneous conclusion t moisture supplies over the entire surface of the earth varied in unison. It

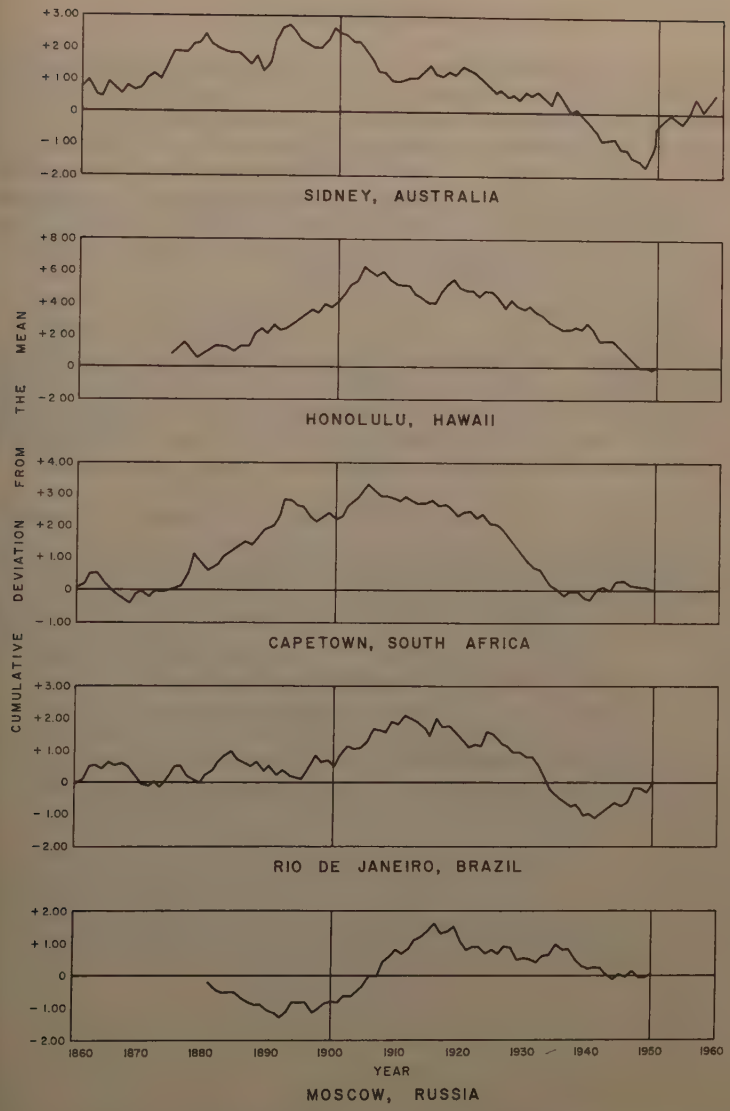

FIG. 10.—LONG TERM CYCLICAL VARIATIONS FOR SELECTED PRECIPITATION STATIONS AROUND THE WORLD

require analysis of perhaps hundreds of records before the limits of region and continental similarities can be defined.

CONCLUSIONS

There is an increasing mass of scientific evidence that solar activity the form of eruptions in the gaseous matter of the sun has immediate effec on the earth's atmosphere. Sunspots which can be seen at times of solar eru tions are visual but perhaps crude indices of this activity. Furthermore, sol activity is evidently cyclical as shown by the short-term cycles in sunsp numbers which have an average periodicity of about 11 yr. The numbers sunspots in the 11-yr peaks vary widely, but not entirely at random, becau 11-yr and 22-yr moving averages of the annual numbers reveal a quasi 100-y cycle superimposed on the short-term cycle.

If either precipitation or runoff data are expressed as ratios to the me and then smoothed by a 3-yr moving average, irregular short-term cyclic trends are revealed. The influence of the 11-yr sunspot cycle on short-ter hydrologic cycles is apparent from comparative plots, but other studies ha indicated that the statistical correlation is poor. One reason for the poor corr lation is that there are often twice as many hydrologic peaks as there a sunspot peaks, and the highest hydrologic peak may coincide with a sunsp trough.

If cumulative deviations from the mean (differential mass curves) are co puted for hydrologic data, continuous periods of 10 yr to 35 yr or more w be revealed in which hydrologic records are consistently below or above the means. These long-term hydrologic trends often coincide in length with lon term changes in sunspot numbers but are usually, but not always, opposite direction to the trend in the numbers. For example, there was a general u ward trend in average sunspot numbers from the beginning to the middle of t twentieth century, but in that same period most of the hydrologic reco analyzed for this paper were consistently below their long-term mea

It is hoped that many more studies of this type will be undertaken and th the results will be of assistance in long-range water resources plannin

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GENERAL SOLUTION FOR OPEN CHANNEL PROFILES

By James A. Liggett[1]

SYNOPSIS

A method of computing open channel profiles that may be used quite generally is presented. This method lends itself either to hand calculation or to calculation on the digital computer. Numerical examples are presented to illustrate the procedure.

INTRODUCTION

The classical open channel profile problem is well known and within the reach of the practicing engineer. However, the classical solutions do not allow for the variation of some of the parameters such as inflow or outflow along the channel or variations in channel width or cross-section. Also, many of the present methods are troublesome near points of critical depth. It is the object of this paper to present a more general method for profile calculation. Although the method is evidently somewhat more laborious than the tabular meth-

Note.—Discussion open until April 1, 1962. Separate discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 6, November, 1961.

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ods now in use, it is much more general and accuracy is maintained throug out the calculation. The method lends itself to digital computer operation.

DIFFERENTIAL EQUATIONS OF ONE-DIMENSIONAL, OPEN CHANNEL FLOW

Notation.—The letter symbols adopted for use in this paper are defin where they first appear and are arranged alphabetically, for convenience reference, in Appendix II.

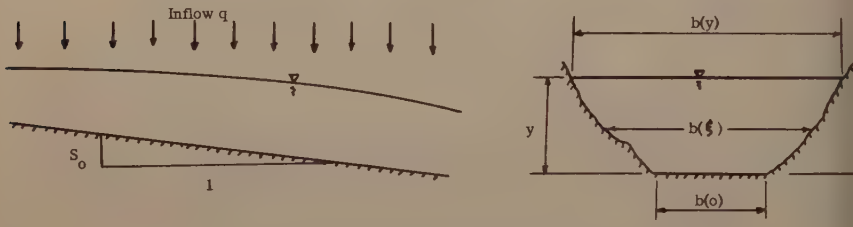


FIG. 1.—CHANNEL WITH SPATIALLY VARIED FLOW

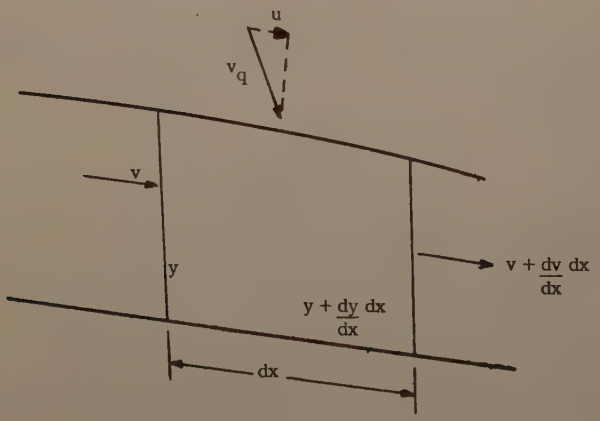


FIG. 2.—DIFFERENTIAL ELEMENT

Fig. 1 shows the general channel under consideration and also gives a d inition of some of the symbols that are to be used. The channel slope is sumed small so that distances measured either parallel to the channel bott or horizontally are essentially equal.

Fig. 2 shows a differential element removed from the channel. The eq tion of continuity is derived by equating the inflow and outflow of this elem

n equation form

$$A\ v + q\ dx = \left(A + \frac{dA}{dx}\ dx\right)\left(v + \frac{dv}{dx}\ dx\right) \quad \ldots\ldots\ldots\ldots (1)$$

r

$$\frac{d}{dx}\ (A\ v) = q \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

n which A is the cross-sectional area, q the lateral inflow in cfs per ft of ength, v the average velocity, and x the downstream distance.

The equation of motion is derived by considering the momentum transport nto and out of the element. The rate of momentum in is equal to the rate of nomentum out minus the sum of the forces acting on the element. In equation orm

$$\rho\ v^2\ A + \rho\ q\ u\ dx = \rho\left(v + \frac{dv}{dx}\ dx\right)^2\left(A + \frac{dA}{dx}\ dx\right) - dF_p$$

$$- \left(S_o\ g\ \rho\ A - S_f\ g\ \rho\ A\right) dx \quad \ldots\ldots\ldots\ldots (3)$$

n which u is the x-component of the velocity of the inflow, S_o is the channel lope, and S_f a resisting "friction slope." The term dF_p is the contribution of he pressure force. (A derivation of the term dF_p is given in Appendix I. This erivation is omitted here to preserve the continuity of the general thought.) implifying and retaining only the terms of first order in dx, Eq. 3 becomes

$$\rho\ q\ u = 2\ \rho\ v\ \frac{dv}{dx}\ A + \rho\ v^2\ \frac{dA}{dx} - \frac{dF_p}{dx}\ g\ \rho\ A\left(S_o - S_f\right) \ldots\ldots (4)$$

he equation of motion may then be written as (using Eq. 55)

$$\rho\ q\ u = \rho\ v\ A\ \frac{dv}{dx} + \rho\ v\ \frac{d}{dx}\ (A\ v) + \rho g\ A\ \frac{dy}{dx} - \rho\ g\ A\left(S_o - S_f\right) \ldots (5)$$

ubstitution of the continuity equation for the second term on the right side nd dividing by ρ A yields

$$v\ \frac{dv}{dx} + g\ \frac{dy}{dx} = g\left(S_o - S_f\right) + \frac{q\ u}{A} - \frac{q\ v}{A} \quad \ldots\ldots\ldots (6)$$

he term S_f may be given by any one of the many empirical relations. For xample, using the Chezy equation, $S_f = \frac{v^2}{C^2 R}$, in which R is the hydraulic ra- us and C is the Chezy coefficient.

WORKING EQUATIONS

The usual procedure from this point is to integrate the continuity equation q. 2) and then to eliminate v in Eq. 6. The result is a difficult ordinary dif-

ferential equation—especially under conditions where the equation conta singularities. Therefore, the following procedure is adopted. Both Eq. 2 a Eq. 6 are integrated separately yielding

$$A\ v = \int q\ dx + \text{constant} \quad \ldots\ldots\ldots\ldots$$

and

$$\frac{1}{2} v^2 + g\ y = g \int \left(S_o - S_f\right) dx + \int \frac{q}{A} (u - v)\ dx + \text{constant} \quad \ldots\ldots$$

Because there are two constants of integration, two boundary conditions m be specified. The boundary conditions may take many different forms. T following cases are all acceptable boundary conditions:

(a) The velocity and depth may be specified at one point. This condit would be a logical choice for flow under a sluice gate, for example.

(b) The occurrence of critical depth $\left(v = \sqrt{g\ y}\right.$ for rectangular channels always a boundary condition. This condition is used in flow over a sudden dr

(c) Although stage-discharge relationships are often not at controls, t sometimes provide acceptable boundary conditions.

All of the preceding and many other conditions may be used.

The procedure for solution is to eliminate the velocity in Eq. 8 through use of Eq. 7 giving

$$\frac{\left(\int_o^x q\ (\xi)\ d\xi + A_o\ v_o\right)^2}{2\ A^2} + g\ y = \int_o^x \left[g\left(S_o - S_f\right) + \frac{q}{A}(u - v)\right] d\xi$$

$$+ \frac{1}{2} v_o^2 + g\ y_o \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

Eq. 9 assumes that the boundary conditions are specified at x = 0. For the c of one boundary condition specified at x = 0 and the other at x = L Eq. 8 co be written

$$\frac{\left(\int_o^x q\ (\xi)\ d\xi + A_o\ v_o\right)^2}{2\ A^2} + g\ y = \frac{1}{2} v_L^2 + g\ y_L - \int_x^L \left[g\left(S_o - S_f\right)\right.$$

$$\left. + \frac{q}{A}(u - v)\right] d\xi \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

Eq. 9 (or Eq. 10) may now be solved using an iteration procedure. Firs trial solution is specified of the form $v_1 = v_1(x)$ and $y_1 = y_1(x)$ in which subscripts indicate the number of the trial. This trial solution is used to tegrate the right side of Eq. 9. Because the left side is a function of y o

e trial solution gives a new function $y_2 = y_2(x)$ and $v_2 = v_2(x)$ (using Eq. 7 so). In general

$$\frac{\left(\int_0^x q\,(\xi)\,d\xi + A_o\,v_o\right)^2}{2\,A_n^2} + g\,y_n = \int_0^x \left[g\left(S_o - S_{f_{n-1}}\right) + \frac{q}{A_{n-1}}\left(u - v_{n-1}\right)\right] d\xi + \frac{1}{2}v_o^2 + g\,y_o = P_{n-1}(x) \quad \ldots\ldots (11)$$

which the subscript n refers to the nth trial solution. Convergence is reached hen the nth iteration is sufficiently close to the n - 1 iteration or that

$$\left|1 - \frac{P_n(x)}{P_{n-1}(x)}\right| < \epsilon \quad \ldots\ldots\ldots\ldots\ldots (12)$$

which ϵ is some small number.

COMPUTER METHOD OF CALCULATION

After substitution of a trial solution into Eq. 9 (or Eq. 10) an equation of the llowing form is obtained.

$$\frac{\left[Q\,(x)\right]^2}{2\left[A\,(y_n)\right]^2} + g\,y_n = P_{n-1}(x) \quad \ldots\ldots\ldots\ldots (13)$$

which the values of $Q(x)$ and $P_{n-1}(x)$ are stored for each increment in x. . 13 may be solved using Newton's method of successive approximations. t

$$f\,(y_n) = \frac{1}{2}\frac{Q^2}{\left[A\,(y_n)\right]^2} + g\,y_n - P_{n-1} = 0 \quad \ldots\ldots\ldots (14)$$

en

$$f'\,(y_n) = -\frac{Q^2}{\left[A\,(y_n)\right]^3}\frac{dA}{dy} + g = -\frac{Q^2}{\left[A\,(y_n)\right]^3}\,b\,(y_n) + g \quad \ldots\ldots (15)$$

which the prime denotes differentiation. From Eq. 14 it can be seen that $(x_1) \leq \frac{1}{g} P_{n-1}(x_1)$. Also for subcritical flow and a channel the sides of ch do not converge upward $f'(y_n) \leq g$ and approaches g as $y_n \to \infty$. There- e, $y_n^{(1)} = \frac{1}{g} P_{n-1}$ is a good first value to use in Newton's method for find-

ing $y_n(x_1)$. Then

$$y^{(2)} = y^{(1)} - \frac{f\left(y^{(1)}\right)}{f'\left(y^{(1)}\right)} \quad \ldots\ldots\ldots\ldots$$

and in general

$$y^{(i)} = y^{(i-1)} - \frac{f\left(y^{(i-1)}\right)}{f'\left(y^{(i-1)}\right)} \quad \ldots\ldots\ldots\ldots$$

The method is the same for finding subsequent values of y (x) except tha better first guess would be the y value of the previous x coordinate. Tha

$$y^{(1)}(x_k) = y(x_{k-1}) \quad \ldots\ldots\ldots\ldots$$

makes a good value to use in Eq. 16.

In order to carry out the calculation, b = b (x, ξ) needs to be specif From this equation A = A (x, ξ) may be derived, or it may be more conven to specify A = A (x, ξ) separately. The method used to specify these funct is left open so that the method that seems most appropriate may be adop Usually it is most convenient to specify these as explicit functions of the v iables x and ξ where this is possible, that is, in cases where the channel sh is constant throughout its length or can be given by a simple equation. In case of a highly irregular channel it is possible only to give b (x, ξ) in the f of a table or array such as

$$[B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1k} & \cdots & b_{1K} \\ b_{21} & & & & & & \\ b_{31} & & & & & & \\ \vdots & & & & & & \\ b_{j1} & & & & b_{jk} & & \\ \vdots & & & & & & \\ b_{J1} & & & & & & b_{JK} \end{bmatrix} \quad \ldots\ldots\ldots$$

Each column represents a different value of x and the elements within a col are width values as a function of depth. Thus, an array of this type would K stations along the channel and J points on the depth-width curve at each tion. The value of the width (or the integral of the width, the area) as a f tion of depth and length can be specified to the computer by such an array a table look-up and interpolation routine.

PROCEDURE FOR HAND CALCULATION

When not using a computer, a graphical method can be used which facilit finding the values of v_n (x) and y_n (x) for each iteration. The quantity

$$z = \sqrt{2\,g\,y} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

introduced to replace the depth. Eq. 11 is written

$$v_n^2 + z_n^2 = 2 \int_0^x \left[g\left(S_o - S_{f_{n-1}}\right) + \frac{q}{A_{n-1}}\left(u - v_{n-1}\right) \right] d\zeta + v_o^2 + z_o^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

he (v, z) plane is prepared by plotting v as ordinate against z as abscissa us-g x as a parameter as in Eq. 7. That is, the area is eliminated in Eq. 7 in

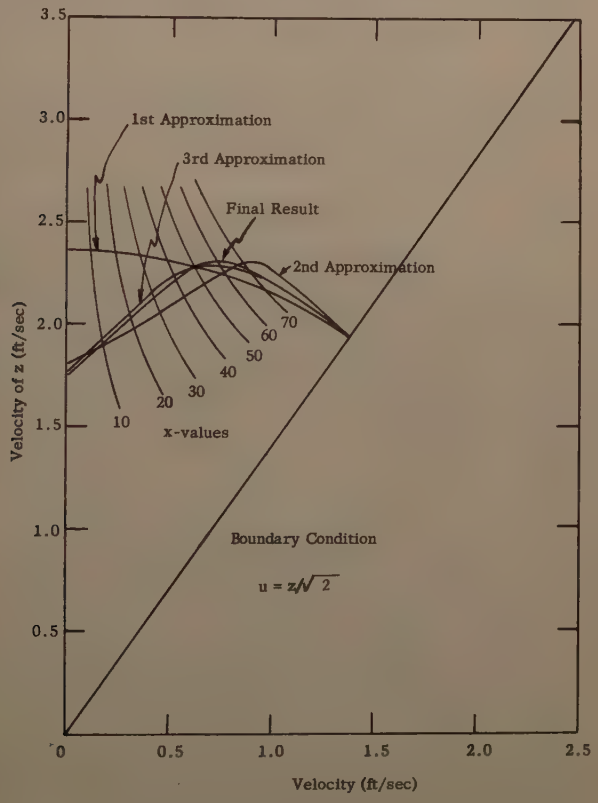


FIG. 3.—(v,z) PLANE FOR HAND CALCULATION

vor of z and a line representing this equation is drawn for each increment of The boundary conditions are also located on this plane so that any of the al solutions will satisfy the boundary conditions. An example of the (v, z) ane is shown in Fig. 3.

When the (v, z) plane has been prepared, a trial solution is plotted on t graph. Eq. 21 now takes the form

$$v_2^2 + z_2^2 = 2\ P_1\ (x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (22)$$

For each x line plotted according to Eq. 7 the values of v_2 and z_2 may be r directly. These values represent a new trial solution and are substituted b into Eq. 21. The integration must be done graphically or by some other merical method. The result is

$$v_3^2 + z_3^2 = 2\ P_2\ (x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (23)$$

This function can easily be plotted on the (v, z) plane because the equatio in the form of a circle for P_2 a constant. However, because P_2 is a funct only of parameter x, Eq. 23 may be plotted by swinging arcs from the origi radius $r = \sqrt{2\ P_2}$. Points are established where these arcs intersect the li of Eq. 7 for that particular value of x. Thus, the third trial solution is plott This procedure is continued until there is no essential change in the trial utions.

NUMERICAL EXAMPLES

The preceeding methods will be illustrated by using several example numerical calculation in varying degrees of detail.

Hand Calculation for Spatially Varied Flow Over a Plane.—In order to lustrate the method of solution without the use of a digital computer, the pr lem of spatially varied flow over a plane (two dimensional flow) has been solv The constants of the problem are

$q = 0.001$ cfs per sq ft $\qquad L = 80$ ft

$S_o = 0.001$ $\qquad C = 100$

with the boundary conditions that there is no inflow to the upper end and the channel ends in a sudden drop (critical depth) which means that $v = z$ at $x = L$ and $a = 0$. Eq. 21 is

$$v^2 + z^2 = 3\ v_L^2 - 2\ g \int_x^L \left(S_o - \frac{2\ g}{C^2} \frac{v^2}{z^2} \right) dx \quad \ldots\ldots\ldots\ (24)$$

The terms containing the momentum transfer between the inflow and the w flowing in the channel have been neglected. The velocity at the end (v_L evaluated by using the equation of continuity. From Eq. 7

$$v_L\ y_L = q\ L \quad \ldots\ldots\ldots\ldots\ldots\ (25)$$

or, because

$$y_L = \frac{z_L^2}{2\ g} = \frac{v_L^2}{g} \quad \ldots\ldots\ldots\ldots\ldots\ (26)$$

om the boundary condition

$$v_L = (g\ q\ L)^{1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

To begin the solution the (v, z) plane is prepared (Fig. 3). The boundary ndition is plotted as a straight line and the point x = L may be found where is line passes through the value of v_L as obtained from Eq. 27. The lines

$$v_L\ z_L^2 = 2\ g\ q\ x \quad \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

e plotted in increments in x of 10 ft. When these lines have been plotted, rial solution may be drawn on the (v, z) plane. Of course, this solution must ss through the previously established point at the downstream boundary. For e present solution the first trial was to take the right side of Eq. 24 as a nstant equal to $3\ v_L^2$. This equation then represents a circle on the (v, z) ane so that

$$\left[v_1\ (x)\right]^2 + \left[z_1\ (x)\right]^2 = 3\ v_L^2 = R_1^2 = \text{CONSTANT} \quad \ldots\ldots (29)$$

which the subscript 1 refers to the first approximation. This circle is la- led "1st approximation" in Fig. 3.

The next step is to use the (v, z) plane to prepare a curve of $(v/z)^2$ versus Such a curve has been plotted in Fig. 4 and labeled as "first approximation." ing the curve thus obtained, the right side of Eq. 24 may be graphically in- grated yielding

$$\left[v_2\ (x)\right]^2 + \left[z_2\ (x)\right]^2 = 3\ v_L^3 - 2\ g \int_x^L \left(S_o - \frac{2\ g}{C^2}\frac{v_1^2}{z_1^2}\right) dx$$

$$= \left[R_2\ (x)\right]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

hough the right side of Eq. 30 is not a constant, it may be solved on the (v, z) ne by simply swinging an arc of radius R until it intersects the curve la- ed with that particular value of x. The points thus obtained are used to plot line labeled "2nd approximation." The second approximation of $(v/z)^2$ sus x is now plotted and the entire process repeated twice more. Because solution changed little between the third and fourth solutions, the fourth ution was taken to be final. The profile curve and wave velocity $\left(c = \sqrt{g\ y}\right)$ m the final result are plotted in Fig. 5.

Computer Calculation for Spatially Varied Flow Over a Plane.—Spatially ied flow over a plane is calculated using a digital computer with the con- nts

q = 0.0115 cfs per sq ft	L = 1200 ft
S_o = 0.002	C = 40

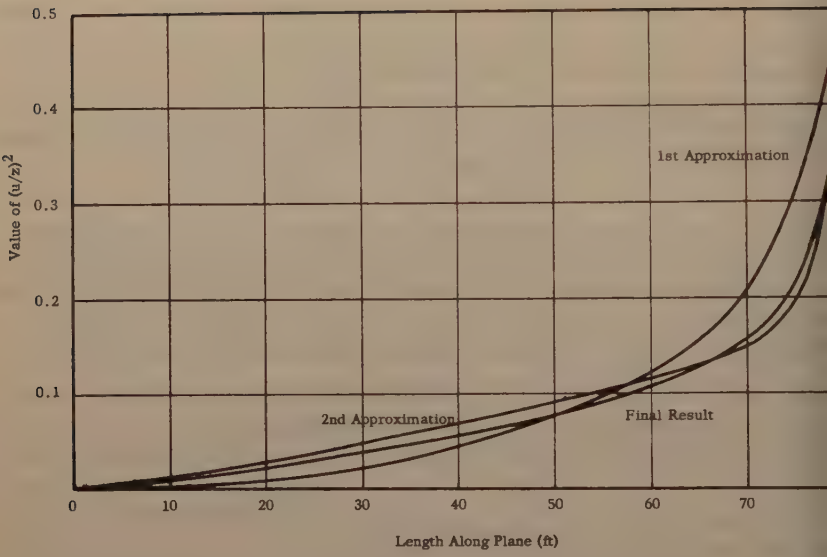


FIG. 4.—PLOT OF THE TERM $\left(\frac{v}{z}\right)^2$

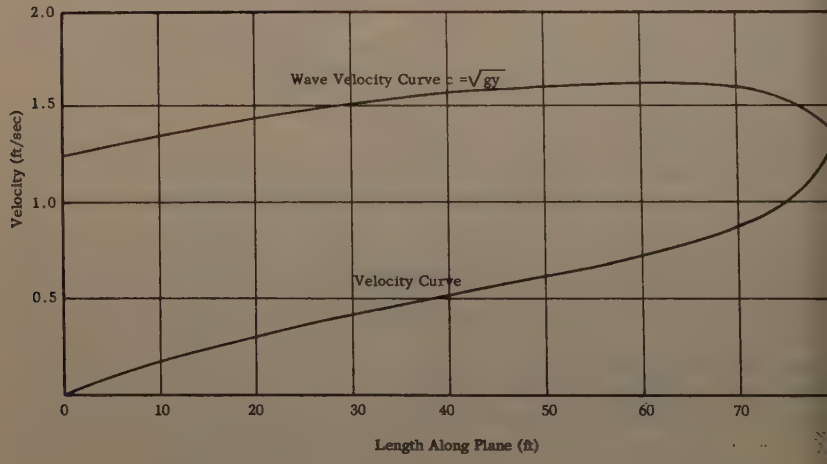


FIG. 5.—VELOCITY AND WAVE VELOCITY CURVES FROM THE HAND CALCULATION

he boundary conditions will be $v = \sqrt{g\,y}$ at $x = L$ and $Q = 0$ at $x = 0$. Eq. 10 is ritten

$$\frac{(q\,x)^2}{2\,y^2} + g\,y = \frac{v_L^2}{2} + g\,y_L - g\int_0^L \left(S_o - \frac{v^2}{C^2\,y}\right) dx \quad \ldots\ldots (31)$$

gain the momentum transfer from the incoming flow is neglected.
The first approximation is

$$\int_x^L \left(S_o - \frac{v^2}{C^2\,y}\right) dx \equiv 0 \quad \ldots\ldots\ldots\ldots (32)$$

us, for the data used Eq. 31 becomes

$$\frac{(0.0115\,x)^2}{2\,y_1\,(x)} + 32.2\,y_1\,(x) = 87.35 \quad \ldots\ldots\ldots (33)$$

d succeeding approximations are

$$\frac{(0.0115\,x)^2}{2\,y_n\,(x)} + 32.2\,y_n\,(x) = 87.35 - 32.2\int_x^L \left[0.002 - \left(\frac{v_{n-1}\,(x)}{1600\,y_{n-1}\,(x)}\right)^2\right] dx \quad \ldots\ldots\ldots\ldots (34)$$

In order to see how the right side of Eq. 31 converged in this case, the com-ter was instructed to print out the right side of Eq. 34 for each iteration. It ok a total of seven intermediate iterations before the tolerance limit on the nvergence was reached. The right side of Eq. 34 is plotted in Fig. 6 for each ration. In this case (as in all cases thus far completed) the value of the al iteration is very nearly given by

$$P_n\,(x) \approx \frac{1}{3}\left[P_{n-2}\,(x) + 2\,P_{n-1}\,(x)\right] \quad \ldots\ldots\ldots (35)$$

is observation was used to accelerate convergence of the computer program case 4 (given subsequently).

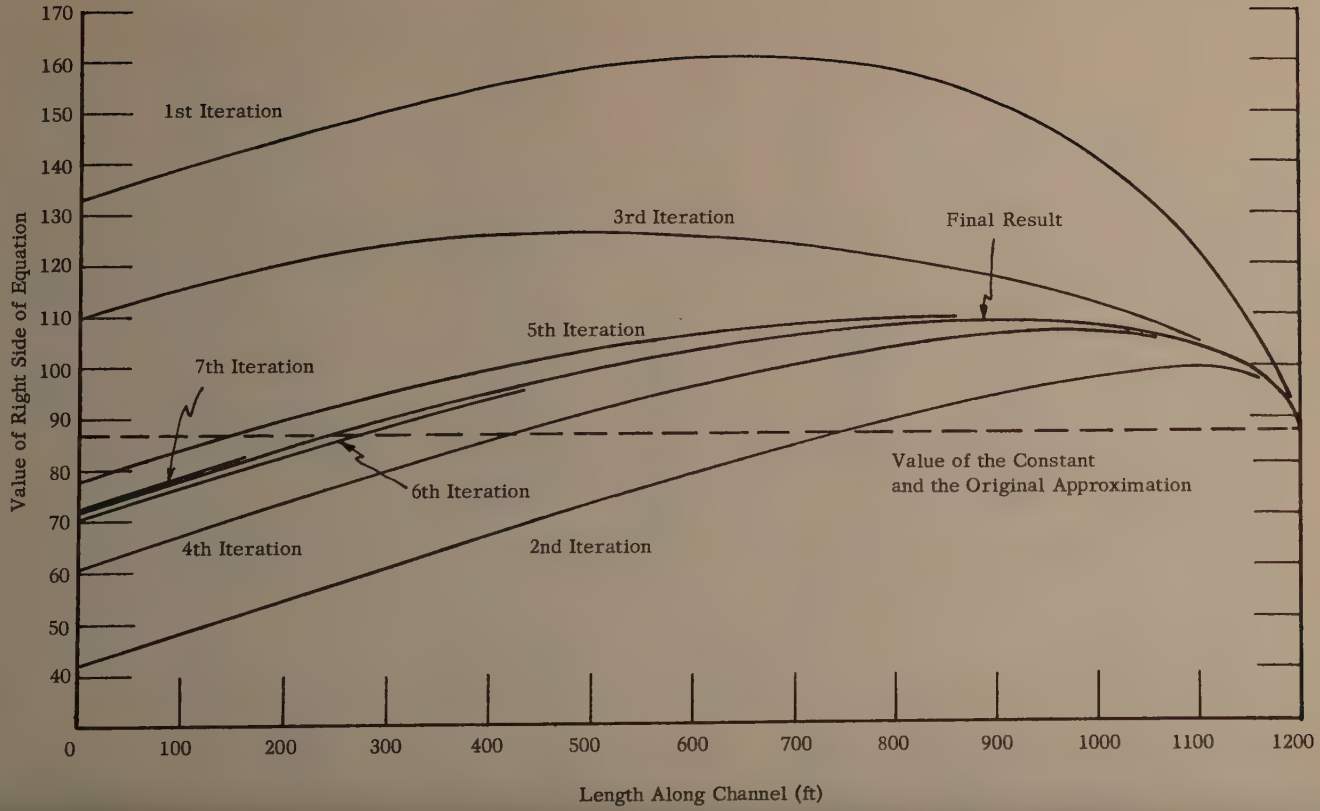
The profile and velocity curves for these data are plotted in Fig. 7.

Flow in a Trapezoidal Channel.—Flow in a trapezoidal channel of 10 ft bot-n with 1:1 side slopes was solved by means of the computer for the data C 0, L = 1200 ft, and S = 0.001. Four different cases were treated as follows:

Case 1. Inflow used in this case was q = 0.0115 cfs per ft. Eq. 10 was solved npletely.

Case 2. Inflow used in this case was q = 0.115 cfs per ft. Eq. 10 was solved npletely.

Case 3. Inflow used in this case was q = 0.115 cfs per ft. The momentum nsfer terms in Eq. 10 were neglected.


170
160
150
140
130
120
110
100
90
80
70
60
50
40
Value of Right Side of Equation
1st Iteration
3rd Iteration
Final Result
5th Iteration
7th Iteration
6th Iteration
Value of the Constant
and the Original Approximation
4th Iteration
2nd Iteration
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
Length Along Channel (ft)

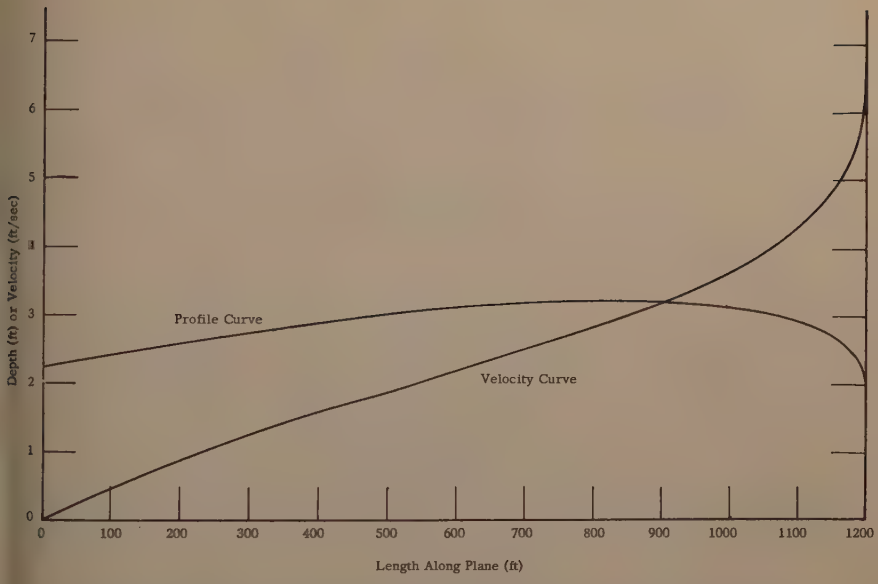


FIG. 7.—PROFILE AND VELOCITY CURVES FOR FLOW OVER A PLANE

Case 4. The flow through the channel is constant at 138 cfs (the same as the flow at x = L for cases 2 and 3).

For case 1, Eq. 10 becomes for each iteration

$$\frac{(0.0115\ x)^2}{2\ A_n^2} + g\ y_n = 18.35 - \int_x^L \left[32.2 \left(0.001 - \frac{v_{n-1}^2}{1600\ R_{n-1}} \right) + \frac{0.0115\ v_{n-1}}{A_{n-1}} \right] dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

in which

$$A = 10\ y + y^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

and

$$R = \frac{10\ y + y^2}{10 + 2.828\ y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

For case 2

$$\frac{(0.0115\ x)^2}{2\ A_n^2} + g\ y_n = 18.35 - \int_x^L \left[32.2 \left(0.001 - \frac{v_{n-1}^2}{1600\ R_{n-1}} \right) + \frac{0.0115\ v_{n-1}}{A_{n-1}} \right] dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (3$$

For case 3

$$\frac{(0.115\ x)^2}{2\ A_n^2} + g\ y_n = 78.81 - \int_x^L \left[32.2 \left(0.001 - \frac{v_{n-1}^2}{1600\ R_{n-1}} \right) + \frac{0.115\ v_{n-1}}{A_{n-1}} \right] dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (4$$

and for case 4

$$\frac{(0.115\ x)^2}{2\ A_n^2} + g\ y_n = 78.81 - \int_x^L \left[32.2 \left(0.001 - \frac{v_{n-1}^2}{1600\ R_{n-1}} \right) \right] dx \ldots (4$$

The resulting profile curves are plotted in Fig. 8 and the velocity curves Fig. 9. As can be seen from Fig. 8 the profile curves obtained from the equ tions which neglect the momentum term are in considerable error. It is i teresting to note that the profile curve first rises then falls when the mome tum term is neglected whereas the curve begins falling immediately with t exact equation.

APPENDIX I.—DERIVATION OF THE TERM dF_p

The force in the downstream direction due to pressure (assuming a hydr static pressure distribution) is given by

$$\Delta F_p = \gamma \int_0^{y_1} (y_1 - \xi)\, b(x_1, \xi)\, d\xi - \gamma \int_0^{y_2} (y_2$$

$$- \xi)\, b(x_2, \xi)\, d\xi + \gamma \int_0^{y} (y - \xi) \frac{\partial b}{\partial x} \Delta x\, d\xi \quad \ldots\ldots (4$$

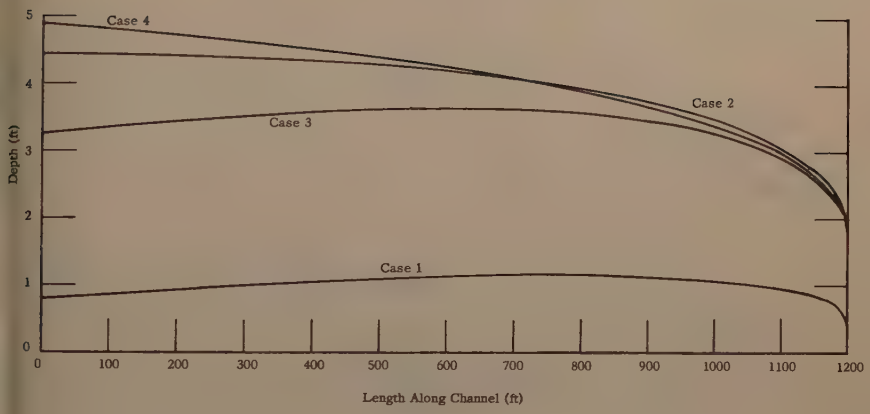


FIG. 8.—PROFILE CURVES FOR CHANNEL FLOW

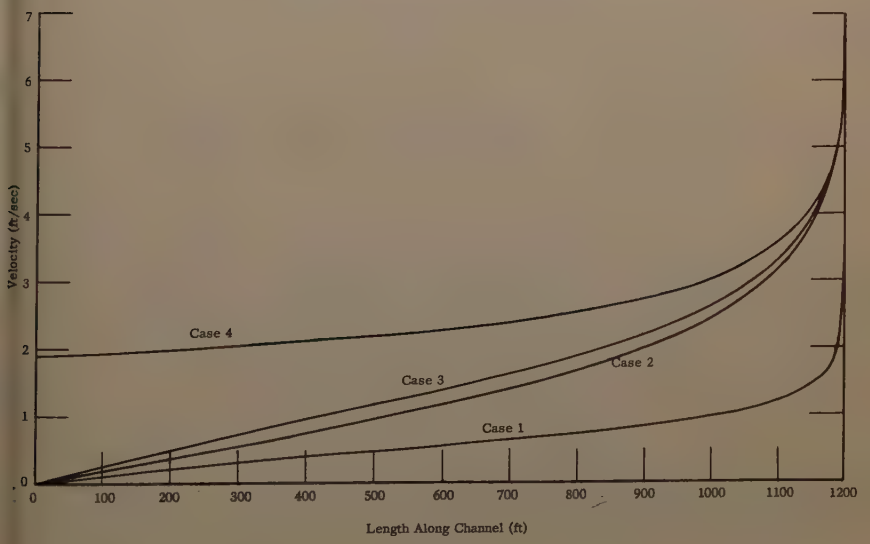


FIG. 9.—VELOCITY CURVES FOR CHANNEL FLOW

in which γ is the specific weight and the subscript 1 refers to the section at and the subscript 2 to the section at $x + \Delta x$. The first and second terms ar contributions from the ends of the sections. The last term is a contributio from the sides of the channel if the channel is not of constant width (that is, $\frac{\partial b}{\partial x} \neq 0$). Because Δx is taken to be small

$$y_2 = y_1 + \frac{dy}{dx} \Delta x \quad \ldots\ldots\ldots\ldots\ldots\ldots (43$$

and

$$x_2 = x_1 + \Delta x \quad \ldots\ldots\ldots\ldots\ldots\ldots (44$$

Consider the second integral

$$\gamma \int_0^{y_2} (y_2 - \xi)\, b(x_2, \xi)\, d\xi = \gamma \int_0^{y_2} (y_1 - \xi$$

$$+ \frac{dy}{dx} \Delta x)\, b(x_2, \xi)\, d\xi \quad \ldots\ldots\ldots\ldots\ldots\ldots (45$$

But

$$b(x_2, \xi) = b(x_1, \xi) + \frac{\partial b}{\partial x} \Delta x \quad \ldots\ldots\ldots\ldots\ldots\ldots (46$$

Therefore, the right side of Eq. 45 becomes

$$\gamma \int_0^{y_2} (y_1 - \xi)\, b(x_1, \xi)\, d\xi + \int_0^{y_2} \gamma \left[\frac{dy}{dx} b(x_1, \xi) \right.$$

$$\left. + (y - \xi) \frac{\partial b}{\partial x} \right] \Delta x\, d\xi \quad \ldots\ldots\ldots\ldots\ldots\ldots (4$$

when carried out to the first order in Δx. The first term combines with th first term of Eq. 42 to form the expression

$$\gamma \int_{y_1}^{y_2} (y - \xi)\, b(x, \xi)\, d\xi \quad \ldots\ldots\ldots\ldots\ldots\ldots (4$$

Here the subscript 1 has been dropped. When the integrand is expanded in Taylor series this integral is of the type

$$\int_a^{a+\epsilon} f(\xi)\, d\xi = \int_a^{a+\epsilon} \left[f(a) + f^1(a)(\xi - a) + \frac{f^{11}(a)}{2} (\xi - a)^2 \right.$$

$$\left. + \ldots \right] d\xi \quad \ldots\ldots\ldots\ldots\ldots\ldots (4$$

Integrating

$$\int_{a}^{a+\epsilon} f(\xi)\, d\xi = \epsilon\, f(a) + \ldots \quad \ldots\ldots\ldots\ldots (50)$$

in which the dots indicate terms that are at least of the order ϵ^2. Applying this result to Eq. 48 yields

$$\gamma \int_{y}^{y+\frac{dy}{dx}\Delta x} (y - \xi)\, b\,(x, \xi)\, d\xi = 0 + \ldots \quad \ldots\ldots\ldots (51)$$

because f (y) is zero in this case.

Thus, Eq. 42 has been reduced to

$$\Delta F_p = -\gamma \int_{0}^{y+\frac{dy}{dx}\Delta x} \left(\frac{dy}{dx}\, b\,(x, \xi) + (y - \xi)\,\frac{\partial b}{\partial x}\right)\Delta x\, d\xi + \gamma \int_{0}^{y} (y - \xi)\,\frac{\partial b}{\partial x}\,\Delta x\, d\xi \quad \ldots\ldots\ldots\ldots (52)$$

or

$$\Delta F_p = -\gamma \int_{0}^{y} \left(\frac{dy}{dx}\, b\,(x, \xi) + (y - \xi)\,\frac{\partial b}{\partial x}\right)\Delta x\, d\xi - \int_{y}^{y+\frac{dy}{dx}\Delta x} \left(\frac{dy}{dx}\, b\,(x, \xi) + (y - \xi)\,\frac{\partial b}{\partial x}\right)\Delta x\, d\xi + \gamma \int_{0}^{y} (y - \xi)\,\frac{\partial b}{\partial x}\,\Delta x\, d\xi \quad \ldots\ldots (53)$$

The second term of the first integral cancels with the last integral. The second integral is at least of the order of $(\Delta x)^2$ and can be neglected. This leaves

$$\Delta F_p = -\gamma \int_{0}^{y} \frac{dy}{dx}\, b\,(x, \xi)\,\Delta x\, d\xi \quad \ldots\ldots\ldots\ldots (54)$$

The derivative of y and x can be moved out from under the integral sign and the remainder is simply the area. Thus

$$\Delta F_p = -\gamma\, A\, \frac{dy}{dx}\,\Delta x \quad \ldots\ldots\ldots\ldots\ldots (55)$$

APPENDIX II.—NOTATION

A = Cross-sectional area of the wetted channel;

a = constant of integration $a = Q$ at $x = 0$;

$b(\xi)$ = width of the channel at the distance ξ above the bottom;

b_o or $b(0)$ = bottom width of the channel;

b or $b(y)$ = width of the channel at the water surface;

C = Chezy coefficient;

c = velocity of infinitesimal gravity waves $\left(c = \sqrt{g\ y}\right)$;

g = acceleration of gravity;

i = superscript denoting the number of the iteration in Newton's method for finding the roots of a cubic equation;

J = subscript denoting the last ξ value in the table of width versus depth;

j = subscript denoting the particular ξ value;

K = subscript denoting the last x-coordinate;

k = subscript denoting the particular x-coordinate;

n = subscript denoting the number of iterations used in evaluating the integral;

P or $P(x)$ = the right side of the integral equation;

q or $q(x)$ = inflow in cubic feet per second per foot of length or cubic feet per second per square foot in the case of flow over a plane;

R = hydraulic radius;

r = radius of arcs used in the graphical solution;

S_f = "friction slope";

S_o = channel slope;

u = horizontal velocity of the inflow;

v = velocity of flow in the channel;

v_o or v_L = boundary values of velocity at $x = 0$ or $x = L$;

x = distance along the channel;

y = depth of water in the channel;

y_o or y_L = boundary values of depth at $x = 0$ or $x = L$;

z = derived variable $\left(z = \sqrt{2\ g\ y}\right)$;

γ = specific weight of the fluid flowing;

ξ = vertical distance from the bottom of the channel; and

ξ' = variable of integration.

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Proceedings of the American Society of Civil Engineers

STABILITY OF ALLUVIAL CHANNELS

By Francis M. Henderson,[1] M. ASCE

SYNOPSIS

By making a slight extension to E. W. Lane's tractive-force theory of stable channel design, and by combining it with Strickler's formula, equations are deduced similar in form to G. Lacey's "regime" equations. In particular it is shown that Lacey's width-discharge relation, $P \propto \sqrt{Q}$, is true for the narrow channel developed in Lane's theory. It is pointed out that the similarity is of limited significance because the regime condition implies that the bed is live, whereas the tractive force criterion assumes that the bed is only on the threshold of motion.

The latter criterion requires that the shear stress, and hence RS, is constant at all points along the channel. Lacey's observations on the other hand were to the effect that $R^{\frac{1}{2}} S$ is constant along the channel, and it is shown by using the Einstein bed load function that this condition is fulfilled in a stable channel with a live bed.

These results are then applied to the consideration of longitudinal profiles of stable rivers and canals. It is pointed out that only two stable channel equations can be obtained from consideration of bed conditions and flow resistance, and that a third equation, such as Lacey's $P \propto \sqrt{Q}$, can be true only if there is a certain slope-discharge relationship, that is a certain longitudinal profile. The nature of this relationship is determined both for fixed (threshold) and live bed conditions, and the former is found to fit well to some data of L. B. Leopold, F. ASCE, and M. G. Wolman on natural rivers in coarse alluvium. Hence, it is argued that in such rivers the relations obtaining at the threshold

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of motion substantially determine the processes of natural river formatio
both in cross section and in longitudinal profile, and that the results obtain
herein may form a basis for a general attack on the problem.

INTRODUCTION

In 1930 Lacey published his "regime" theory[2] of the design of stable cha
nels in incoherent granular material. In this and subsequent papers[3,4] he pr
sented equations which purported to describe channels in which the bed w
live but stable, that is, sediment load was being supplied from upstream a
rate sufficient to balance any scour due to bed movement. In this condition t
channel was said to be "in regime"; no attempt was made in the original theo
to specify just how much sediment supply was needed to maintain the regir
condition.

Notation.—The letter symbols adopted for use in this paper are defined whe
they first appear, in the illustrations or in the text, and are arranged alph
betically, for convenience of reference, in the Appendix.

The equations were based on observations made on irrigation canals
India. The following three independent equations were derived:

$$v = 1.17 \sqrt{f\,R} \quad \text{(}$$

$$v = 16\, R^{\frac{2}{3}}\, S^{\frac{1}{3}} \quad \text{(}$$

and

$$P = 2.67 \sqrt{Q} \quad \text{(}$$

in which f is a "silt factor" related only to the size of the bed material, by t
equation

$$f = 8\sqrt{d} \quad \text{(}$$

in which d is the sediment size in inches. The term R refers to the hydrau
radius, S is the channel longitudinal slope, Q denotes the discharge, P describ
the wetted perimeter of the channel cross section, and v is the mean water v
locity. By making the substitution $v = Q/(P\,R)$, and eliminating P and R fr
the first three equations, a fourth equation can be derived

$$S = \frac{f^{\frac{5}{3}}}{1750\, Q^{\frac{1}{6}}} \quad \text{(5}$$

2 "Stable Channels in Alluvium," by G. Lacey, Proceedings, ICE, Vol. 229, 1929-
p. 259.

3 "Uniform Flow in Alluvial Rivers and Channels," by G. Lacey, Proceedings, I
Vol. 237, 1933-34, p. 421.

4 "A General Theory of Flow in Alluvium," by G. Lacey, Journal, ICE, Vol. 27, 19
p. 16.

The equation actually given by Lacey corresponding to Eq. 5a was

$$S = \frac{f^{\frac{3}{2}}}{2580\, Q^{\frac{1}{9}}} \qquad \text{(5b)}$$

This, however, was deduced from Manning's equation and presented[2] before Eq. 2 was developed in a later study.[3] Eq. 2 is, in essence, derived from a resistance equation in which the exponent of R is $\frac{1}{4}$, and not $\frac{2}{3}$ as in the Manning equation. The confusion between Eqs. 5a and 5b is due to the use of two different resistance formulas in different parts of the theory.

The points worthy of notice are that neither form of Eq. 5 can be independent of Eqs. 1, 2 and 3; and that Eq. 5a is consistent with these equations, whereas Eq. 5b is not. In fact Eq. 5a is also more consistent with the original canal data. For the one set of observations on which slope values were given by Lacey (the lower Chenab canal) a plot of S against Q shows a significantly better fit to Eq. 5a than to Eq. 5b.

The theory proved popular in the British Commonwealth countries, where it was (and still is) used in circumstances quite unlike those in which Lacey's original data were collected. Elsewhere its reception was less enthusiastic, and it was criticized on two principal grounds: That it took no account of the magnitude of the sediment charge, and that it was based on observations covering only a narrow range of silt sizes. The first objection has been partially met by subsequent work relating the silt factor to the sediment charge.

Although there has been much discussion in the literature on whether Eqs. 1 through 3 are right or wrong, less attention seems to have been paid to the question of how many independent equations there should be, and whether three are too many or too few.

More recently, other and more rational approaches to the problem of stable channel design have been devised. The United States Bureau of Reclamation Dept. of Interior (USBR)[5,6,7] presented methods based on the concept of the limiting tractive force at the threshold of motion, and Ning Chien[8] used the Einstein bed load function to plot charts from which channel dimensions could be obtained, given the discharge, slope, bed material and sediment load. Ning Chien also[9] re-examined Lacey's canal data and used the charts referred to previously, to determine the relationship between the Lacey silt factor and the sediment load.

The theory developed by the USBR is commonly referred to as the "tractive-force" theory, although a better name would perhaps be the "threshold" theory, because it postulates a granular bed on the threshold of motion. The use of the tractive force concept is not confined to the USBR theory, it is also implicit in Ning Chien's use of the Einstein bed-load function.

5 "Stable Channel Profiles," USBR, Hydr. Lab. Report Hyd - 325, Sept. 1951.

6 "Progress Report on Design of Stable Channels," USBR, Hydr. Lab. Report Hyd - 352, June, 1952.

7 "Critical Tractive Forces on Channel Side Slopes," USBR, Hydr. Lab. Report Hyd - 366, Feb., 1953.

8 "Graphical Design of Alluvial Channels," by Ning Chien, Transactions, ASCE, Vol. 121, 1956, p. 1267.

9 "A Concept of the Regime Theory," by Ning Chien, Transactions, ASCE, Vol. 122, 1957, p. 785.

On the one hand there is the Lacey regime theory, which is suspect for t reasons previously mentioned; on the other hand are the two major attemp at a more rational theory, one based on a stable bed at the threshold of motio the other on a "live bed" condition. The question remaining is whether one the other of the rational theories may be found to have something in comm with the Lacey theory. The work of D. B. Simons, M. ASCE, and M. L. Alber son,[10] F. ASCE, attempting to relate the USBR "threshold" theory to the Lac theory is indicative of the present interest in this problem.

It must be conceded that there is good reason for the continued interest the Lacey theory, despite all the criticisms levelled at it. The theory st commands respect because it appears to fit fairly well to river and canal da outside India, and outside the range of sediment sizes found in the canals fro which Lacey's original data were collected. In particular, the observations Leopold and Thomas Maddocks, Jr.,[11] F. ASCE, on rivers in the United Stat have furnished some evidence for the form of Eq. 3 although there is dou about the size of the coefficient, 2.67.

The purpose of this paper is to explore the "rational" theories to see wheth any theoretical justification can be found for any of the Lacey equations, a to see whether these equations have any feature that identifies them with eith the threshold or the live bed theory. Some comments will then be made on t determinacy of the problem, that is, on how many equations are necessary a sufficient for a complete solution.

This last question leads naturally into a discussion of the inter-relationsh between equations such as Eq. 3 and Eq. 5b, that is, between the channel secti and the longitudinal profile, and of the fact that adjustments to each of the cannot be independent, but must be regarded as part of the same process. this connection the evidence afforded by natural rivers is at least as interesti as that afforded by artificial canals, because natural rivers are more like than artificial canals to have completed the simultaneous adjustment of chan section and longitudinal profile. In fact, if the "rational" theories can provi a basis for such empirical formulas as Eq. 3—and, necessarily, Eq. 5a—th also provide a basis for understanding the processes of natural river form tion generally.

No new data will be presented. Reference will be made to the data of Lac Leopold and Maddocks, Simons and Albertson, and Leopold and Wolman.

THE LACEY EQUATIONS AND THE "THRESHOLD" THEORY

The theory of the stable channel profile developed at the USBR along t lines laid down by Lane will be referred to as Lane's theory.

The problem dealt with was the derivation of a channel cross-section su that when it was flowing full the bed material would be just on the threshold movement at every point on the bed cross section. On the side slopes the f that the shear stress is lower than at the center will be offset by the fact t both lateral slope and shear are combining their influence to dislodge the

10 "Uniform Water Conveyance Channels in Alluvial Material," by D. B. Simons M. L. Albertson, Proceedings, ASCE, Vol. 86, No. 5, May, 1960, p. 33.

11 "The Hydraulic Geometry of Stream Channels and Some Physiographic Impli tions," by L. B. Leopold and Thomas Maddock, Jr., U.S. Geol. Survey, Professio Paper No. 252, 1953.

material. The critical shear stress, τ, on a side slope of angle θ to the horizontal, is related to the critical shear stress τ_c on a bed with no side slopes by an equation derived by Lane and Carlson:

$$\frac{\tau}{\tau_c} = \cos\theta\sqrt{1 - \frac{\tan^2\theta}{\tan^2\phi}} \quad \ldots\ldots\ldots\ldots (6)$$

in which ϕ is the angle of repose of the material.

If now it is assumed that the local value of shear stress τ varies directly as the local depth y (Fig. 1) Eq. 6 yields a y - θ relation that can be integrated to give the following x-y equation:

$$\frac{y}{y_o} = \cos\frac{x\tan\phi}{y_o} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

so that the required profile is a sine curve. A further integration shows that the area, A, of the cross section is equal to

$$A = \frac{2y_o^2}{\tan\phi} \quad \ldots\ldots\ldots\ldots\ldots (8)$$

The assumption that τ varies directly as y ignores the lateral transfer of shear stress across vertical faces such as AD and BC in Fig. 1; that is, it is

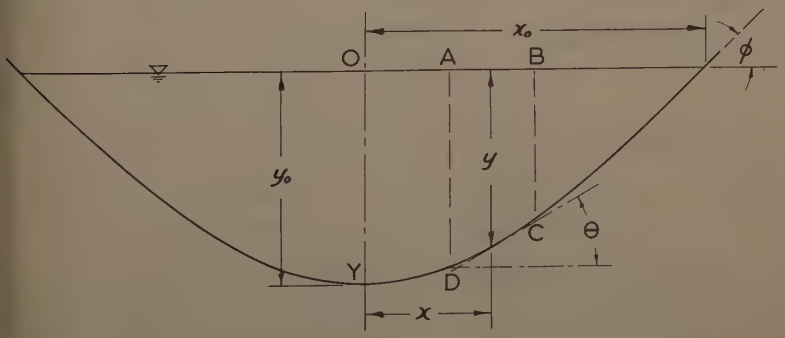

FIG. 1.—DEFINITION SKETCH-LANE'S TRACTIVE FORCE THEORY OF STABLE CHANNEL DESIGN

assumed that the shear stress on the surface element DC is due only to the weight of the prism whose section is ABCD resolved down the longitudinal slope of the channel. A more refined theory was also developed allowing for the lateral transfer of shear stress, but it was found that solutions obtained by the refined theory agreed closely with the sine-curve solution. It was accordingly suggested[5] that the sine curve solution might give equally satisfactory results with considerably less work. This suggestion will be followed in the present study.

A slight extension of the theory gives an expression for the wetted perimet P. It is equal to

$$2\int_0^{x_o}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = 2\int_0^{x_o}\sqrt{1+\tan^2\phi\,\sin^2\frac{x\tan\phi}{y_o}}\,dx \quad \ldots (9$$

The substitution $\alpha = \frac{x\tan\phi}{y_o}$, and some further manipulation leads to the int gral

$$\frac{2y_o}{\sin\phi}\int_0^{\frac{\pi}{2}}\sqrt{1-\sin^2\phi\,\cos^2\alpha}\,d\alpha$$

Because the limits of integration are $\frac{\pi}{2}$ and 0, $\cos^2\alpha$ may be replaced by $\sin^2\alpha$ without changing the value of the integral. Hence, finally,

$$P = \frac{2\,y_o}{\sin\phi}\int_0^{\frac{\pi}{2}}\sqrt{1-\sin^2\phi\,\sin^2\alpha}\,d\alpha \quad \ldots\ldots\ldots (1$$

The integral is the standard complete elliptic integral of the second kind, us ally denoted by E. For any given value of ϕ its value is readily obtained fr tables. Eq. 10 is rewritten as

$$P = \frac{2\,y_o\,E}{\sin\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1$$

Hence, the hydraulic mean radius R is equal to

$$R = \frac{A}{P} = \frac{2\,y_o^2}{\tan\phi}\;\frac{\sin\phi}{2\,y_o\,E} = \frac{y_o\cos\phi}{E} \quad \ldots\ldots\ldots\ldots (1$$

The key values for a typical value of ϕ, = 35° may be computed as a su mary.

Surface width	$= 2x_o = \frac{\pi\,y_o}{\tan\phi}$	$= 4.49\,y_o$	
Area, A	$= \frac{2\,y_o^2}{\tan\phi} = \frac{2\,y_o^2}{0.698}$	$= 2.86\,y_o^2$	
Perimeter, P	$= \frac{2\,y_o\,E}{\sin\phi} = \frac{2\,y_o \times 1.432}{0.574}$	$= 4.99\,y_o$	$\ldots$(
Hydraulic Radius, R	$= \frac{y_o\cos\phi}{E} = \frac{y_o \times 0.818}{1.432}$	$= 0.572y_o$	

The y - θ relationship which leads to Eq. 7 does not uniquely define a complete channel cross section. The y - θ equation would be equally satisfied by inserting a section of constant depth between the two curved banks (Type A in Fig. 2), or by removing a section from the center (Type C in Fig. 2). The type B section consists of the complete curved banks with $\theta = 0$ at $y = y_o$, but without a central insert. It can be thought of as a critical, or limiting section, and will subsequently be referred to as such. The significance of the three types of section will be commented on at a later stage.

We now introduce the size, d, of the bed material. The experimental work of Shields and others has shown that for values of the particle Reynolds number above approximately 400, the following relation holds:

$$\frac{\tau_c}{\gamma(s_s - 1)d} = 0.056 \text{ (approximately)} \quad \ldots\ldots (14)$$

in which τ_c is the critical shear stress, γ denotes the fluid specific weight and s_s is the ratio of solid to fluid density. The expression on the left of the

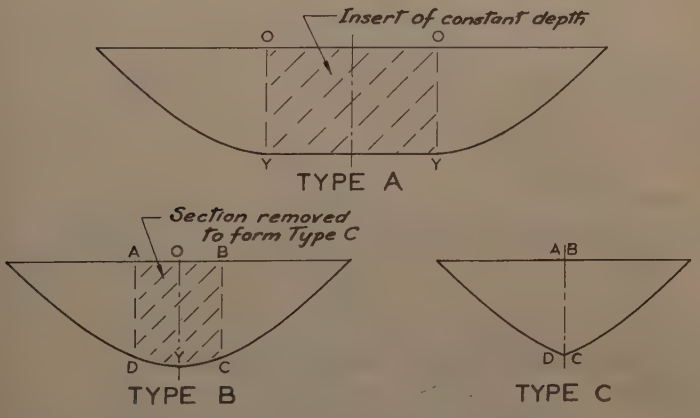

FIG. 2.—ALTERNATIVE TYPES OF STABLE CHANNEL PROFILE

equation is known as the entrainment function. For a section wide enough for τ to be constant over the entire bed, $\tau/\gamma = R\,S$, in which S is the longitudinal slope. Taking a typical value of s_s, for example 2.6, Eq. 14 shows that the condition

$$d = 11\,R\,S \quad \ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

holds at the threshold of movement. Eq. 15a appears in the literature in various forms, such as in general form as the Ishbash formula. It appears to offer the simplest and most convenient way of applying the threshold tractive-force criterion in practice. The lowest value of d to which the equation applies is obtained by eliminating τ_c between Eq. 14 and the equation $R_e^* = 400$; if the kinematic viscosity ν is assumed to be 1.2×10^{-5} sq ft per sec, it is found that d approximates 0.24 in. Hence Eq. 15a is true only when $d > \frac{1}{4}$ in. approximately.

When the channel section is comparatively narrow it is the maximum rath than the mean shear stress which is of interest. The maximum shear stre is approximately equal to $\gamma\ y_o\ S$, in which y_o is the maximum depth, thus E 15 should be rewritten

$$d = 11\ y_o\ S \quad \ldots\ldots\ldots\ldots\ldots\ldots (15$$

In particular, for the limiting Type B cross section developed in Lane theory, $R = 0.572\ y_o$, from which

$$d = \frac{11\ R\ S}{0.572} = 19\ R\ S \quad \ldots\ldots\ldots\ldots\ldots (15$$

Eqs. 15 can now be combined with Strickler's formula relating the bed m terial size d to the Manning, n

$$n = 0.034\ d^{\frac{1}{6}} \quad \ldots\ldots\ldots\ldots\ldots (1$$

from which

$$v = \frac{1.49\ R^{\frac{2}{3}}\ S^{\frac{1}{2}}}{0.034\ d^{\frac{1}{6}}} \quad \ldots\ldots\ldots\ldots\ldots (1$$

It is now possible to substitute for d from Eqs. 15, yielding, for wide channe (d = 11 R S)

$$v = 29\ R^{\frac{1}{2}}\ S^{\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots (18$$

and for the Type B channel derived in Lane's theory (d = 19 R S)

$$v = 27\ R^{\frac{1}{2}}\ S^{\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots (18$$

that are both very similar in form to the Lacey equation

$$v = 16\ R^{\frac{2}{3}}\ S^{\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots$$

There are some reservations to be made about the coefficients in Eqs. Both equations rest on the assumption that the bed material is uniform; i is not uniform, allowance must be made for the fact that Strickler's form is based on the median bed size, whereas Eqs. 15 are based on a kind of "dor nant" stone size - the size of stone with which the bed tends to become armor It is the writer's observation that in natural rivers this dominant size appe to be around the D-80 to D-90 size, which may be two or three times as gr as the median size.

The difference involved is not very great, because of the low value of exponent in Strickler's formula. However, it should be observed that if t formula were to be based on the dominant stone size instead of the medi its coefficient would be reduced from 0.034 to approximately 0.030, and coefficients of Eqs. 18 would accordingly be raised by approximately 10%.

question will not be pursued any further here. The emphasis in this study is on exponents rather than on coefficients, which in any case would have to be verified by experimental and field observations.

If attention is confined to the Type B channel described in Eq. 18b, enough information can be extracted to determine the channel cross-section completely. If, as suggested previously, the coefficient of Eq. 18b is arbitrarily increased from 27 to 30, it is possible to write, from this equation and from Eqs. 13 and 15c

$$Q = 30\, P\, R^{1\frac{1}{2}}\, S^{\frac{1}{3}}; \quad R = \frac{d}{19\, S}; \quad P = 8.75\, R;$$

Eliminating P and R from these equations yields

$$S = 0.44\, d^{1.15}\, Q^{-0.46} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

an equation that gives the limiting value of slope at which the limiting, or Type B, channel shape derived in Lane's theory would just be stable. If the slope is greater than this value, the wide Type A channel of less scouring capacity is required. If the slope is less than that given by Eq. 19 then a Type C channel is apparently required. Whether such a channel section can be realized in practice is a question which will subsequently be examined.

Meanwhile an equation will be obtained for the surface width, w, from Eq. 13

$$w = 4.49\, y_o = \frac{4.49\, d}{11\, S} = \frac{0.41\, d}{S} \qquad \ldots\ldots\ldots\ldots (20)$$

hence, from Eq. 19

$$w = 0.93\, d^{-0.15}\, Q^{0.46} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Similarly for the wetted perimeter P

$$P = 1.03\, d^{-0.15}\, Q^{0.46} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

These equations are remarkably similar in form to the Lacey equation

$$P = 2.67\, \sqrt{Q} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

although they are more general in that they take account of the bed material size. Eqs. 22 and 3 would yield the same result for d = 0.02 in. approximately, which is well within the range of silt sizes of the Indian canals on which the Lacey equations are based. In fact for this size the silt factor, f, can be calculated from Eq. 4, giving a value of 1.1 approximately, close to the value of unity appropriate to what Lacey calls "standard silt."

For the sake of completeness an equation corresponding to the first of the Lacey equations,

$$v = 1.17\, \sqrt{f\, R} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

should be sought. Such an equation would express v in terms of d and R, and can be simply obtained by eliminating S between Eqs. 15c and 18b. The result-

ing equation is

$$v = \frac{30\ R^{\frac{1}{2}}\ d^{\frac{1}{2}}}{(19\ R)^{\frac{1}{3}}} = 11\ d^{\frac{1}{3}}\ R^{\frac{1}{6}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23$$

which, perhaps somewhat disappointingly, is substantially different in form from the corresponding Lacey equation.

In summary, from Lane's tractive force theory the following equations have been deduced, applying to the limiting, or Type B, channel cross section, and to a bed material having a specific gravity of 2.6 and an angle of repose of 35°

$$v = 11\ d^{\frac{1}{3}}\ R^{\frac{1}{6}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23$$

$$v = 30\ R^{\frac{1}{2}}\ S^{\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18c$$

and

$$w = 0.93\ d^{-0.15}\ Q^{0.46} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21$$

Eq. 18c is obtained by arbitrarily increasing the coefficient of Eq. 18b by 10%. For other properties of the bed material the coefficients of these equations would change, but the exponents would be unaltered. The two latter equations are remarkably similar in form to the corresponding Lacey "regime" equations.

Although the three equations are derived for the Type B channel section the first two will be true for all width depth ratios with only a slight change to the coefficients. The third equation however applies only to the Type B section.

Particular values of ϕ and s_s were assumed in the preceding argument in order to leave the outline of the argument uncluttered by detail and to show it as simply as possible the similarity with the Lacey equations. However, for the sake of completeness, the general forms of the equations will be given in Eqs. 24 through 29, setting $r = s_s - 1$. On the left of each equation is set the number of the particular form of the equation, that is, that for which $\phi = 35$ and $r = 1.6$.

(15) $$d = \frac{18\ R\ S}{r} \quad \text{or} \quad \frac{18\ y_o\ S}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24$$

(18a) $$v = 30.6\ r^{\frac{1}{6}}\ R^{\frac{1}{2}}\ S^{\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25a$$

(18b) $$v = 30.6\ R^{\frac{1}{2}}\ S^{\frac{1}{3}} \left(\frac{r \cos \phi}{E}\right)^{\frac{1}{6}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25b$$

(19) $$S = 0.24\ r^{1.15} \left(\cot \phi \sqrt{\frac{\cos \phi}{E}}\right)^{0.46}\ d^{1.15}\ Q^{-0.46} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26$$

$$w = 0.73 \frac{\cos\phi \cot\phi}{E} \left(\tan\phi \sqrt{\frac{E}{\cos\phi}}\right)^{0.46} r^{-0.15} d^{-0.15} Q^{0.46} \quad \ldots(27)$$

$$P = 0.47 \cot\phi \left(\tan\phi \sqrt{\frac{E}{\cos\phi}}\right)^{0.46} r^{-0.15} d^{-0.15} Q^{0.46} \quad \ldots\ldots(28)$$

$$v = 11.7 \, d^{\frac{1}{3}} R^{\frac{1}{6}} \left(\frac{r \cos\phi}{E}\right)^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

A further generalization is possible which takes care of those cases in which e sediment size is less than $\frac{1}{4}$ in., and it is necessary to deal with the dip in ields' curve of entrainment function versus particle Reynolds number, R_e^*, here $R_e^* < 400$ and the entrainment function, $\frac{1}{\Psi}$, is accordingly less than 0.056, r instance 0.056 k. Then the coefficient 11 in Eq. 15a is increased to 11/k nd the coefficients in the other equations altered accordingly. The effect may e summed up as follows:

Equation Number	Multiply right hand side by:
15, 24	k^{-1}
18, 25	$k^{\frac{1}{6}}$
19, 26	$k^{1.23}$
21, 27	$k^{-0.23}$
22, 28	$k^{-0.23}$
23, 29	$k^{\frac{1}{2}}$

LACEY EQUATIONS AND THE BED-LOAD FUNCTION

As mentioned previously, workers in this field have recently become in- rested in establishing a connection between the regime theory and the thres- old tractive force theory. The similarity deduced in the previous section be- ween the corresponding equations of the two theories certainly encourages elief in the existence of such a connection.

However it must still be recognized that no one theory can embrace both e threshold condition, in which the bed is on the point of movement, and the egime condition, in which the bed movement is fully established. This latter atement can easily be verified by calculating the value of Shields' entrain- ent function

$$\frac{1}{\Psi} = \frac{\tau}{(s_s - 1)\gamma \, d} = \frac{5 \, R \, S}{8 \, d} \left(\text{for } s_s = 2.6\right) \ldots\ldots\ldots(30)$$

r various of the canal data cited by Lacey. In all cases the function is several mes greater than is required by the Shields criterion for the threshold of mo-

tion. In fact the value of the function is high enough to produce a value on th straight-line portion of the Einstein bed load function curve, and from this fac may be deduced a simple explanation for the basic form of the regime equa tions.

Before advancing this explanation it would be useful to attempt a critica assessment of just how far the Lacey equations are supported by the India canal data which Lacey himself presented. The firmest conclusions that th data have to offer relate to variations within a single canal system; the diffi culties that are characteristic of the regime theory arise only when one cana system is compared with another by means of the silt factor. In the origina theory these difficulties were met simply by assigning to each canal syste the value of f required to make $v/\sqrt{fR}$ the same for all systems, it having bee established that $v/\sqrt{R}$ was constant within each particular system. Only as a afterthought was f related to the silt size d. The intention seems to have bee that a designer seeking to find a value of f for a certain area would be guide primarily by knowledge of the behavior of $v/\sqrt{R}$ in canals already in that area and only secondarily by the silt size d.

This attitude toward the silt factor has influenced subsequent work by th Lacey school on its relation to the sediment charge q_s. The work of T. Blench,[1] F. ASCE, is prominent in this field. Blench has modified the original Lace equations, and has produced further equations approximately relating his newl defined silt factor to the sediment charge. However he expresses the sam reservations about these equations as Lacey originally did about Eq. 4, an indeed the same doubts as to whether the practicing engineer really needs ex plicit rules relating f to sediment properties. He states: "The engineer wh uses regime equations, and acquaints himself with canal and river behaviou in the field, will acquire the ability to assess likely bed factors for almost an case. At present, however, no simple rules can be presented to replace fiel experience and engineering judgment."[12]

A statement like this would certainly not be challenged by anyone who di not share the experience on which it is based. Nevertheless it does place cer tain difficulties in the way of the theoretical worker seeking an explanation events in fundamental terms.

Despite these difficulties the fact remains that firm conclusions of gener interest may be reached by a study of variations within a particular canal sys tem. It was pointed out by H. E. Hurst[13] that Eq. 1 is suspect because the ve locity v was not measured directly, but deduced from a resistance equatio

$$v = K R^{\frac{3}{4}} S^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3$$

rather similar to Manning's. Hurst suggested that the only firm conclusion be drawn from the data would be one that related directly observed quantiti such as R and S. Now if R is plotetd against S for any given canal system

12 "Regime Behaviour of Canals and Rivers," by T. Blench, Butterworth's Scientifi Publications, London, 1957.

13 Discussion by H. E. Hurst of "Stable Channels in Alluvium," by G. Lacey, Proceedings, ICE, Vol. 229, 1929-30, p. 259.

found that

$$R^{\frac{1}{2}} S = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

one proceeds downstream through the system, although the discharge and e channel dimensions vary over a wide range. Eq. 32 is true for all systems, though the constant is different for each system, and the conclusion is statistically quite a firm one, given the limits of accuracy normally accepted in is type of study. Now the threshold theory requires the shear stress, which proportional to R S, to be constant along the canal, that is, that

$$R S = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

there is little point in hoping that this theory might be stretched so as to clude the regime theory.

The form of Eq. 33 can be accounted for by referring to the Einstein bed ad function,

$$\Phi = \frac{q_s}{F d \sqrt{g\left(s_s - 1\right) d}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

which q_s is the sediment discharge per unit width of the channel. Einstein owed that Φ was related to the entrainment function

$$\frac{1}{\Psi} = \frac{\tau}{\left(s_s - 1\right)\gamma d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

this way $\Phi \to 0$ as $\frac{1}{\Psi} \to 0.056$; and for $\frac{1}{\Psi} > 0.1$ approximately, then

$$\Phi = 40\left(\frac{1}{\Psi}\right)^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

the straight line portion of the $\Phi - \frac{1}{\Psi}$ curve previously referred to. Because $= \gamma$ R S, Eq. 36 can be rewritten

$$\frac{q_s}{F d \sqrt{g d}} = \frac{40}{\left(s_s - 1\right)^{2\frac{1}{2}}} \left(\frac{R S}{d}\right)^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

which F is a function of grain size and fluid properties, so that for any one nal whose bed material is the same along its whole length

$$q_s \propto R^3 S^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Further, along any one stable canal system the sediment concentration $\frac{q_s}{q}$ ll be constant as well as d. In order to use this fact q, the water discharge

per unit width, must be introduced into Eq. 38. To do so a resistance equati such as Eq. 31 is needed. H. K. Liu, M. ASCE and S. Y. Hwang, A. M. ASC have shown[14] how the exponents X and Y in the equation

$$v = C R^X S^Y \quad \ldots\ldots\ldots\ldots\ldots\ldots (3$$

vary with bed formation and sediment size. To a first approximation it is po sible to write

$$q = v R = C R^{1+X} S^Y \quad \ldots\ldots\ldots\ldots\ldots\ldots (4$$

whence from Eq. 38

$$\frac{q_s}{q} \propto R^{2-X} S^{3-Y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4$$

so that for any one stable canal system, with d and $\frac{q_s}{q}$ constant, it follows th

$R^{2-X} S^{3-Y}$ will be constant, that is that $R^{\frac{1}{2}} S$ is constant if $\frac{2-X}{3-Y} = \frac{1}{2}$. Th

condition is exactly fulfilled if X and Y have the values of $\frac{5}{7}$ and $\frac{4}{7}$ assumed Lacey. The condition is very nearly fulfilled if X and Y have the more pr cise values assigned to them by Liu and Hwang, as shown in Table 1.

TABLE 1

X	Y	$\frac{2-X}{3-Y}$	$X-Y\frac{2-X}{3-Y}$	$\left(\frac{2(1+X)(3-Y)}{2-X} - 2Y\right)^{-1}$	Remarks
5/7	4/7	9/17	0.412	0.188	Plane bed; d < 0.2 mm
5/7	0.55	0.525	0.426	0.184	Plane bed; d = 0.4 mm
2/3	1/2	0.533	0.4	0.190	Manning equation: d > 10 r
0.35	0.3	0.61	0.167	0.262	Dune bed; d = 0.01 mm
0.4	0.3	0.59	0.223	0.241	Dune bed; d = 0.1 mm
0.5	0.33	0.56	0.315	0.214	Dune bed; d = 0.4 mm
0.6	0.35	0.53	0.414	0.216	Dune bed; d = 0.7 mm
2/3	0.4	0.513	0.462	0.175	Dune bed; d = 2 mm

The conclusion is that Lacey's observations that $R^{\frac{1}{2}} S$ = constant in a "r gime," that is live-bed, channel are well supported by the Einstein theory. contrast to this case is the "threshold," or stable bed, channel in which R constant. Each of these relationships provides in its particular case a ba on which the designer can operate, and each must be thought of as distinct fr the other.

Lacey's first equation, $v \propto \sqrt{R}$, does not stand up well to the same exa nation as was given previously to the relation $R^{\frac{1}{2}} S$ = constant. Lacey's ori nal derivation of the equation consisted, essentially, of eliminating S betw

14 "Discharge Formula for Straight Alluvial Channels," by H. K. Liu and S. Y. Hw Proceedings, ASCE, Vol. 85, No. 11, Nov. 1959, p. 65.

qs. 31 and 32. If Liu and Hwang's more general form of the resistance equation is considered S should be eliminated between the equations

$$R^{\frac{2-X}{3-Y}} S = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

nd

$$v \propto R^X S^Y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

ading to the result

$$v \propto R^{X-Y\frac{2-X}{3-Y}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

nd this exponent of R is also tabulated in Table 1. It is seen that it varies idely, and approaches the value $\frac{1}{2}$ only in certain cases.

These conclusions are indirectly supported by the work of Ning Chien[9] who vestigated the dependence of the "silt factors"

$$f_{VR} = \frac{v^2}{1.325R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

nd

$$f_{RS} = 193\left(R^{\frac{1}{2}} S\right)^{\frac{2}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

d and $\frac{q_s}{q}$. These parameters were regarded as identical in the original presentation of the regime theory but have since been shown by C. C. Inglis[15] to e different. Ning Chien found that whereas f_{RS} could be simply expressed as

$$f_{RS} = 2.2\, d^{0.45}\left(\frac{q_s}{q}\right)^{0.052} \quad \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

er the range 0.25 mm. $< d <$ 25 mm., the variation of f_{VR} with d was somehat erratic and inconsistent. These results tend to support the previous conusion that the relation $R^{\frac{1}{2}} S$ = constant for a given canal is well founded, t the relation $v \propto \sqrt{R}$ is not.

This section is concerned in essence with the differences between a live-ed theory and a threshold, or fixed bed, theory. Consideration should therere be made of the live-bed equivalent of the Type B section deduced in the reshold theory. In the absence of the simple criteria governing the shape the threshold channel, it is difficult to see how the appropriate theory could constructed. However, the following rather speculative argument is suggsted as a possible approximation to the facts.

Consider a Type B channel section, as given by Lane's theory, flowing full the correct threshold condition. Now consider the channel to be tilted, so at its longitudinal slope is increased. If it is still to run full, the velocity

15 "The Behaviour and Control of Rivers and Canals," by C. C. Inglis, Central Waterwer Irrig. and Navg. Research Sta., Research Publication No. 13, Poona, India, 1949.

and discharge must increase, and enough sediment must be fed in at the u stream end to maintain the channel stable despite its newly-found transporti capacity.

The question now is how will the channel section adjust itself, if at all? Th answer can only be guessed here. It might be, for instance, that material wi be eroded from the banks and deposited in the bed, making the channel wid and shallower. But, however the width or depth may change, it seems reaso able to assume that it will change in a ratio which is related only to the sed ment concentration, q_s/q, and that the relation $P \propto \sqrt{Q}$ will still be true, wi some power of q_s/q included in the equation as well as the sediment size

$$P \propto d^m \left(\frac{q_s}{q}\right)^n \sqrt{Q} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4$$

The exponent m would probably be negative, as it is in Eq. 22. The exp nent n may well be negative also, if there is little change in the channel secti after its being tilted and made into a live-bed section as previously describe This would mean that an increase in Q had made little difference to P; henc the ratio $P/\sqrt{Q}$ would have decreased with an increase in $\frac{q_s}{q}$.

Whatever may be the merits of the preceding argument, and whatever t correct form of the equation may be, it will, like Eq. 22, be true only when certain relationship, such as given by Eq. 19, holds between slope, discharg and the bed conditions. This question will be elaborated in the next sectio

DETERMINACY OF STABLE CHANNEL EQUATIONS

Every engineer who has attempted to use the Lacey equations will have m the difficulty that the equations are overdeterminate, and apparently try to t the designer too much. Three equations are presented to deal with a syst in which there are five independent quantities in all (Q, P, R, S, and f). Henc only two quantities may be fixed initially, although in practice it may well ha pen, particularly in river work, that three quantities (Q, S, and f) are fix initially by factors beyond the designers control. When this happens Eqs. 2 and 3 cannot, in general, be satisfied simultaneously. This can only be do in the particular case in which the values of Q, S, and f satisfy the relati

$$S = \frac{f^{\frac{5}{3}}}{1750\, Q^{\frac{1}{6}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5$$

The difficulty is not removed by the argument that there are two quantiti d and q_s/q, implicit in the silt factor f, because the designer will, in gener wish to fix both these quantities in advance as well as Q and S. The introdu tion of q_s/q does not, therefore, give him another degree of freedom. T flexibility that he really wants is that which allows him to make the chan wider, and therefore less able to transport sediment, if the slope is grea than the critical value given by Eq. 5. This flexibility is realized in the thre hold tractive force theory, as mentioned previously. If S is greater than t value given by Eq. 19 the channel should be widened by placing an insert of co

ant depth between the curved banks. The shape of the curved banks is de-
ved from Lane's theory.

In this connection it is useful to recall the arguments leading to Eqs. 19,
, and 22. Although these equations apply only to threshold channels, they
pear to clarify the whole question of the logical structure of stable channel
uations. For the conditions relating to bed movement (or its absence) only
e equation, such as Eq. 15 in the threshold case or Eq. 32 in the live-bed
se, will result. Consideration of flow resistance either on its own or in com-
nation with the first equation leads to a second equation such as Eq. 2 or Eq.
. As has been pointed out, Lacey's first regime equation is actually a com-
nation of Eq. 32 and a resistance equation.

The two equations examined, thus far, will be true without restriction as to
annel shape or slope, provided only that the slope is great enough. A third
uation, such as Eq. 3 or Eq. 22, can only be derived if some further condi-
n, relating slope to discharge and bed material, is imposed. To complete
e theory there should clearly be some such condition giving the minimum
ope at which a channel can keep itself clear. In the threshold theory this
ndition is given by Eq. 19 which describes the Type B section. The Type C
ction (Fig. 2) would be meaningful only if the channel is to carry clear water
d there is no supply of bed material from upstream. However, a general
eory applicable to river work as well as canal design must allow for the case
which there is some supply of material coming from upstream and some
ocess of continuous adjustment going on. In this case the Type C section
uld hardly keep itself clear, because at the point D, C, at which the two side
opes meet, the bed material will not be dislodged as it would be at the cor-
sponding point on the Type B section.

The Type B section is therefore a truly limiting one, being the narrowest
ssible section consistent with bank stability. The value of slope at which it
just stable, given by Eq. 19, is the minimum slope at which the given dis-
arge is capable of maintaining a clear channel in bed material of the given
e.

Now there is no reason why the logical outline of a fixed-bed theory should
fer from that of a live-bed theory. It seems, therefore, that Eq. 5a, the live-
d analogue of Eq. 19, is properly interpreted as giving a critical minimum
ue of slope, below which a regime channel cannot be formed. Eq. 3 will,
refore, only be true when the slope has this minimum value.

The river engineer, as suggested previously, often has to deal with reaches
t do not conform to Eq. 5a, and in which any process of slope adjustment is
remely slow. In such cases, even if Eqs. 1 and 2 were well established,
re would be no possible justification for using Eq. 3. It is in this kind of
uation that the designer needs the flexibility in the choice of channel width
ich is allowed him by the threshold tractive force theory, but not by the Lacey
gime theory if all three equations are included. However, if the third equa-
n is disregarded, then the first two equations can be solved for v and R if f
d S are specified in advance. The value of P then can be obtained if Q is also
ecified. By this means the designer achieves the required freedom of choice.
may be this line of reasoning that is implied in the often-quoted remark, "All
cey channels are regime channels; but all regime channels are not Lacey
annels."

It must be emphasized that this freedom of choice can only be achieve abandoning Eq. 3; its only function now becomes that of verifying that the ac P is greater than the "critical" P given by this equation, that is, that the ac slope is greater than the critical slope given by Eq. 5a. Unfortunately n engineers in the British Commonwealth tend to regard Eq. 3 as the cor stone of the Lacey theory. It is certainly the one most quoted in report river works; whereas it is logically the one most likely to be redundant. haps it is so popular because it can be used so easily, without the need to ev ate the silt factor?

There remains the question of the behavior of a channel designed as lined previously, with S and P values greater than the Lacey equations requ The answer implied in the original presentation of the theory and since plicitly stated by Blench, among others, is that the slope will adjust itself it reaches the minimum value given by Eq. 5a, and that the width will be sim taneously reduced by build-up of the banks. This means that material is sco from the bed and deposited on the banks as well as on the bed further do stream. However, there is nothing in the approach outlined so far which account for this phenomenon. There is in fact nothing in the Shields or Eins equations to suggest that a wide channel on a steep slope is any less st than a narrow channel on a gentler slope.

Unfortunately experimental evidence on this point is not as explicit might be. The data originally presented by Lacey are confounded (in the tistical sense) by the initial choice in the matter of slope exercized by the d designers. On the grounds of flow resistance alone, designers tend to ch greater slopes for smaller discharges, in the interests of economical pro tioning of the cross section. Therefore, it is not clear how far the obse values of slope are self-adjusted, and how far they merely reflect the ch of the canal designer. It would have been helpful if the data had included in as well as final values of slope. The same remarks might be made abou supporting evidence for Eq. 3 such as is presented by Chien.[9] This evid does not really help to validate this equation unless it is made clear to extent the slopes are self-adjusted.

The previous remarks are not meant to imply skepticism as to the fa slope adjustment, but to emphasize that because it is hard to account fo theoretical grounds, there is a corresponding need for experimental evid that cannot be challenged in any way. Undoubtedly natural rivers, even in co alluvium, do adjust their slopes over long periods of time, and artificial ca carrying heavy charges of fine silt may do so quite quickly. Some such e tion as Eq. 5 or Eq. 19, therefore, may well describe the ultimate conditi a graded river or canal.

Eq. 19 will now be considered in particular. The case for this equati an indicator of ultimate (minimum) slopes in rivers of coarse alluvium, no means completely established. It is open to the objection that there reason to assume that a river will cut itself a channel conforming to the B section. One outstanding property of this section is that the width: depth depends only on the angle of repose, and is equal to about 4 or 5. In na rivers the width: depth ratio is usually much greater than this, and quite nitely increases with the discharge.

The most that can be confidently claimed for Eq. 19 is that it gives the engineer some guidance in deciding whether an aggrading river reach is agg ing because it does not have enough slope, or merely because it has been all

wander over too wide a bed. If the slope is less than as given by Eq. 19 the mer reason is correct, and the only hope of correction lies in increasing e slope by means of cut-offs. If the slope is greater than as given by Eq. 19 en it is sufficient for the formation of a stable single thread channel, which n be created by reducing the width between banks.

Nevertheless it will be seen in the next section that some data from natural vers give support at least for the form of Eq. 19, if not for the numerical lue of its coefficient.

To complete this analysis of the dependence of equations such as Eq. 3 on uations such as Eq. 5, it would be logical to consider how this dependence erates in the live-bed theory developed in the previous section. Again con-

dering a channel in which d and $\frac{q_s}{q}$ are constant, Eq. 41 yields

$$R^{2-X} S^{3-Y} = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

d

$$v = C R^X S^Y \quad \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

ting $Q = v P R$ and eliminating R yields

$$P \propto Q S^{\frac{(1+X)(3-Y)}{2-X} - Y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

the condition for $P \propto Q^{\frac{1}{2}}$ to be true, is that

$$S \propto Q^{-\frac{1}{\frac{2(1+X)(3-Y)}{2-X} - 2Y}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5c)$$

is exponent is also listed in Table 1, and it is seen to be quite close to $-\frac{1}{6}$, in Eq. 5a, for a plane bed. For a dune bed it ranges between $-\frac{1}{4}$ and $-\frac{1}{6}$.

APPLICATIONS AND EVIDENCE OF FIELD DATA

Artificial Channels.—In principle a channel could be designed for either eshold or live-bed conditions whatever the size of the bed material, but in ctice threshold conditions are more likely to be associated with a bed of rse material, and live-bed conditions with fine material. It is to channels coarse alluvium that one tends to look for verification of a threshold theory. A most comprehensive collection of data has been published by Simons and ertson.[10] The only data clearly relating to coarse material are from the BR canals in the San Luis Valley of Colorado. These data were used by ons and Albertson to plot v against $R^2 S$, in line with Lacey's Eq. 2. But values of velocity may equally well be plotted against $R^{3/2} S$, as in Fig. 3. traight line having a slope of $\frac{1}{3}$ can be drawn through the plotted points; the

scatter is no worse than in Simons and Albertson's graph, and the resu equation

$$v = 24\, R^{\frac{1}{2}}\, S^{\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots$$

is fairly close to Eq. 18. The lower value of the coefficient represents, i fect, a factor of safety against the onset of bed motion.

The remaining data plotted on the v - R^2 S plane show approximate firmation of Eq. 2, with a fair degree of scatter. Because Eq. 2 is base an approximate resistance formula this is only to be expected. A combin of Eq. 32 with the more exact equations given by Liu and Hwang would le a formula that, although more complex than Eq. 2, might fit the data closely.

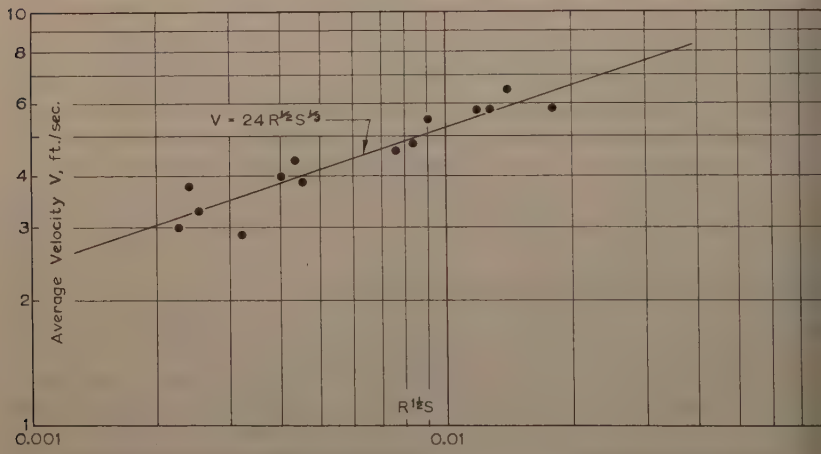

FIG. 3.—VARIATION OF V WITH $R^{\frac{1}{2}}S$, USBR CANAL DATA

In the same paper P is plotted against Q for all canals whether of coar fine material and it was found that in all cases

$$P = G\, Q^{0.512} \quad \ldots\ldots\ldots\ldots\ldots$$

the size of the constant G depending on the bed material, ranging from 1.7 the coarse-bed canals to 3.5 for those with sand bed and banks. This beh of G is, at least an approximate confirmation of Eq. 22, according to wh should decrease with increasing size of the bed material. There is one system however - the Imperial Valley canals - for which G is low (1.7 though the bed material is fine, and similar to that in another system for G was approximately 2.6. The explanation may lie in the unusually high ment load carried by the Imperial Valley canals. If this is so, it is a confi tion of the conjecture that in Eq. 48 the exponent n may be negative.

However, one has to resist the temptation to see these data as confirma- n of Eq. 22, which relates to the Type B section of substantially fixed width: th ratio equal to 4 or 5, which is much less than in any of the canals being sidered here. With the Lacey theory, on the other hand, one can easily show m Eqs. 1 and 3 that

$$R \propto Q^{\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

that R increases more slowly with Q than P does, and P/R increases with The data for all canals, with fine and coarse beds, show that

$$R \propto Q^{0.361} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

esult which appears to support the Lacey equations. But the support is not decisive as it may at first appear.

It has been pointed out in the previous section that the observations on which s. 52 and 54 are based do not necessarily validate the Lacey theory unless an be shown that the corresponding slope-discharge relation is the result of f adjustment by the channel. However it is still possible to deduce equa- ns relating P, R, and Q, which will be true for any slope, and these may be mpared with those obtained by combining Eqs. 52 and 54:

$$P R = A \propto Q^{0.873} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

$$P R^{1.35} \propto Q \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

Now by substituting a v-R equation into the identity $P R v = Q$ an equation responding to Eq. 56 is obtained. In the threshold theory, Eq. 23 yields

$$v \propto d^{\frac{1}{3}} R^{\frac{1}{6}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

m which

$$P R^{1\frac{1}{6}} \propto d^{-\frac{1}{3}} Q \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

this is true without restriction as to channel shape or slope. (Eq. 23 is particular form of Eq. 57 which applies to the Type B section. However, 57 is true for any other section, with only slight adjustment to the numeri- coefficient.)

The corresponding equation in the Lacey theory is

$$P R^{1\frac{1}{2}} \propto f^{-\frac{1}{2}} Q \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

ch again is true without the restrictions such as are imposed by Eq. 5. The ι of Simons and Albertson indicate that

$$P R^{1\frac{1}{6}} \propto Q^{0.933} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

and

$$P R^{1\frac{1}{2}} \propto Q^{1.053} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(($$

which means that in this respect the data do not really discriminate betw the Lacey theory and the threshold theory. However, other equations can derived which show up the difference between the two theories more clear Eliminating R between Eqs. 15a and 18a - both applicable to wide channels s as occur in the data being examined - yields

$$P = \frac{1.14\ Q\ S^{1\frac{1}{6}}}{d^{3/2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(($$

Eq. 62 shows clearly that if the relation $P \propto \sqrt{Q}$ is to hold in a certain ca system, there must also be a slope-discharge relation of the form

$$S^{1\frac{1}{6}} \propto Q^{-\frac{1}{2}}$$

that is

$$S \propto Q^{-0.46} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(($$

Similarily, in order that the relation

$$P \propto d^{-0.15}\ Q^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(($$

(See Eq. 22) should be true, the relation

$$S \propto d^{1.15}\ Q^{-0.46} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(($$

would have to be true. (See Eq. 19). Hence, the dependence of a P-d-Q re tion on an S-d-Q relation has the same form for wide channels as for nar Type B channels.

A live-bed theory on the other hand requires that

$$S \propto Q^{-\frac{1}{6}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(($$

whether by the Lacey equations or by the approach developed in a prev section of this paper. The marked difference between the exponents of Eqs. 5a and 54 suggest a more satisfactory way of testing whether a given of data fits better to a threshold theory or a live-bed theory.

The data of Simons and Albertson do not however lend themselves to sort of test, since equations such as Eq. 5 relate to variations within one c system having the same d and q_s/q throughout its length, and the data deal a number of different canals with widely varying d and q_s/q. On the other Eqs. 2 and 18 may be satisfactorily checked against those data because equations purport to be independent of any parameters describing bed co tions.

Natural Rivers.—Natural rivers in coarse alluvium should provide useful
ta for a check on the threshold tractive force theory, despite certain diffi-
ities in collecting and assessing the data.

The first difficulty is in obtaining a definition of the so-called "dominant
scharge," that is, the constant discharge to which the varying flow of a natu-
l river is equivalent for the purposes of channel formation. It is generally
cepted that the dominant discharge can be defined in terms of flow frequency,
t the question is by no means satisfactorily resolved yet. In the case of riv-
s possessing a stable single-thread channel there is a physical indicator in
e form of the discharge required to fill this channel, and it is generally as-
med that this "bank-full discharge" may be defined as the dominant discharge.
e observations of Wolman and Leopold[16] in the United States and of Marshall
xon[17] in England and Wales have attempted to relate the bank-full definition
a statistical definition. Wolman and Leopold obtained a mean return period
about 1.4 yr, based on annual floods, the latter of about 6 months, based on
count of all floods. In both cases there was considerable scatter about the
ean. Further, because the two return periods are defined in different ways
e American and English results cannot be easily compared. It appears that
re statistical work on river flow distribution is required before the matter
n be made clear. Meanwhile it can only be assumed that in testing equations
ch as Eq. 3, river discharges are comparable if they are of the same fre-
ency.

The best known observations are those of Leopold and Maddocks,[11] who
nd that for a number of rivers in the United States the width and depth varied
h discharge of constant frequency as one moves downstream:

$$\text{width} \propto Q^a \quad \ldots\ldots\ldots\ldots\ldots\ldots (66)$$

d

$$\text{depth} \propto Q^b \quad \ldots\ldots\ldots\ldots\ldots\ldots (67)$$

which a and b had the following values:

	Maximum	Minimum	Mean
a	0.76	0.43	0.5
b	0.45	0.10	0.4

These results are often cited in support of Lacey's Eq. 3. They do indeed
nish substantial evidence in favor of this equation, and by implication, in
or of the view that the rivers concerned have adjusted their slope-discharge
ationships, that is their longitudinal profiles, in such a way as to validate
equation of the same form as Eq. 3.

The natural question to ask now is; just what form do the longitudinal pro-
s of these rivers take? The appropriate information is not included by
opold and Maddocks.[11] However, some light is thrown on the problem, al-

16 "River Flood Plains: Some Observations on Their Formation," by M. G. Wolman
L. B. Leopold, U.S. Geol. Survey, Professional Paper No. 282-C, 1957.

17 "A Study of the Bank-full Discharges of Rivers in England and Wales," by Marshall
on, Proceedings, ICE, Vol. 12, 1959, p. 157.

though rather indirectly, by the work of Leopold and Wolman[18] in which d are presented on natural river channels in coarse alluvium. The aim of th study was to find a criterion which distinguished between braided and meand ing channels. It is well known that the braided formation - the spreading the river into many channels which endlessly divide and rejoin - is associa with steep slopes, and the meandering form with gentler slopes. The presur tion is that a river becomes braided because it possesses surplus transport power, which it dissipates by lateral attack on its banks. On the other han meandering river retains a fairly stable single thread channel because it d not have enough transporting power to attack its banks. Although the mean channel formation is not absolutely stable, because the meander loops ten move downstream, the channel cross-section itself is relatively stable. can suggest, at least tentatively, that a well formed meander channel

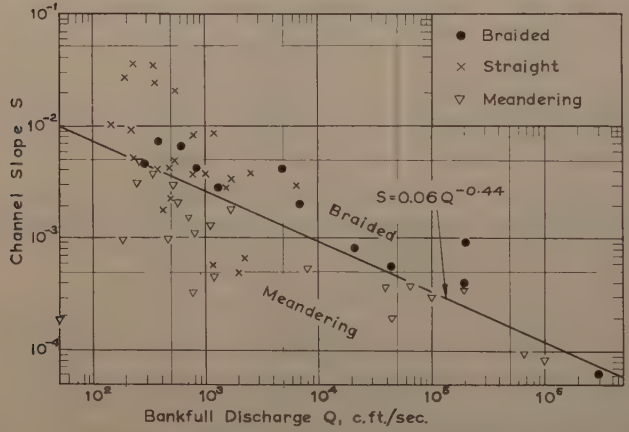

FIG. 4.—LEOPOLD & WOLMAN'S DATA ON NATURAL RIVERS, UNITED STATES AND INDIA

reached a quasi-stable condition that, if the bed material is coarse, wil governed by the threshold criterion.

Points representing a large number of rivers in the United States and I were plotted by Leopold and Wolman on the S-Q plane, as shown in Fig. 4, wi is a reproduction of their Fig. 46. The discharge used was, in the cas single thread channels, the bank-full discharge, that is, the discharge that fills the low-flow channel before overflowing on to the berms, and in the of braided channels, as the discharge of equivalent frequency to the bank discharge in single thread channels.

It was found possible to draw a line on the S-Q plane such that all po representing braided channels were above the line and those representing

18 "River Channel Patterns: Braided, Meandering, and Straight," by L. B. Leo and M. G. Wolman, U.S. Geol. Survey, Professional Paper 282-B, 1957.

ndering channels were below it. This line has the equation

$$S = 0.06\ Q^{-0.44} \quad \ldots\ldots\ldots\ldots\ldots\ldots (68)$$

oints representing straight channels occurred both above and below the line. There are two points to be made about this interesting result: (1) The exponents of Q in Eqs. 19 and 68 are very nearly equal; and (2) if transporting power is to be the criterion that distinguishes braided from meandering channels then the size of the bed material, as well as the slope and discharge, ought to be taken into account. A way of doing this would be to relate the size of the bed material to the distance of each point in Fig. 4 above or below the line, that is, the ratio $S/0.06Q^{-0.44}$. This has been done in Fig. 5, using median bed sizes as given by Leopold and Wolman.

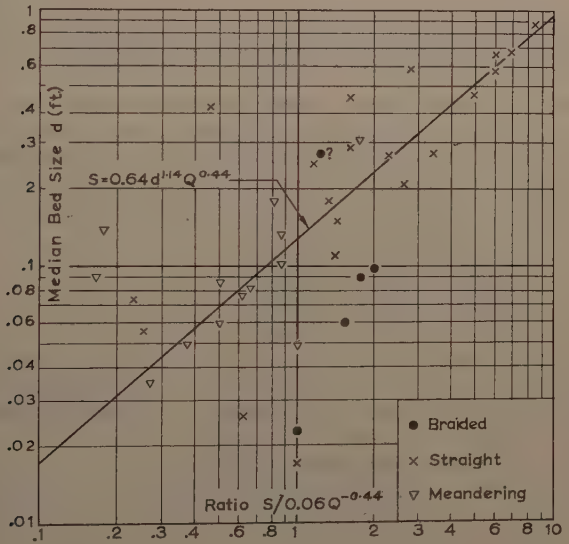

FIG. 5.—S-d-Q RELATIONSHIP, NATURAL RIVER CHANNELS

This representation reduces the scatter of the points to a great extent. In ct a straight line can be drawn, with the equation

$$S = 0.64\ d^{1.14}\ Q^{-0.44} \quad \ldots\ldots\ldots\ldots\ldots\ldots (69)$$

ch that eight out of thirteen meander channel points, and thirteen out of twenty raight-channel points, lie very close to the line. All except one of the braided-annel points lie in a group well below the line. The one that appears above e line is marked with a query in Leopold and Wolman's appended list of riv-s, and so is similarly marked on Fig. 5. The senior author has confirmed in ivate correspondence that this river reach - the Yellowstone River at Bil-gs - might be classified as either braided or straight.

The similarity between Eqs. 19 and 69 is noteworthy, despite the uncertai ties that must attach to empirical equations such as Eq. 69. The coefficien of the two equations differ, as one would expect: First, because the "d" of E 69 is the median, or D-50, bed size, whereas that of Eq. 19 is the "dominan (D-80 to D-90) bed size; second, because Eq. 19 applies only to the Type channel whereas natural rivers have a much greater width:depth ratio than th type of channel.

A further point of interest emerging from Fig. 5 is that both straight a meandering channels conform equally well to Eq. 69; this is quite natural, b cause they both belong to the same family of single-thread channels as distin from braided channels. Although straight channels usually have a symmetric cross-section, and meandering channels an asymmetrical section, the diffe ence in shapes does not appear to be significant in determining a criterion su as Eq. 69. In Fig. 4, on the other hand, straight channels are associated random with both braided and meandering channels. The conclusion is, would be expected, that by taking account of the bed material size one refin and makes more realistic the distinctions between the various kinds of natur river channel.

Quite apart from the question of distinguishing between braided and singl thread channels, Eq. 69 is of interest in suggesting that single-thread channe in coarse material tend to adjust their slopes so as to conform with equatio such as Eq. 19, derived from the threshold criterion. Further, the comparis implied in Eq. 69 holds good between different rivers, as well as between di ferent reaches of the same river.

Now according to the threshold theory, Eq. 19 is true only if

$$P \propto d^{-0.15} Q^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6$$

and the evidence of Eq. 69 would be consolidated if this latter equation we also shown to be true for the group of rivers from which Eq. 69 was derive Unfortunately Leopold and Wolman[18] do not give enough information to enab this check to be made. Even if they did, and Eq. 64 were verified, an explan tion of why this equation should be true in wide channels would be no neare

However the physical evidence embodied in Eq. 69 is at least worthy of no although it would be unwise to attach too much importance to the evidence such a comparatively small sample. It does at least suggest a basis for furth investigation.

Practically all the data on which Eq. 69 is based are on rivers in coar alluvium; in fine alluvium the conditions would presumably be live bed rath than on the threshold of motion, and it might be expected that in the S-Q equ tion corresponding to Eq. 69 the exponent of Q would be - $\frac{1}{6}$, as in Eqs. 5a a 5c. Such an equation would also have to include q_s/q as well as d.

This difference between the exponents of Q in Eqs. 69 and 5c suggests t in a river in very fine material the slope does not decrease so markedly w increasing discharge downstream as in a river in coarse material - that the concavity in a river's longitudinal profile would be more pronounced i river in coarse alluvium. However, this conclusion rests on the assumpt that the bed material size does not change in the downstream direction. T

s not true, and it is possible that changes in bed material size may mask the ffect suggested by the difference in the exponents of discharge.

ANALYSIS AND CONCLUSIONS

It has been shown that by introducing Strickler's formula into the argument, quations similar to Lacey's may be deduced from those defining the threshold f bed movement. The first two of these equations are substantially independent of the width:depth ratio of the channel, provided that ratio is reasonably arge; the third, as originally derived, is dependent on the channel having the imiting Type B section as derived in Lane's theory (in which case a certain -d-Q relation must hold). But this third equation, which relates P to d and Q, an also be true for a wide channel provided there exists an S-d-Q relation, imilar in form to the first.

The similarity between these threshold equations and those of the Lacey egime theory is however of rather limited significance. The threshold theory equires RS to be constant everywhere in a channel having constant bed condiions, whereas Lacey's observations showed that $R^{\frac{1}{2}}$ S was constant throughout ach "regime" channel, that is one having a live bed of fine sediment. It is hown from the Einstein bed load formula and Liu and Hwang's work on flow esistance in alluvial channels that $R^{\frac{1}{2}}$ S should in fact be constant in a channel aving the same d and q_s/q throughout its length. This conclusion is suggested s a rational basis for live-bed equations similar to Lacey's.

It has been seen that in the threshold theory only two equations can be absoutely true for a wide range of channel shapes and slopes. The truth of the hird equation depends on a further relation involving S, Q, and the bed condiions. This limitation applies also to the live-bed theories such as Lacey's. n fact it is in the form of the required S-Q equation that the most distinct difference shows up between the threshold theory, which requires that

$$S \propto Q^{-0.46} \text{ approximately} \quad \ldots\ldots\ldots\ldots\ldots (63)$$

nd the live bed theory, which requires that

$$S \propto Q^{-\frac{1}{6}} \text{ approximately} \quad \ldots\ldots\ldots\ldots (5a, 5c)$$

he bed conditions being assumed constant in each case.

Although there is evidence in favor of the third Lacey equation,

$$P \propto \sqrt{Q} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

nd of Eq. 63, in natural rivers, a general proof of these equations remains lusive. The contribution made by this paper to the problem has been to esablish Eq. 22 and then to lessen its importance by pointing out that its applicaon is restricted to one particular channel shape and width:depth ratio, much maller than those found in natural and artificial channels in the field.

However the following argument may at least indicate a line of attack on the roblem. It has been shown that eliminating R from the first two equations f the threshold theory yields

$$P = \frac{1.14\, Q\, S^{1\frac{1}{6}}}{d^{\frac{3}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (62)$$

A similar procedure applied to the Lacey equations yields the relation

$$P = 1.68 \times 10^{10} \frac{Q S^3}{f^5} \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

Consider now a long stable channel, either threshold or live bed, carr a discharge Q_1, and having a slope S_1 and a width P_1. At some point along length of the channel further flow is now introduced making a total discha of $Q_2 \left(> Q_1\right)$. The question is how the channel downstream of this point adjust itself to the increased discharge. If the slope remains steady, then 62 and 70 simply indicate that

$$\frac{P_2}{P_1} = \frac{Q_2}{Q_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

that is, that the width will increase in direct proportion to the discharge. H ever, in fact the erosion induced by the increased discharge will not only on the banks, thereby increasing the width, but also on the bed, thereby aff ing the slope. The erosion on the bed will result ultimately in material b deposited at the foot of the slope, reducing the slope value in the lower re It is legitimate to speak of the slope's having a foot, because in natural ri no channel is infinitely long.

The net effect is that $S_2 < S_1$, so that

$$\frac{P_2}{P_1} < \frac{Q_2}{Q_1}$$

and the width will increase in proportion to some power of Q less than u To prove that this power is one-half would require a more searching exam tion, and perhaps experimental studies, of the relative importance of bank bed erosion in the situation previously described. However the details work out, it seems that a proof of Eq. 3 can be obtained only by conside the longitudinal profile of a river as an organic whole, including the ef (neglected in the preceding argument) of abrasion and sorting in produ variation of bed conditions in the downstream direction.

The conclusions may be summarized as follows:

1. The Shields criterion for the threshold of motion provides, via La theory, a theoretical basis for the Lacey relation $P \propto \sqrt{Q}$ in the special of the narrowest possible channel consistent with bank stability. In this a certain slope-discharge relation must also be true.

2. The Einstein bed-load formula and Liu and Hwang's resistance form provide, in the case of a live bed, a theoretical basis for the Lacey rel

$R^{\frac{1}{2}} S$ = constant, but tend to show that the relation $v \propto \sqrt{f R}$ is not well fou

3. Whether the bed is fixed (threshold conditions) or live, only two e tions can be deduced immediately from the considerations of bed movemer flow resistance. A third equation, such as $P \propto \sqrt{Q}$, can only be true if a

ain slope discharge relation is true. No general proof of the equation $P \propto \sqrt{Q}$ s obtained, but some supporting arguments can be constructed. In the case of wide channel and threshold conditions, the required slope-discharge relation s of the same form as for the narrow channel of Lane's theory. For a live ed channel the required slope discharge relation is materially different from ne corresponding relation under threshold conditions.

4. The threshold slope-discharge relation referred to previously fits well o the data of Leopold and Wolman on natural rivers in coarse alluvium. The esults derived in this paper may, therefore, form a basis for the rational exloration of the processes by which natural rivers form themselves, both in ross section and in longitudinal profile.

ACKNOWLEDGMENTS

The writer is indebted to L. B. Leopold, United States Geological Survey, ept. of Interior for reading the manuscript and making some most helpful uggestions and criticisms.

APPENDIX.—NOTATION

The following symbols, adopted for use in this paper, conform essentially ith "American Standard Letter Symbols for Hydraulics" (ASA Z10.2-1942), repared by a committee of the American Standards Association with Society epresentation, and approved by the Association in 1942:

= Area of channel cross section;

= coefficient in resistance equation, $v = C R^X S^Y$;

= bed material size;

= complete elliptic integral of second kind;

$$= \sqrt{\frac{2}{3} + \frac{36\nu^2}{gd^3\left(s_s - 1\right)}} - \sqrt{\frac{36\nu^2}{gd^3\left(s_s - 1\right)}}$$

= Lacey silt factor;

= coefficient in equation $P = GQ^{0.512}$;

= coefficient in resistance equation $v = KR^{\frac{3}{4}} S^{\frac{1}{2}}$;

= Manning resistance coefficient;

= wetted perimeter of channel cross section;

= discharge;

q = discharge per unit width;

q_s = sediment discharge per unit width;

R = hydraulic radius, A/P;

R_e^* = particle Reynolds Number $\sqrt{\frac{\tau}{\rho}\frac{d}{\nu}}$;

r = $s_s - 1$;

S = channel longitudinal slope;

s_s = solid: fluid density ratio;

v = mean water velocity;

w = surface width of channel cross section;

$X)$
$Y)$ = exponents in resistance equation, $v = C R^X S^Y$;

x = lateral horizontal distance from channel center line;

y = local water depth;

y_o = water depth at channel center line;

α = $\frac{x \text{ Tan } \phi}{y_o}$

γ = fluid specific weight;

θ = inclination of channel bank to the horizontal;

ν = fluid Kinematic viscosity;

ρ = fluid mass density;

τ = shear stress at the bed;

τ_c = critical shear stress (at threshold of motion);

Φ = Einstein bed load function, $\frac{q_s}{F d \sqrt{g(s_s - 1) d}}$;

ϕ = angle of repose of bed material; and

$\frac{1}{\Psi}$ = shields entrainment function, $\frac{\tau}{\gamma(s_s - 1) d}$.

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TOTAL SEDIMENT TRANSPORT IN THE LOWER COLORADO RIVER

By John R. Sheppard, A. M. ASCE[1]

SYNOPSIS

Construction of major dams on the Lower Colorado River below Grand Canyon has caused continuous channel adjustments between these structures. Channel degradation below each dam and aggradation above each reservoir have created problems of river channel control. To aid in evaluating these problems, the United States Bureau of Reclamation, in 1955, initiated a total sediment load sampling program. This program consists of periodic sampling at a series of stations between Davis and Imperial Dams.

The sampling program, which was established to provide comprehensive data for evaluation and solution of the channel problems, is presented. The methods of sampling, computation, and analysis are described; results of the program over the first 4 yr are given; and new developments and procedures, derived from program evaluation, are discussed.

INTRODUCTION

During 1955, the Bureau of Reclamation, United States Dept. of Interior (USBR) established a series of total sediment load sampling stations on the Lower Colorado River between Davis Dam and Imperial Dam (Figs. 1 and 2). These stations have been operated to determine the quantity and type of sediment being transported by the river within various reaches of the river be-

Note.—Discussion open until April 1, 1962. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 6, November, 1961.

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FIG. 1.—LOWER COLORADO RIVER, LEES FERRY TO GULF OF CALIFORNIA

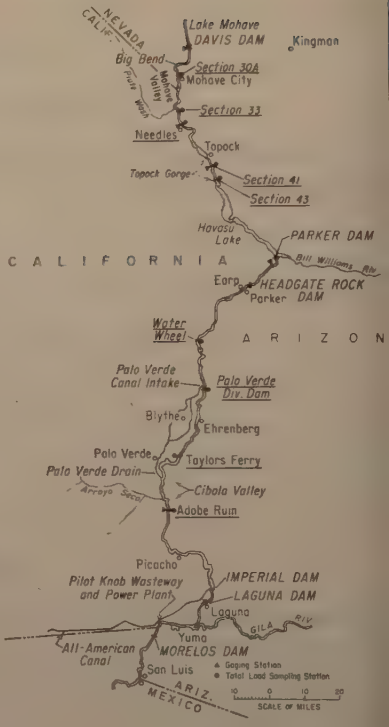

FIG. 2.—LOWER COLORADO RIVE DAVIS DAM TO INTERN TIONAL BOUNDARY

tween major control structures. The data from the stations, when analyz aid in making operation and maintenance estimates, provide data for chann ization design, and will help in evaluation studies of constructed channel c trol work.

HISTORY

The need for and establishment of a total sediment sampling program on Lower Colorado River is based on a long and involved history. At best, it be described only briefly herein. A broader and more detailed descriptio the background history is available.[2,3,4]

2 "Silt in the Colorado River and Its Relation to Irrigation," by Samuel Fortier Harry F. Blaney, Tech. Bulletin No. 67, U. S. Dept. of Agric., February, 1928.

3 "Annual Reports on Retrogression Observations, Salinity Studies, and Suspe Load Investigations—Colorado River Basin," U. S. Bur. of Reclam., Denver, Colo.

4 "Reports of River Control Work and Investigations, Lower Colorado River Bas Office of River Control, U. S. Bur. of Reclam., Boulder City, Nev.

Before the construction of any storage dams on the Colorado River, the regimen of the lower river below Grand Canyon was typical of a river carrying a heavy sediment load. Below Lee's Ferry (Fig. 1), the Lower Colorado River flows through many miles of comparatively narrow canyons that restrict the river more than is usual in a stream of this type. However, the canyons are interrupted by several large valleys, including the Mohave, Palo Verde, and Cibola Valleys (Figs. 1 and 2). Within these valleys the river was free to follow a natural alluvial pattern. The river moved back and forth across the valley bottoms, either slowly by accretion or abruptly by avulsions. The river was actively building up the valleys both by aggradational meander methods and by deposition from the large spring floods on the adjacent flood plain.

With the closure of Hoover Dam in 1935, Parker and Imperial Dams in 1938, and Davis Dam in 1950, a controlled artifical type of regime was forced upon the river (Fig. 1). The large sediment loads carried by the snowmelt floods were deposited in Lake Mead behind Hoover Dam. The clear water releases from each dam picked up a sediment load from the channel banks and bed causing degradation below each dam and aggradation in the backwater area of each reservoir downstream. These river adjustments caused problems in each reach of the river by either lowering or raising the water surface (Fig. 3). These problems have been described[5] by Whitney, M. Borland, M. ASCE and Carl R. Miller, M. ASCE. Figs. 1, 2, 4, and 5 have been taken all or in part from this earlier paper with the kind consent of the authors.

The Lower Colorado River below Hoover Dam is almost a closed system in relation to sediment inflow. The two major tributaries that enter the system are the Bill Williams River and the Gila River.

The Bill Williams River flows into Havasu Lake behind Parker Dam and the Gila joins the Colorado between Imperial Dam and Yuma (Fig. 1). The Gila has not contributed runoff to the Colorado in recent years partially due to upstream storage on the Gila. The sediment loads in the Colorado between Davis and Imperial Dams are predominantly from bed and bank sources. Small tributaries along the river do occasionally move coarse sediment into the river as a result of infrequent, high-intensity rainstorms. The primary source of sediment, however, is that which the river can erode from its own alluvial channel.

Ultimately the Colorado River would naturally adjust itself to the imposed regime. The degradation and aggradation cycle in each river reach between major structures would eventually decrease the channel slope to establish a form of stability. The shifting of bed material sediment by clear water releases would tend to move the smaller bed particles downstream leaving the coarser particles to armor and stabilize the bed. The period of time for such stabilizing action to occur is far too long, however, to help in the immediate solution of the many channel problems.

The USBR has to date (1961) channelized the Colorado River from Big Bend to Topock. A comprehensive plan of channelization is in the planning stage for the Cibola Valley. Additional planning and work may result in a controlled

5 "Sediment Problems of the Lower Colorado River," by Whitney M. Borland and Carl R. Miller, Proceedings ASCE, Vol. 86, No. HY 4, April, 1960.

channel from Davis Dam to Imperial Dam, and possibly on to the Mexic Border (Fig. 2).

PURPOSE

As the problems of channel control developed, the USBR used maintenan procedures at local trouble areas. The aggradation occurring between Needle Calif., and Topock Gorge, however, increased the water surface until pa of the town of Needles were endangered by high water levels (Fig. 2). To relie this situation, the USBR purchased a 20-in. cutter-head dredge and complet rectification of a river channel from Needles to Topock, Arizona, in June 19 It soon became apparent that in order to control this aggradation, the areas the upper Mohave Valley from which the sediment load was derived, would al have to be controlled. To develop a comprehensive plan for rectification of t total length of channel between Davis and Parker Dams, a technical analysis the sediment loads picked up and carried by the river was necessary. The were no quantitative records available of either suspended or bed mater sediment loads for the river reach between Davis and Imperial Dams exc periodic suspended sediment samples at Taylor's Ferry. Sediment moveme estimates were based on resurveys of river cross sections, that determin relative amounts of scour and fill. Technically these figures could not be a justed for channelization design or to determine total quantities of sedim movement for operation and maintenance estimates.

The total sediment sampling program was initially begun to help solve t pressing channel control problem at Needles, but has been expanded sin that time to cover the total reach of channel adjustment length between Da and Imperial Dams. The program will be extended to cover the river rea between Imperial Dam and the border of Mexico.

During the planning and execution of a comprehensive rectification progra the river channel adjustment and associated sediment loads may pass throu four phases:

1. The adjusted channel regime of degradation, aggradation, bank cutti and channel realinement forced on the channel by structural control.
2. The changing channel conditions and sediment loads associated w channelization and rectification construction.
3. The channel adjustment period after channelization construction completed.
4. The stability regime after complete channel adjustment to channeliz tion.

For the first phase, the sampling program helps to evaluate the cause a effect of channel adjustment, aids in estimating maintenance and operat costs, and provides data for technical analysis and design of rectificat plans. During construction activity, the sampling frequency is reduced, becau technically, the program provides only information on loads carried or cau in part by construction activity. The last two phases of the program aid evaluation of the channelization and rectification and provide data for estima of operation and maintenance costs.

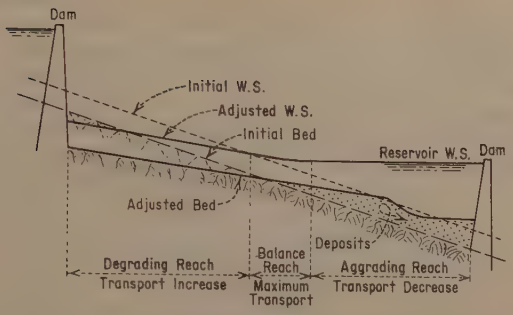

FIG. 3.—DEGRADATION - AGGRADATION CHANNEL ADJUSTMENTS FROM DAM CONSTRUCTION

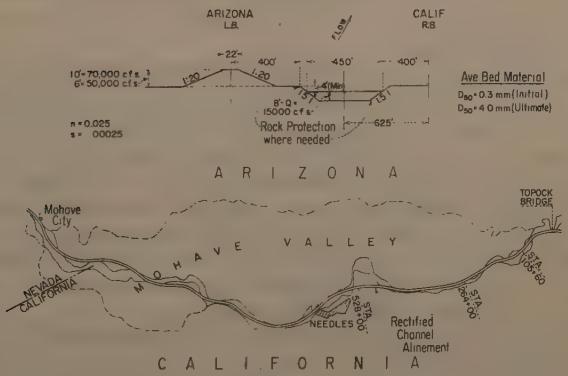

FIG. 4.—COLORADO RIVER CHANNELIZATION BIG BEND TO TOPOCK, ARIZ.

FIG. 5.—PROPOSED COLORADO RIVER CHANNELIZATION TAYLORS FERRY TO ADOBE RUIN

Between Davis and Parker Dams, a comprehensive channelization plan w completed in April 1955.[6] Channelization construction was completed in Ju 1960 (Fig. 4). The sampling program in this reach is in Phase 3 as the chan adjusts to construction. Between Parker and Imperial Dams, a comprehens plan has been completed for channelization in the Cibola Valley and bank pr tective works in the Palo Verde Valley.[7] Construction of this work is schedu to begin in fiscal year 1962 (Fig. 5). The channel above Palo Verde Divers Dam is under surveillance (as of 1961) as a future area of channelization necessary.

METHODS

A total of nine total load sampling stations have been established in length of river between Davis Dam and Imperial Dam. Five of these stati are located between Davis and Parker Dams and four between Parker Imperial Dams (Fig. 2). No stations have been located between Hoover Davis Dams because this reach of the river is almost entirely in the ba water area of Lake Mohave.

One sampling station in each reach has been established near the balance ar of the reach. Because both degradation and aggradation occur in each rea there is a certain length of channel in which degradation changes to aggra tion and a balance is reached. This balance area may shift up or down river with different hydraulic conditions or levels of downstream reserv storage but with present conditions is within the same general location. T balance area also represents the location at which the maximum total load carried by the river (Fig. 3). In the Davis to Parker reach, the balance are upstream from Needles. In the Parker to Imperial reach, the balance area near or downstream from Taylor's Ferry (Fig. 2).

Additional sampling stations upstream and downstream from the bala area in each reach were established to create a comprehensive picture sediment transport within each reach as well as provide for possible shift of the balance area under changing conditions of channel development.

In order to adequately determine channel shape and hydraulic propert compatible with sediment and water transport, it was necessary to know total sediment transport at the strategic locations and the size distributio the suspended load and bed material.

Because the suspended load carried by the Colorado River below Davis Parker Dams is not a sufficiently large percentage of the total load tra ported, it was necessary to use some procedure to estimate or compute total load transported. After some consideration, the modified Einstein p cedure was chosen.[8] This method was adaptable to a single cross section had received wide usage by the USBR and Geological Survey, United Sta

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6 "Report on Comprehensive Plan, Colorado River Channelization, Big Ben Needles Division, Colorado River Front Work and Levee System," U. S. Bur. of Recl Boulder City, Nev., April, 1955.

7 "Report on Comprehensive Plan, Colorado River Channelization, Palo Verde Cibola Valley Divisions, Colorado River Front Work and Levee System," U. S. Bu Reclam., Boulder City, Nev., December, 1959.

8 "Computations of Total Sediment Discharge, Niobrara River near Cody, Nebras Water Supply Paper 1357, U. S. Geol. Survey, Dept. of Interior, Washington, D. C., 1

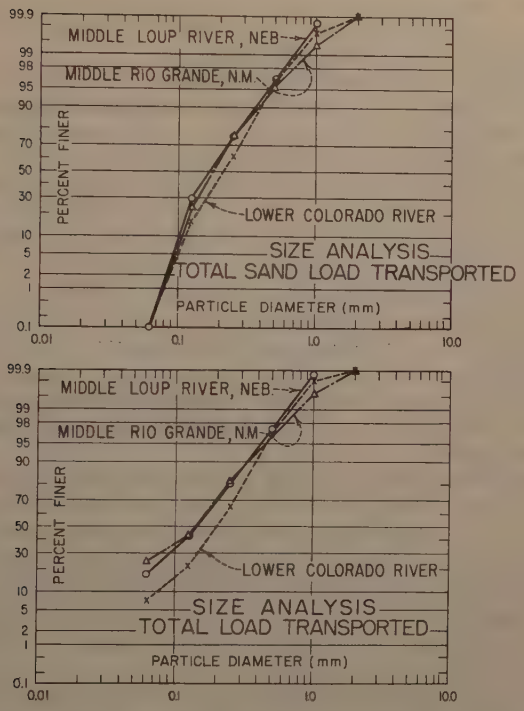

FIG. 6.—SIZE OF MATERIAL IN TRANSPORT LOWER COLORADO, MIDDLE LOUP AND MIDDLE RIO GRANDE

Dept. of Interior (USGS) in the sand hills region of Nebraska and in the Middle Rio Grande.[9] The transported materials on the Colorado and the other streams are quite similar (Fig. 6).

The data needed to carry out the modified Einstein procedure consist of the following:

a. Water discharge during the sampling period;
b. channel width, area, and average depth;
c. water temperature;
d. average depth of sampling verticals;
e. average suspended concentration;
f. size distribution of suspended sediment; and
g. size distribution of bed material.

The hydraulic parameters (a and b) are determined by standard procedures for stream gaging using current meter measurements. The water surface slope over a 1,500-ft length near the sampling section is also measured. The

9 "Application of the Modified Einstein Procedure for Computation of Total Sediment Load," by K. B. Schroeder and C. H. Hembree, Transactions, Amer. Geophysical Union, Vol. 37, No. 2, April, 1956.

slope is used to compute Manning's "n" roughness values associated with discharge and computed total load. These "n" values can then be used in draulic computations for rectification design.

The procedure for suspended sediment sampling was selected to give most representative data with a minimum of laboratory analysis. Although cross sections within the two reaches show some degree of regularity, obs vations indicated an irregularity in sediment transport. Large boils or clo of material from the bed swirl up into suspension at irregular spaced interv across the width of the stream. To obtain, as nearly as possible, a represen tive sample of the suspended sediment condition, the equal-transit-r (ETR) method of sampling was adopted.

The ETR method of sampling was originated in 1946, by B. C. Colby.[1] was developed to eliminate approximate and arbitrary methods of suspen sediment sampling. Some of these methods required a detailed knowledg the discharge distribution across the cross section where a sample was to collected. The ETR method eliminates this requirement by providing a s pended sediment concentration weighted for discharge distribution during sampling measurement.

A sampler that samples at stream velocity, if held in the center of an cremental area of the cross section, will collect a sample volume proportion to the velocity at the center. If the velocity at the center of each increme area is representative of that area, the volume of sample is proportionat the discharge as well. The volume of sample from each equal incremental a composing the cross section can be added together to give a total sample p portionate to the average velocity and the total discharge, provided the samp were all collected during the same time interval.

For the ETR method of sampling, the stream width should be divided sections of equal width. It is assumed that the suspended concentration at middle of the section is representative of the concentration throughout the s tion. The sampler is lowered and raised at the same rate at the cente each section. The sample collected in this way is integrated throughout depth because the sampler is in each unit of depth for the same length of ti The samples collected at each section centerline can be combined into sample weighted for average velocity and total discharge. This sampl representative of the depth sampled. A small portion of the total flow is sampled, however, because the sampler does not sample to the bed. A sr area of the suspended flow is, therefore, unmeasured (Fig. 7).

On the Lower Colorado River where the channel width is between 300 f 500 ft, 25 equally spaced verticals are usually sampled using a US-D-49 d integrating sampler.

The sampling sections were chosen in reaches of the river where unif conditions of flow (and hence uniform sediment transport) were the best. T was done so that only one computation of total load for each measurement necessary. If a sampling station has different types of hydraulic condition different parts of the flow section, a separate total load computation shoul made for each part. Even though the sampling sections were selected, som

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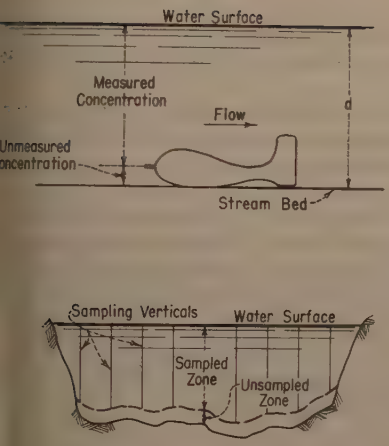
10 "A Study of Methods Used in Measurement and Analysis of Sediment Loa Streams, Measurement of the Sediment Discharge of Streams," Report No. 14 (Rev of Report No. 8) Subcommittee on Sedimentation, Inter-Agency Committee on W Resources (scheduled for future publication).



IG. 7.—MEASURED AND UNMEASURED LOAD SUSPENDED LOAD SAMPLING

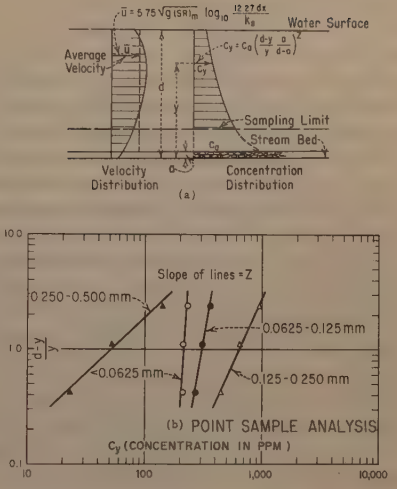
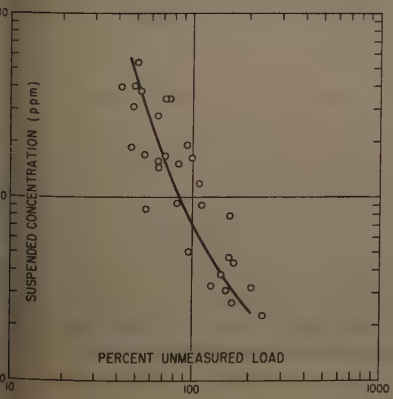


FIG. 8.—VELOCITY AND CONCENTRATION DISTRIBUTION MODIFIED EINSTEIN PROCEDURE



G. 9.—PERCENT UNMEASURED LOAD AND SUSPENDED CONCENTRATION WATER WHEEL SAMPLING STATION

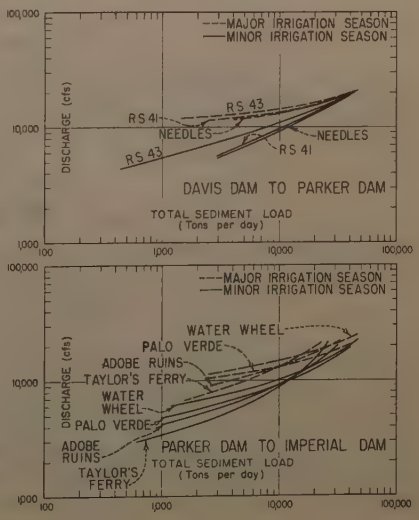


FIG. 10.—TOTAL LOAD RATING CURVES DAVIS DAM TO IMPERIAL DAM

the sections have different types of bed material in parts of the section varyi from gravels to sands. The gravels do not transport under normal discharg however, and the sands on the bed are used in the computation as the bed mat rial in transport.

A scoop-type bed material sampler is used to collect the bed sample at t sampling locations across the stream width. The ten samples are composit into one total sample for analysis.

Size distribution of both suspended and bed material samples is analyz by the use of standard sedimentation sieves. The percentage of material geometric progressive size fractions from the sieve analysis is used in t computation of total load.

The modified Einstein procedure for computing total sediment load i complicated mathematical composite of theory, laboratory flume experim results, and field study modifications based on an original paper by H. Einstein, F. ASCE.[11] When the procedure has been checked by various me ods, it appears to result in reasonable values of total load.[9] The necessa data for computation of total load have been described previously. The pr cedure basically integrates suspended sediment concentration distribution w depth, velocity distribution with depth (Keulegan), and a bedload function arrive at a total load in transport.

Fig. 8(a) is a graphical interpretation of the basic functions involved in modified Einstein procedure. The suspended concentration at any distanc from the bed is represented by C_y. If the concentration at any distance fr the bed, such as at a, is known, the concentration at any depth can be compu based on the formula

$$C_y = C_a \left(\frac{d-y}{y}\,\frac{a}{d-a}\right)^Z \quad \ldots\ldots\ldots\ldots$$

in which Z is the logarithmic slope of concentration with depth [Fig. 8(b)]. T velocity distribution with depth is based on the formulas proposed by Keule and developed by Einstein[11] with a modification using the shear velocity

$$u_m = \sqrt{g\,(SR)_m} \quad \ldots\ldots\ldots\ldots$$

and, for the transition between hydraulic rough and hydraulic smooth be results in the expression

$$\bar{u} = 5.75\sqrt{g\,(SR)_m}\,\log_{10}\frac{12.27\,dx}{k_s} \quad \ldots\ldots\ldots\ldots$$

in which $\bar{u}$ is the average velocity of streamflow, g denotes the accelerat due to gravity, $(SR)_m$ describes the slope-hydraulic radius function, d is average depth of flow, x refers to the parameter for transition from smooth rough boundaries, and k_s denotes the roughness diameter. In the modif Einstein procedure, the rate of bedload transport per unit width for one-s fraction is

$$i_b\,q_b = 1{,}200\,D^{3/2}\,i_b\frac{\phi_*}{2} \quad \ldots\ldots\ldots\ldots$$

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11 "The Bed-Load Function for Sediment Transportation in Open Channel Flows, Hans Albert Einstein, Tech. Bulletin No. 1026, SCS, U. S. Dept. of Agric., September, 1

in which D denotes the geometric mean diameter of a size fraction, i_b refers to the percent by weight of bed material in a size fraction, and ϕ_* is the intensity of bedload transport.

Using the collected field data, the modified Einstein procedure determines the rate of bedload transport for a size fraction. This rate determines the concentration C_a at reference level a above the bed. The distance a is used as two-grain diameters (D) above the bed. The shear velocity is determined by using the measured average velocity $\bar{u}$ in the average velocity formula by assuming values of the rough-smooth parameter x (trial and error). Using the roughness conditions determined by the average velocity formula, the reference concentration C_a, and the average suspended concentration (field data), a trial and error determination of the concentration gradient with depth (Z) is found for a size fraction. This concentration gradient is used to determine the suspended sediment load in the unsampled zone, which, when combined with the measured suspended load, results in total suspended load. The total suspended load is combined with the bedload function which results in total sediment load.

The USBR uses the flow-duration, sediment-rating curve method of analysis to investigate sediment movement in streams.[12] This method is particularly adaptable to the Lower Colorado because it can be used to compute sediment loads for different types of operating conditions once a sediment-discharge relationship has been determined for a particular channel condition.

Besides providing basic data for determining total sediment loads, the collection and computations provide a great deal of background information that aids in analyzing the sediment problems of river control and that also may be used to examine various sediment movement theories and formulas. The data give a complete picture of the size of material in suspension, on the bed, and in total transport for the changing transport conditions for degrading sections to aggrading sections.[13] Other relationships can be investigated such as the variation of unmeasured load with suspended concentration (Fig. 9) or the change in size of transported material with discharge or concentration. Other sediment transport equations either old or newly developed can be evaluated by the use of the basic data and the results of the total load computations.

RESULTS

The primary result of the total sediment load program on the Lower Colorado River has been the establishment of sediment transport conditions in the reaches of the river between major structures. These transport conditions have been used to evaluate the magnitude of the problems in river control, develop designs for channelization and rectification plans, evaluate the plans on the basis of sediment control, and estimate maintenance work required on the river reaches. The program in the future will result in evaluation of channelization construction and the final stability of the channel.

The total sediment load rating curves developed from the program results are shown in Fig. 10. Because the river is controlled between Davis and Im-

12 "Analysis of Flow Duration, Sediment-Rating Curve Method of Computing Sediment Yield," Sedimentation Sect., Hydrology Branch, U. S. Bur. of Reclam., Denver, Colo., April, 1951.

13 "Interim Report, Total Sediment Transport Program, Lower Colorado River Basin," Sedimentation Sect., Hydrology Branch, U. S. Bur. of Reclam., Denver, Colo., January, 1958.

perial Dams, the normal flows in this part of the river are releases for down stream irrigation requirements and occasionally for power production re quirements. The major irrigation requirements are at or below Imperial Da and releases from upstream reservoirs are based on advanced orders for th water. During the months of April through September irrigation demands ar higher and releases in the river are larger. During October through Marc irrigation demands drop to some extent and the river carries lower flows. Th higher flows of the major irrigation season carry a relatively smaller sed ment load than the lower flows of the minor irrigation season. This relatio ship is shown as two rating curves for each station in Fig. 10. Basic reaso for the change in the sediment-discharge relationship are probably (1) avai ability of sediment materials in the banks and bed of the stress and (2) tem perature variations which coincide with irrigation demands. If the bed and ba materials are available for transport at a constant rate, the higher flows wou logically carry a small sediment load relative to the lower flows. Howeve bank instability, caused by bank drainage, is more pronounced after recessi of the higher flows making available a larger supply of sediment for transpo by the lower flows. The seasonal temperatures are higher from April to Se tember when irrigation releases are high, and are lower from October throu March when releases are low. The increased viscosity of the water duri lower temperatures would tend to increase the concentration carried. The sed ment rating curves also show the increasing sediment load carried by a flo down the river until the degradation-aggradation balance area is reache Stations below the balance area show the decrease in sediment load carri by a flow.

A secondary result of the program has been the use of the sediment tran port relationship at Adobe Ruins to evaluate sediment conditions at Imperi Reservoir and the required releases from Imperial for river regulation pu poses between Imperial and the boundary with Mexico. The flow into Imperi Reservoir is divided into diversions to the All-American Canal, the Gila Ma Canal, and releases to the river. The sediment inflow to Imperial Reservoi computed at the Adobe Ruins total load station, is also divided into the pa that deposits in the reservoir and the part that is outflow. The major part the sediment outflow is returned to the river from the All-American Can desilting basins, the Gila Canal settling basin, and the California sluicew releases. A certain amount of the incoming flow is necessary to carry t returned sediment loads on down the river without causing channel deterior tion from aggradation or degradation. The sediment problems below Imperi are being carefully studied, and the total sampling program has been inval able to these studies.

During the progress of the program, some new developments have be investigated and adopted that improve the program's effectiveness. The developments were necessary to correct deficiencies in the computation pr cedure for the total sediment load.

The modified Einstein procedure, as developed by the USGS,[8] uses general relationship of the 0.7 power of the fall velocity ratio for two si fractions, to determine the Z exponent of suspended sediment concentrati change with depth. During the first part of the Lower Colorado program, u of this relationship resulted in total size fraction loads for coarser sand siz that were less than the load measured in the suspended sediment sampli

zone. Because the computed loads for these size fractions were obviously in error, other methods of determining Z values were investigated.

If the bed material in transport and the material in suspension are similar, the size analyses will show an overlap in several size fractions. This is the case on the Lower Colorado where the "wash load" is small and the largest part of the suspended materials is derived from the smaller bed materials. (Wash load is defined as the finer part of the suspended sediment load which

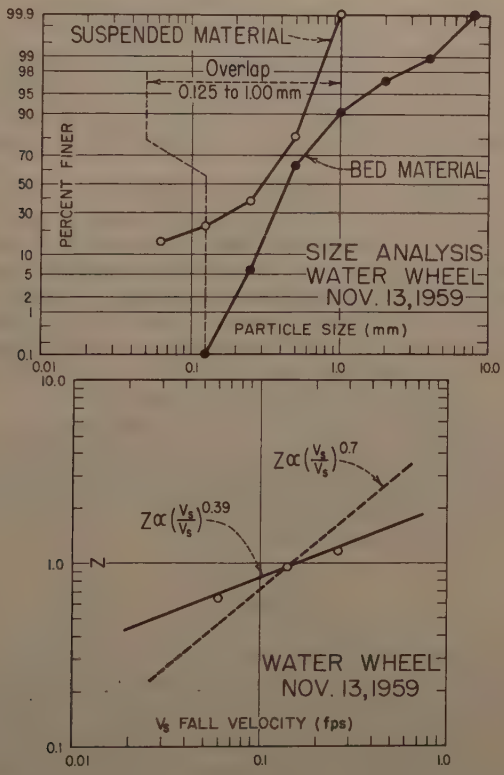

FIG. 11.—OVERLAP FOR Z RELATIONSHIP SUSPENDED MATERIAL AND BED MATERIAL

s not found in the bed materials and is, therefore, transported entirely in uspension.) With this size overlap of suspended and bed materials, a Z value or more than one size fraction can be computed using the measured suspended oad and the bedload function of each size fraction. The Z values determined n this manner can then be plotted with fall velocities of the size fractions to how the relationship between the size fractions as opposed to the 0.7 power elationship (Fig. 11). The USBR has found the Z relationship varies for each

measurement and even that the Z relationship between the size fractions f the same measurement may not be a straight logarithmic function. Unfortunat ly, enough time has not been available to evaluate how other variables m effect the determined Z relationship.

The calculation of total load by the modified Einstein procedure requir several hours for one sampling measurement. Determination of the Z value f one or more size fractions requires the most time within this period. process the many sampling measurements efficiently, the USBR, with init assistance from the Omaha Division of the Corps of Engineers, has develop an electronic computer program for the IBM 650 to calculate total loads fr sampling measurement data.[14] A short description of this program is ava able.[15] The program was developed with as much flexibility as possible that it could be used on most alluvial streams. It was also developed for r search purposes in evaluating the modified Einstein procedure, and is ada able to improvements in suspended load theory or other improvements in t procedure itself.

The computer program is divided into two phases (or actually two separ programs). The first phase uses the sampling data to compute Z values a fall velocities for as many size fractions as desired. The relationship of t Z values as computed by the procedure can be analyzed and final Z valu selected. The final Z values are then inserted into the second phase which co putes values of both total suspended sediment load and total sediment load f each size fraction. Any improved methods of determining suspended concentr tion with depth as expressed by Z values can be used in the second phase wi out consideration of the results of the first phase.

Some attempts have been made to determine the actual Z values associa with a sampling measurement by the collection of point samples throughout t flow. With the small concentrations carried by the flows and the uneven tra port of the suspended materials in the Lower Colorado River, a point sam analysis does not give dependable results.

SUMMARY

The change in river regime on the Lower Colorado River caused by construction and operation of Hoover, Davis, Parker, and Imperial Dams, created problems of channel control. To evaluate the hydraulic and sedim transport conditions associated with these channel problems and to aid channelization and rectification planning, the USBR beginning in 1955, es blished a total load sampling program using nine total load sampling stati on the river between Davis Dam and Imperial Dam.

The sampling program consists of periodic measurements of suspen sediment, bed material sediment, and hydraulic properties at the sampl stations. The sampling stations are located to determine sediment transp in degrading, balanced, and aggrading areas of the river in reaches betw major structures.

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14 "Determination of Total Sediment Load in a Stream by the Modified Einstein P cedure," Electronic Computer Program Description No. HY 100, Commr. Office, U Bur. of Reclam., Denver, Colo., September, 1959.

15 "Electronic Computers Used for Hydrologic Problems," by Francis E. Swain Herbert S. Riesbol, Proceedings, ASCE, Vol. 85, No. HY 11, November, 1959.

Results of the sampling measurements are used to compute total sediment loads by the modified Einstein procedure. The total loads and associated discharges are used in a flow-duration, sediment-rating curve analysis to estimate past or future sediment transport.

In evaluating the program and its results, improvements have been developed primarily in the modified Einstein procedure computation. The improvements are (1) an individual measurement analysis of the relationship of suspended concentration with depth for various size fractions and (2) the adaptation of the procedure for use on the electronic computer.

The program has accomplished its primary purpose of estimating total sediment transport conditions in the river for evaluation and design of rectification measures as well as providing a store of information on the changing regime of the river.

The program does not result in a perfect solution of total sediment transport but is based on the best methods available to accomplish its purpose.

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THEORETICAL AND PRACTICAL ASPECTS OF WELL RECHARGE[a]

By Paul Baumann,[1] F. ASCE

SYNOPSIS

It is the purpose of this paper to show that hydraulically a recharge well is far more complex than a water well in reverse; that in order to accomplish the desired purpose its performance must be clearly understood; and that features of practical nature such as its construction, operation, and maintenance may decisively affect its economy and success.

INTRODUCTION

Contrary to water extraction wells of multitudinous design, henceforth to be referred to simply as water wells, that have been in use since the dawn of history and probably before, wells for the purpose of replenishment of groundwater or repressuring of aquifers or oil strata, generally known as recharge wells, are of more recent origin. Their use has been accelerated through a sustained period of drought especially in southwestern United States, coinciding with enormously increased extraction and consequent depletion of groundwater resources. This gave rise to the intrusion of sea-water into groundwater basins along the sea coasts of our continent and those of nearby islands. In the interior also, the recharge of aquifers and oil sands through wells by means of fresh water, reclaimed water, or sea-water as the case may be, has

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[a] Presented at the August 16-18, 1961 ASCE Hydr. Div. Conf., at Urbana, Ill.

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gained momentum. There, intrusion of saline water, not necessarily of mari origin into fresh water aquifers, has in certain parts gained alarming pr portions.

Nevertheless, the literature on recharge wells and their design, operati and maintenance is, in light of their short historical background, quite limite It is hoped that this paper which will be confined to water-bearing aquife will supplement if only to a small degree, current knowledge on respecti recharge wells and will aid in the understanding of their idiosyncrasies

THEORETICAL ASPECTS

In treating theoretical aspects of recharge through wells, the writer w confine himself to recent publications and to the interpretation of salient fe tures therein presented. No new theories will be advanced. However, present tion of new concepts based on existing theories will be included.

Comparison Between Water Wells and Recharge Wells.—

Horizontal Aquifers.—In view of the long history of water wells and the sh history of recharge wells, it is understandable that the latter are often co sidered as simply being water wells in reverse. Actually recharge wells ha their own idiosyncrosies and characteristics quite distinct from those of wa wells.

A water well is considered in equilibrium if the free surface—or piez metric draw-down remains constant for a constant rate of pumping. The sha of the cone of depression then remains unchanged. If the well extends to bottom of an open aquifer, to the impervious stratum as shown in Fig. 1, a its perforations are continuous, a steady increase in the rate of pumping a therefore, of draw-down will eventually lead to a maximum or critical r beyond which the well is unable to deliver more water irrespective of capacity of the pump. This is because the water below the cone of depress has reached the critical depth h_c, that is to say, the limiting depth necessa for the conveyance of flow corresponding to the maximum rate of pumpi to the well.

Hence, any water well in an open or confined aquifer under natural con tions, is subject to this limitation. It was previously shown[2] that the criti depth,

$$h_c = \frac{q_{max}}{K} \quad \text{.....} \quad ($$

in which q_{max} is the maximum discharge at well in cu ft per hr, and K is Darcy coefficient of permeability in ft per hr for unit gradient.

As also indicated in Fig. 1 the draw-down of a well in a pressure aqui may extend below the confining clay cap. This has been observed in the coas plain of Los Angeles County, Calif., where nearly all aquifers, open and c fined, have been pumped in excess of their safe yields. This, evidently, is unhealthy situation not only from a standpoint of ground-water hydrology also from that of soil stability. Clay caps, deprived of their support by artis pressure, are subject to subsidence under the weight of over-burden.

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2 "Ground-Water Movement Controlled Through Spreading," by Paul Baumann, Tra actions, ASCE, Vol. 117, 1952, p. 1042.

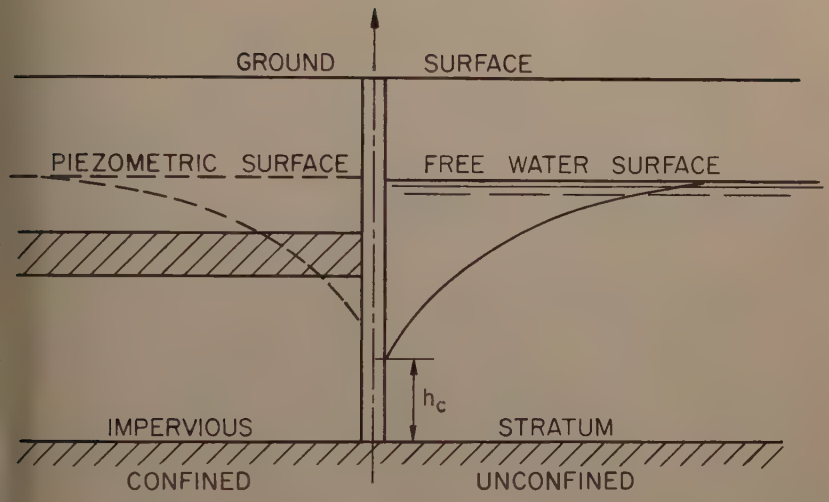

FIG. 1.—WATER WELL

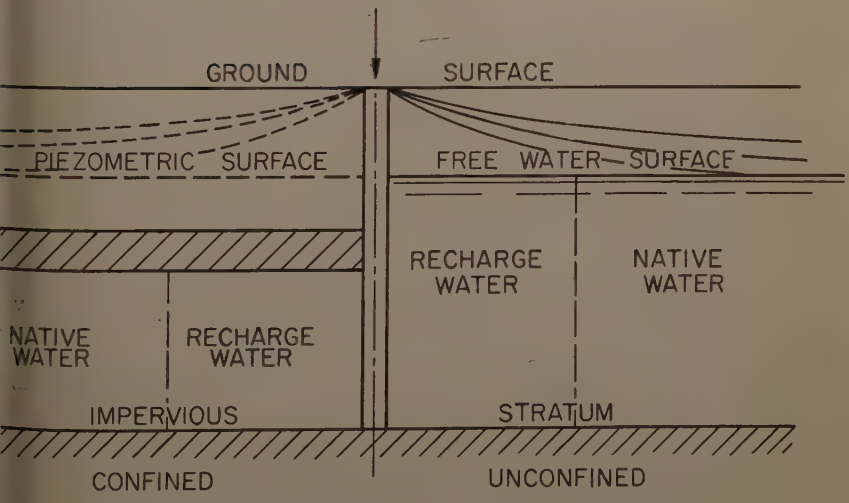

FIG. 2.—RECHARGE WELL

Fig. 2 shows, in as simplified form as Fig. 1, a recharge well in both open and confined, initially undisturbed, semi-infinite horizontal aquifer. It at once apparent that the two salient features typical for a water well, nam equilibrium and critical depth do not apply to a recharge well. For a ste rate of recharge the shape of the cone of impression does not remain cons but undergoes continuous flattening of its slopes as the mound it envelo expands if the aquifer is unconfined. Likewise, if the aquifer is confined, piezometric cone of impression steadily flattens its slopes as the bul recharge water expands and thereby displaces the native ground-water. flattening of the cone of impression as recharge continues is indicate Fig. 2. It is important to note that its curvature is concave upward in open and confined aquifers so long as the aquifer is (quasi) homogenuous there is no obstruction controlling flow. If the head in the recharge wel limited by the ground surface, as is usually the case, then recharge will when the slope of the real or piezometric cone of impression is zero.

It will be shown that in stemming intrusion of saline or other water greater density by means of a fresh-water barrier, the curvature of the "co reverses itself as the flow changes from recharge to discharge.

It has previously been shown[3] that the ground-water mound created thro recharge of an open horizontal aquifer moves away from the center of recha that is from the well, radially in form of a wave. The celerity of this wave greatly exceed the velocity of ground-water particles. An observation w quite remote from the recharge well may, therefore, register a rise in water table long before any part of the recharge water has reached the obse tion well. That this wave effect may be of great importance, where the t element in stemming the intrusion of undesirable waters is crucial, is s evident. On an example it was shown[4] that in an open aquifer inclined slope of 0.0025 the time required for the two dimensional mound wave to re a target would have been about 1/300 of the time required for a water part of the native stream to reach the same target. Although the "fan-out" of th dimensional flow would cause deceleration of the mound wave, its tim reaching a target would still be only a fraction of that of a water part

Inclined Aquifers.—The foregoing reference to horizontal aquifers mus considered chiefly academic, used primarily because of the simplicity it l itself to, in demonstrating fundamental differences between water wells recharge wells. Actually no major, natural aquifer, open or confine horizontal. All of them extend from an area of natural recharge to an are natural discharge. Recharge water springs from rain and runoff from Areas of recharge are deposits of high porosity and permeability along slopes of mountains and foothills and areas of discharge are in the for seepage surfaces above and below, or both, rivers, lakes, or the oce In general all major aquifers change from open to confined as they ex from the area of recharge to the area of discharge.

The slope of important aquifers is normally small, selfom in excess 50 ft per mile. Under these conditions the performance of a water well not differ significantly from that in a horizontal aquifer. The contours o cone of depression will be slightly elliptical instead of circular, but eq

3 Ibid, p. 1047.

4 "Basin Recharge," by Paul Baumann, a part of "Ground-Water Developmen Symposium, Transactions, ASCE, Vol. 122, 1957, p. 461.

rium can establish itself after a period of unsteady flow; the same holds rue for a horizontal aquifer.

In contra-distinction thereto, the performance of a recharge well in an nclined aquifer differs fundamentally from that in a horizontal aquifer. Paradoxically, however, it is only with an inclined aquifer that a recharge ell can attain equilibrium. Theoretically the time required to attain it is nfinite which, incidentally, is also true for water wells. Strictly speaking, ne should, therefore, refer to quasi equilibrium that will establish itself ithin finite time.

The principal distinctions from a water well are that with a recharge well point of stagnation will establish itself upstream from the recharge well and stream will emanate therefrom on the downstream side, the boundaries of hich approach two asymptotes. For a given rate q_o of recharge and of initial round-water flow corresponding to the transmissibility $K a_o i$ per unit width f aquifer the distance X_o from the recharge well to the point of stagnation nd the distance W between the two asymptotes can at once be determined. espective equations read:[5]

$$X_o = -\frac{W}{2\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

nd

$$W = \frac{q_o}{K a_o i} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

n which a_o is the depth of undisturbed ground-water flow; i represents the lope of aquifer; and K is as given previously.

Fig. 3 shows in plan the configuration of the ultimate recharge stream and f three intermittant transitory stages in form of bulbs thereof, superposed on ne streamlines and the equipotential contours of the parent ground-water flow. It is based on recharge conditions which are believed to be more or less ypical, namely

$K = 5.00$ ft per hr
$a_o = 100$ ft
$i = 0.0025$
$q_o = 4000$ cu ft per hr

Therefore, W = 3200 ft and $X_o = -510$ ft.
he scale of Fig. 3 is thereby established. The interval of ground-water con- ours = 1 ft.

Although Fig. 3 depicts the performance of a recharge well in an inclined pen aquifer, the essential features apply to a confined aquifer as well. These re: the controlling influence of the recharge water on the native ground-water; ne separation between the two causing the recharge water to act as a semi- nfinite "island"; the extent of the back-up effect upstream from the recharge ell; and the acceleration of native ground-water around the lateral boundaries f the "island." The principal difference between recharge of open and confined quifers is the absence of storage in the latter. Hence, the pressure trans- ission in a confined aquifer is more immediate.

5 "Ground-Water Movement Controlled Through Spreading," by Paul Baumann, Trans- ctions, ASCE, Vol. 117, 1952, p. 1050.

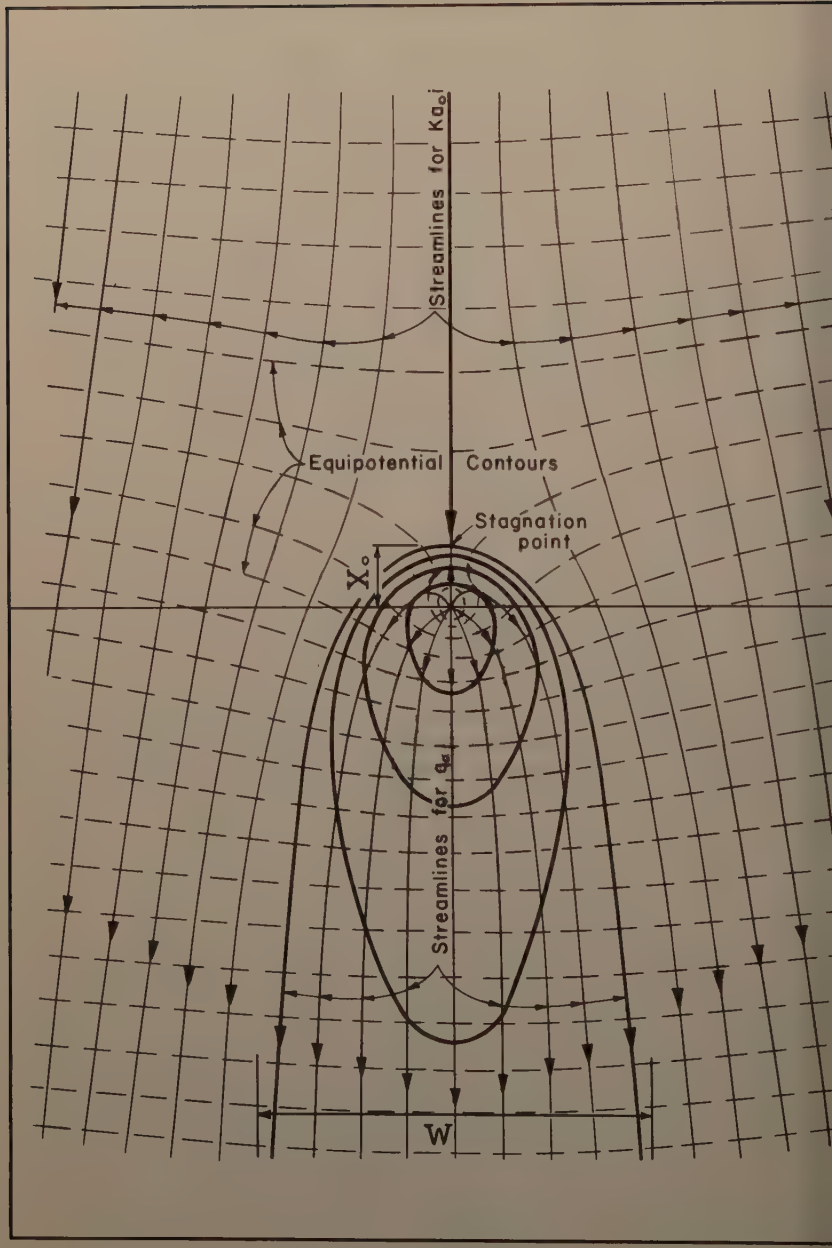

FIG. 3.—RECHARGE THRU WELL IN OPEN, INCLINED AQUIFER

Control of Native Ground-Water in Confined Aquifers.—The profound effect single recharge well may have on a native ground-water stream in an open hifer would of course be multiplied if a row of such wells were placed in a hfined aquifer. In fact the native ground-water flow could be completely ecked, especially if the recharge wells were spaced less than W feet apart l the boundaries of adjacent "islands" did intersect. In this case the com- ed effect of the individual recharge wells would be comparable to the ect of one recharge well of the combined capacity. Hence, by means of re- arge wells so arranged in a confined aquifer, a barrier can be established t will prevent movement of qualitywise undesirable native ground-water st this barrier. Its application to stem intrusion of foreign ground-water, mely sea-water will be treated below.

In light of the foregoing it follows that recharge wells in confined aquifers, t intended for the control of flow but for recharge only, must be spaced a fficient distance apart to avoid undesirable constriction of native ground- ter flow and back-up effect resulting therefrom. Hence, such spacing ould be not less than 2 W.

Control of Native Ground-Water in Open Aquifers.—If in Fig. 3 the contours the native ground-water table were parallel to and a distance D below the ound surface, only incomplete control of native ground-water flow could be ained by means of recharge wells alone even if the apex of the cones of pression should be at the ground surface. This because native ground- ter would be caused to rise and to flow between the cones of impression of e recharge wells similar to surface water flowing between bridge piers. e lack of control would be enhanced if the native ground-water happened to saline or other undesirable water of higher density than fresh recharge ter.[6] Under these conditions control could only be accomplished by means an intercepting open trench or covered drain located upstream from the charge wells, combined with dewatering facilities capable of draining native ound-water flow above the internodal points between the recharge wells. nce, the minimum depth of the intercept between two adjacent recharge lls would have to be equal to D minus the height of the internodal point ve the undisturbed ground-water table.

Spreading Strip Versus Recharge Wells.—From the preceding description costly drainage facilities it might appear that a continuous spreading strip, her than recharge wells should be resorted to in controlling native ground- ter flow in open aquifers. This could well be more economical if the maxi- m potential of the native ground-water were known to be such as to prevent rise to the surface of the ground. Otherwise similar drainage facilities as scribed above would have to be provided, although most likely at lesser pth and therefore lesser cost.

If otherwise open aquifers were intermittently topped by near impervious rficial layers of such thickness as to render open trenching for strip reading uneconomical, the use of closely spaced seepage pits might be sorted to. They would consist of bucket-drilled holes, some 36 in. in dia- ter which would penetrate the tight, surficial layers and which would be ed with clean pea gravel. To facilitate maintenance of satisfactory opera- n a "mushroom" of sand and pea gravel, acting as a filter should be de-

[6] "Ground-Water Hydrology," by David K. Todd, John Wiley and Sons, Inc., New York, 9, p. 278.

posited on the surface over the hole. This "mushroom" could be periodic removed and cleaned if necessary.

Straight recharge of open aquifers would normally be most economic accomplished by means of shallow basin spreading grounds.

Foreign Ground-Water.—Of foreign invaders of ground-water basins, s water must be considered the most notorious of all. Not only is there an exhaustible supply of sea-water, but its potential horizon contrary to tha ground-water remains substantially unchanged. Therefore, its maxim potential is known contrary to that of many native ground-waters.

In a previous paper[7] the fresh-water barrier of the Los Angeles Cou Flood Control District to stem sea-water intrusion into the West Basin aqu of Los Angeles County, Calif., by means of recharge wells, was treated constructively examined. This barrier will, henceforth, be referred to as W Basin Barrier. Reference is made especially to the piezometric hydra gradient between the barrier and the ocean, a distance L = 2000 ft±. As sta previously, this gradient is concave upward where associated with recha changing to reversed curvature where associated with discharge.

Also, in the referenced paper it was shown[8] that sea-water had intruded aquifer and completely displaced the fresh water for a distance of clos 2000 ft in from the ocean. Beyond this distance sea-water had crept landw for an additional 1500 ft in form of a wedge. After 18 months of recha operation the fresh-water bulb had expanded seaward about 500 ft maxim immediately below the clay cap. At the bottom of the aquifer the bulb, wh may be roughly described as an inverted truncated cone, showed little o seaward expansion. Hence, it must be concluded that the bulbs of adja wells, 500 ft apart, had merged near the top but were still separated at near the bottom of the aquifer. The fresh water barrier at that time w therefore, incomplete. The much expressed concern about wasting fresh wa to the ocean as an argument against respective barriers, thus appears to h less justification than the concern about continued intrusion of sea-water was still in evidence after prolonged recharge operation. Of special inte in this connection are the isochlor lines[9] and the barrier profile[10] as of J 24, 1954, after some 16 months of operation, during 8 months of which barrier had not been elevated above sea level. From the isochlor lines it m be concluded that sea-water continued to intrude roughly half way betw seven of the eight operating wells, although the piezometric surface at internodal points was 4 ft or more above sea level except between wells 1A J (+1 ft) and J and K (-1 ft).

In the appraisal of the control of sea-water intrusion by means of a bar a number of factors must be considered. Salient among them are the so-ca syphon effect; the irregularities in the impervious sole; the location of internodal point in light of relative recharge rate of the two adjacent we the possibility of turbulent rather than laminar flow at and some distance f the recharge well; and the effect of a transition zone between fresh water sea-water.

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[7] "Basin Recharge," by Paul Baumann, a part of "Ground-Water Developmen Symposium, Transactions, ASCE, Vol. 122, p. 466.

[8] Ibid, p. 470, Fig. 5.

[9] Ibid, p. 469, Fig. 4.

[10] Ibid, p. 472, Fig. 8.

The syphon effect of sea-water under bodies of fresh ground-water was rst observed in Holland at least a century ago. There, fresh water had been ored in the sand dunes along the North Sea coast with rain water as the tural and pumped water as the artificial source of supply. Thus, the sand nes had been converted into fresh water "bags." Because the aquifers under- ng the dunes are too deep for the fresh water head to form a complete rrier, the fresh water "floats" on the sea-water, and the interface between e two is concave upward similar to an inverted syphon. Hence, if landward om the dunes the ground-water is lowered through pumping, sea-water may syphoned under the fresh water "bag." This would also happen if the fresh ater head of the subject barrier were lowered to less than that required control sea-water to the effective bottom or sole of the aquifer.

It must, however, be realized that the effective fresh water head may be duced through the pressurizing of intruded sea-water in the process of its splacement by the fresh water bulb. This would be most pronounced immedi- ely seaward from the recharge well. Intruded sea water than can flow both ays, that is, seaward and landward unless the barrier head at all points is ch as to balance pressurized sea-water and thereby prevent landward flow. he profile[10] of the barrier and the cross section[11] at Well G on June 24, 54 show the piezometric surface of intruded sea-water, some 500 ft sea- ard from the recharge well at an elevation approximately equal to that of the ternodal points on both sides of that well, namely about 4 ft above sea level. applying the equation for balanced pressure, based on the Ghyben-Herzberg inciple,[12] the incremental fresh water head, ΔH, should be added to the cremental sea-water head, H_d (in this case 4 ft $\pm$) in order to balance sea- ater pressure a_o ft (in this case 110 ft) below sea level. The same reasoning so applies in principle to the invasion of saline water, other than sea-water inland aquifers.

The terms ΔH and H_d are interrelated. Likewise ΔH and q_o, the rate of charge. An increase in the height of the barrier through an increase in the te of recharge will increase the expansion of the bulb and therefore the ad, H_d, of intruded sea-water. At first glance this may appear to be a vicious rcle. For a well spacing of $>W$ it may well be. However, in reducing the ll spacing to $<W$ and thereby effecting a reduction in the recharge rate per ll and, on a two-dimensional basis, per unit length of barrier, the proper lance can be established. For such a system no internodal point would exist.

If the entire volume of displaced sea-water is forced toward the ocean, en at any time T, the following relation must approximately hold true

$$K M \frac{H_d}{l} = q \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

which M is the thickness of aquifer, in feet; l represents the distance from ean to average, seaward boundary of fresh water bulb, in feet; $H_d/l = s_s$ = erage seaward gradient; q is the seaward, fresh water flow from well per it length of barrier in cu ft per hr and K is as noted previously. From Fig. 3 e seaward portion q of the total rate of recharge q_o may be approximated by

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[11] Ibid, p. 471, Fig. 7.

[12] Discussion by David K. Todd of "Ground-Water Movement Controlled Through reading," by Paul Baumann, Transactions, ASCE, Vol. 117, 1952.

the ratio c_1. Thus, for the well spacing B

$$q = \frac{c_1 q_o}{B} \quad \ldots\ldots\ldots\ldots\ldots$$

From Eqs. 4 and 5

$$H_d = \frac{c_1 q_o l}{B K M} \quad \ldots\ldots\ldots\ldots\ldots$$

Because q_o is essentially the discharge due to the head ΔH through submer orifices (perforations) of aggregate area A,

$$q_o = c_2 A \sqrt{\Delta H} \quad \ldots\ldots\ldots\ldots\ldots$$

in which c_2 is a coefficient of discharge and conversion. To satisfy the Ghyl Herzberg principle, previously referred to

$$\Delta H = 0.026 (a_o + H_d) \quad \ldots\ldots\ldots\ldots$$

Introducing Eqs. 7 and 8 into Eq. 6, there results

$$H_d = \frac{c\left(1 + \sqrt{1 + \frac{4 a_o}{c}}\right)}{2} \quad \ldots\ldots\ldots\ldots$$

in which

$$c = 0.026 \left(\frac{c_1 c_2 l A}{B K M}\right)^2 \quad \ldots\ldots\ldots\ldots$$

In evaluating Eq. 9 the following figures were used which are believed to proximate conditions on June 24, 1954 at Well G of the subject barrier, 0.40; $c_2 = 0.365 \times 3600 = 1315$ sec per hr; l = 1500 ft; B = 500 ft; A = 1.00 s K = 5.00 ft per hr; M = 80 ft; and $a_o = 110$ ft.
There results

$$c = 0.405$$

and

$$H_d = 6.875 \text{ ft}$$

which would, based on prior approximations and two-dimensional flow, established itself, had the entire displaced sea-water been forced to the oc Therefore, to prevent landward flow under these conditions of displaced water, the total height H of the fresh water column would have to be

$$H = a_o + H_d + \Delta H = 1.026 \left(a_o + H_d\right) = 120 \text{ ft} \pm \text{ and } \Delta H = 3.125 \text{ ft}$$

The actual rate q_o of recharge at Well G on above date, namely 0.64 c 2300 cu ft $hr^{\pm}$ is thereby reasonably satisfied, whereas the average sea recharge flow q amounted to 1.84 cu ft $hr^{\pm}$

In order to reduce H_d to the observed value of 4 ft and no landward fl saline water, c_1 would have had to be reduced to 0.234. Under this cond

$$H = 117 \text{ ft}$$

and

$$\Delta H = 3 \text{ ft}$$

owever, because the rate of recharge approximately corresponded to , = 0.40, it stands to reason that $\frac{0.40 - 0.234}{0.40} \times 100 = 40\% \pm$ of the displaced ea-water could have been flowing landward, although such flow would actually e governed by the average landward gradient

$$s_l = \frac{(a_o + H_d)\,1.026 - (a_o + H_d + \Delta H)}{L - l} \quad \ldots\ldots\ldots \quad (11)$$

The preceding is intended to be descriptive rather than exact. It is presented o show in principle under what conditions in a two-dimensional system, land-ward flow of displaced sea-water can be avoided. It will also be shown below hat since June 24, 1954, the operation of the West Basin Barrier by the Los ngeles County Flood Control District has progressed so as to eliminate any ppreciable landward flow of displaced sea-water.

The displacement of intruded sea-water is an essential requirement for the stablishment of a complete fresh water barrier. If in its early stage part of he sea-water so displaced flows landward its rate will diminish as l decreases, o long as the rate q_o of recharge is maintained. Such landward flow will ease when the landward gradient disappears, that is when

$$(a_o + H_d)\,1.026 = a_o + H_d + \Delta H \quad \ldots\ldots\ldots\ldots \quad (12)$$

In substance, temporary landward flow of displaced sea-water upstream rom the barrier will add to the temporary landward flow of displaced sea-ater downstream from the barrier and will thereby enhance the effect of the aline wave, referred to below.

Relations between the elevation of barrier above sea level, $H - a_o$; the sea-ard recharge flow, c, q_o; and the differential sea-water head, H_d are shown n Fig. 4.

Requirement for Avoiding Seaward Flow of Recharge Water.—From Fig. 4 nd Eq. 9 it follows that for $H_d = 0$; thus, for no seaward flow of either re-harge or intruded sea-water c, and, therefore, c must theoretically be zero. o have no seaward recharge flow a stagnation front upstream from the barrier ells would have to exist. Physically such a condition is conceivable, provided he barrier be treated as a three-dimensional system which, indeed it is. The ritical cross section of the barrier will then be at the internodal point.

If in Fig. 3 recharge wells <W feet apart were added on both sides of the ne shown and the upstream gradient would disappear then, before being eversed, there would be no flow either seaward or landward upstream from the arrier wells. Hence, at this time a stagnation front would exist. The hydraulic riterion of pressure and continuity would be satisfied. Pressure along the arped interface, convex seaward, between the fresh water bulb and the in-uded sea-water, would be balanced. Continuity of flow would be maintained rough the extra fresh-water head at each well which would serve in conveying echarge water laterally along the (curved) barrier, toward the internodal ection. In its respective progress, discharge at diminishing rate in a down-tream direction would take place. As evidenced by the cross section at Well of the West Basin Barrier[11] this condition appears to have been passed rough between June 15 and October 11, 1953. The practical requirement for educing c, from 0.40 to zero will be discussed subsequently.

In view of possible irregularities in the impervious sole and other uncertai ties such as deeper-seated channels, fresh water barriers should be provid with a reasonable "safety factor," similar to other engineering structur

Contrary to a water well the operation of which being always governed atmospheric pressure, so far as open aquifers are concerned, a recharge w may be operated, theoretically, under any head. Because the velocity along stream lines increases with the head it follows that flow will no longer

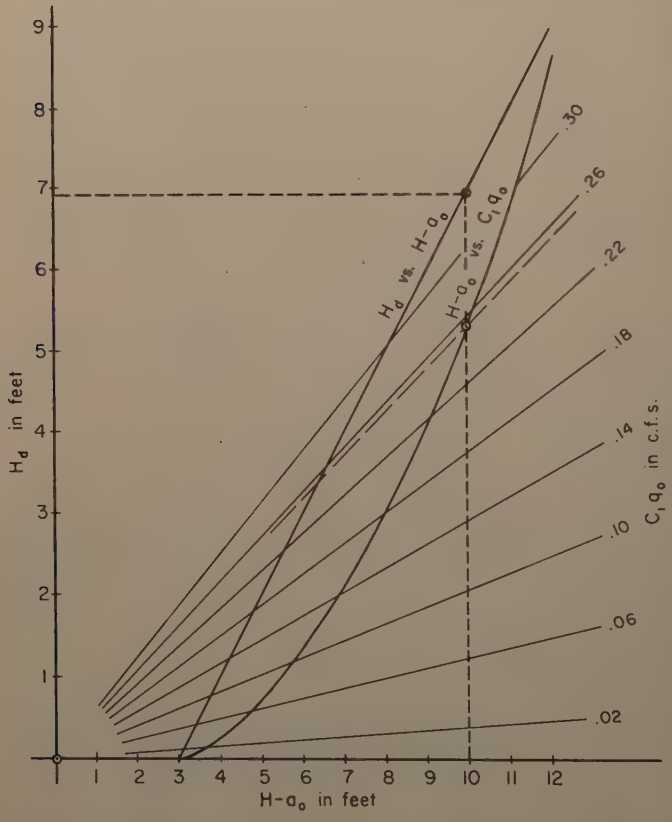

FIG. 4.—RELATION OF $H-a_o$, C_1q_o AND H_d FOR SEAWARD FLOW

laminar once the head exceeds a critical height. The velocity head may t no longer be negligible and effective pressure (static head minus velocity he no longer adequate for complete control. However, it is hardly conceiva that this influence could ever be important.

In view of the premise that in connection with a fresh water barrier continuity of flow to the ocean, the initially abrupt interface will change in

ransition zone[13] the significance thereof should be investigated. This transition one which is primarily a dispersal phenomenon could, according to Jacob ear and David R. Todd's equations,[13] vary in thickness from a few tens of eet at the West Basin Barrier location to several feet at the ocean. The change rom fresh water to sea-water within this zone is accomplished through mixing ut the direction of flow of this mixture is toward the ocean, once respective ontinuity has been established. Unitl such is the case there is no fresh water low to the ocean but merely to the slowly expanding bulb unless such expansion ere prevented as outlined previously. Hence, the transition zone must than be inor or zero in thickness and could not conceivably upset the performance of he barrier.

Saline Wave.—If the sea-water wedge extends beyond the line of recharge ells it will be separated from the main body of sea-water and forced land-ard by the recharge water. This will give rise to a so-called saline wave, hat is to a more or less acute increase in salinity at wells along the flow aths, followed by a gradual decrease in salinity as the recharge water follows ehind the wave. All depending on the volume of sea-water in the respective edge, relative to the rate of recharge per unit width of barrier, will the ntensity and duration of the saline wave be more or less damaging at various istances from the recharge wells. Regardless of how severely wells in the ath of flow may be affected by the saline wave, however, such affect will be emporary if recharge continues and will diminish approximately with the quare of the distance from the barrier. Hence, wells in close proximity o the barrier, although temporarily most severely affected would, in the bsence of the barrier, be overtaken by sea-water and thereby permanently ut out of commission. Therefore, although the saline wave caused by recharge hould be avoided if possible by locating the barrier wells landward of the sea-ater wedge, it is by far the lesser of the evils if compared with no saline ave because of no recharge barrier and therefore uninhibited invasion of sea-ater.

Recent Barrier Development.—Since preparation of the writer's contribution o the referenced symposium[14] the performance record of the West Basin arrier was brought up to date by the Flood Control District. It is shown in igs. 5 and 6. It includes the cross section and profile of the barrier on June 4, 1954 previously referred to but also shows the effect of a northerly and outherly extension of the barrier completed subsequent to that date. Tne temming of sea-water intrusion over a distance of at least $1\frac{1}{2}$ miles and to a epth of about 200 ft below sea level now appears assured. Isochlors recently stablished show 250 ppm of chloride ions (Cl^-) immediately upstream and lose to 2000 ft average, downstream from the barrier. The actual effective epth below sea level of the aquifer, originally estimated[14] at 110 ft was ecently revised to 130 ft. This seems to confirm the prior reference to

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13 "The Transition Zone Between Fresh and Salt Waters in Coastal Aquifers," by acob Bear and David K. Todd, Water Resources Center, Contribution No. 29, Univ. of alifornia, Berkeley, Calif., September 1, 1960.

14 "Basin Recharge," by Paul Baumann, a part of "Ground-Water Development," a ymposium, Transactions, ASCE, Vol. 122, p. 468.

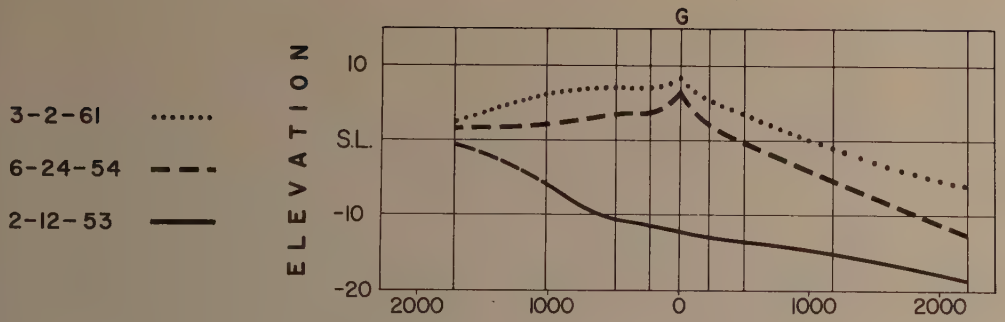
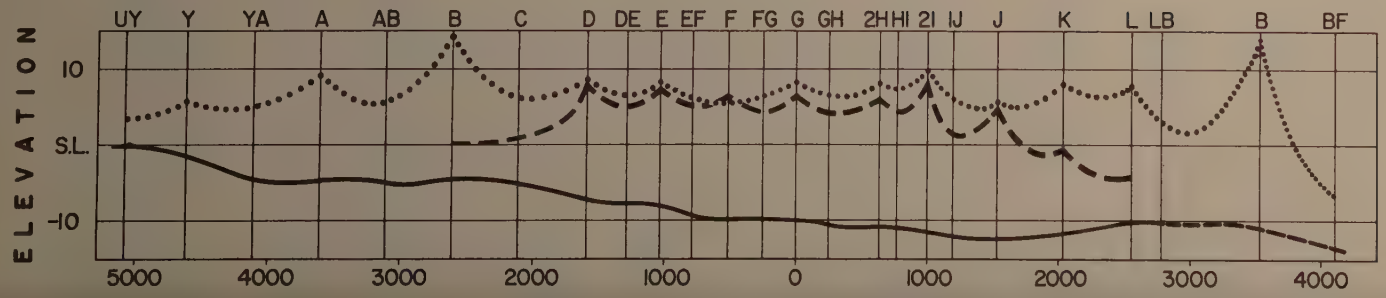


FIG. 5.—NORMAL TO LINE OF RECHARGE THRU WELL G



uncertainties regarding the configuration of the effective, impervious sole and the wisdom in the use of safety factors.

PRACTICAL ASPECTS

Although as outlined previously the performance of a recharge well differs in many respects from that of a water well there are no principal differences in the methods of construction. One proviso, however, must be emphasized, namely the indispensable requirement of gravel jackets for recharge wells.

Where recharge wells are used to create a fresh-water barrier to stem sea-water intrusion into a confined aquifer the essential requirements, besides the gravel jacket, should include an effective seal in form of a grouted concrete collar at the base of the conductor pipe, that is to say the bottom of the clay cap or by other means. It should be such as to prevent leakage under the differential operating pressure between the conductor pipe, or its equivalent, and the soil of the clay cap.

Among the theoretical aspects, the need to reduce seaward recharge flow in order to curtail the corresponding rate of seaward or landward flow of displaced sea-water through reduction of its gradients, was examined. Also, the requirement for avoiding or at least curtailing seaward flow of recharge water.

It is possible that the practical requirement rests with the reduction in the number and size or both, of perforations in the seaward one-half of the well casing (Fig. 3). Whether or not such procedure would lead to the desired result could, however, only be ascertained through test. In any event the effectiveness of the fresh-water barrier in stemming sea-water intrusion would remain intact so long as the Ghyben-Herzberg principle were satisfied.

Should it be necessary to chlorinate the recharge water in order to prevent bacterial activity in the pores of the soil and obstruction to the movement of water resulting therefrom, or for reasons of sanitation, proper precaution should be taken to prevent corrosion of the well casing. This can only be fully accomplished through the use of such materials for the well casing that will be immune to the attack of free chlorine or chlorine compounds. The Los Angeles County Flood Control District has recently undertaken to solve this problem through the use of perforated transite pipe for well casings.

Important also is the avoidance of air bubbles in the recharge water. This was originally accomplished through an insert pipe, not immune to chlorine attack, equipped with a foot valve operated from the top of the well. In this manner free fall of recharge water and resulting air bubbles could be prevented. Recent experience proved the use of plastic insert tubing, without foot valve but immune to chlorine attack and completely filled with water, preferable.

Close observation of piezometric levels and sampling of ground-water by means of special wells is necessary to ascertain whether or not the individual recharge wells of a barrier are functioning properly. Possible existence of, or change in ground-water above the clay cap must be carefully and frequently checked. Facilities for the regulation and shut-off of flow into the insert pipe or tube of each well are indispensable.

Originally, the Los Angeles County Flood Control District used 12 in. welded steel casings with 24 in. conductor and 4 in. insert pipes for its gravel jacketed wells. For reasons stated previously, perforated transite pipe was substituted for steel casing and plastic tubing for steel insert pipe. No conductor pipe is used in the new type well. Instead, a 32 in. hole is drilled by

rotary, reverse circulation method, the transite pipe set and gravel jacket Between the ground surface and the bottom of the clay cap two 4 in. plas pipes are set and embedded in a cement slurry. Through these two pipes gra may be tremied for refill as needed.

Automatic dosing of chlorine gas proved successful whereas automa control of constant rate of recharge did not.

CONCLUSIONS

From the foregoing study on theoretical and practical aspects of rechar through wells the following conclusions may be drawn so far as fresh wa aquifers are concerned:

1. Although structurally recharge wells may differ little if any from gra jacketed water wells, there is a fundamental difference in their hydrau idiosyncrasies. Therefore, recharge wells cannot be treated simply as wa wells in reverse.

2. Contrary to the operation of a water well in an open aquifer which always governed by atmospheric pressure, a recharge well in one and the sa aquifer can be operated under any arbitrary pressure so long as the flow fr the well remains laminar. If chlorination of recharge water is necessary prevent bacterial clogging of the soil or for reasons of sanitation, all materi installed in the well should be immune to chlorine attack.

3. The spacing of wells in open or confined aquifers for recharge o should be not less than 2 W, whereas for the control of undesirable grou water such as sea-water, the well spacing should be $< W$. If a fresh wa barrier is used for the control of sea-water, recharge of the aquifer is a be ficial by-product.

4. To prevent landward flow of the intruded sea-water or waste of recha water to the ocean, the seaward flow of recharge water must be curtailed. I possible that the practical implication in the accomplishment thereof rests w the reduction in the number and size or both of perforations in the seaw one-half of the recharge well casing. However, the validity of this postul could only be established through test.

5. A fresh water barrier will give rise to a saline wave if located wit the intruded sea-water wedge. Hence, if feasible such a barrier should located landward from the sea-water wedge, so as to prevent the initiation saline wave. However, of paramount importance is the existence of a barr for the stemming of sea-water intrusion, in that without it there will not only a temporary saline wave but an uninhibited invasion of sea-water resulting eventually putting all water wells in its path out of commission.

6. Fundamental in regard to the fresh water head required to stem sa water intrusion is the Ghyben-Herzberg principle. To apply it the depth to impervious sole of an aquifer must be known. As quite often this depth ca be precisely established a safety factor, similar to that in connection with ot engineering structures, should be applied.

ACKNOWLEDGMENTS

The writer is indebted to Finley B. Laverty, F. ASCE, and Arthur E. Bru ton, M. ASCE, Assistant Chief Engineer and Division Engineer, Water Cons vation, respectively, Los Angeles County Flood Control District, for valu assistance particularly in the preparation of exhibits for this paper.

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Proceedings of the American Society of Civil Engineers

JET DISCHARGE INTO A FLUID WITH A DENSITY GRADIENT

By William E. Hart[1]

SYNOPSIS

Turbulent liquid jets (the primary fluid) discharging vertically upward into liquid of greater density having a density gradient (the secondary fluid) are nsidered. The investigations were performed with a hydraulic model that nulated some of the conditions common at southern California ocean outls, with primary fluids of densities equivalent to those of sewage and secdary fluids of densities equivalent to those of receiving waters. The major rpose of the study was to determine under what conditions of density gradient the secondary fluid the primary-secondary fluid mixture (corresponding to sewage field) would spread at the surface, beneath the surface, or in both se areas. It was found that the field location could be predicted using modid equations for jets discharging into homogeneous secondary fluids.

INTRODUCTION

Notation.—Symbols adopted for use in this paper are defined where they st appear and are listed alphabetically, for convenience of reference in the pendix.

An important method of waste disposal for coastal communities is the disrge of sewage, raw or treated, into ocean waters. Adequate disposal reements demand that the sewage not be detrimental to public health, aesthet-

Note.—Discussion open until April 1, 1962. To extend the closing date one month, a tten request must be filed with the Executive Secretary, ASCE. This paper is part ne copyrighted Journal of the Hydraulics Division, Proceedings of the American soy of Civil Engineers, Vol. 87, No. HY 6, November, 1961.

[1] Research Engr., Hawaiian Sugar Planters' Assn., Honolulu, Hawaii.

ics, and wildlife. In nearly all practical cases dilution takes an import place in the meeting of these requirements. Ocean outfalls are therefore c sidered here in the dilution aspect.

A significant contribution to the understanding of ocean outfall dilution made by A M Rawn, Hon. M., and H. K. Palmer, F. ASCE,[2] in 1930. T discharged 1/4 in to 2 in. diameter jets of fresh water (the primary liquid) a 2 acre body of salt water (the secondary liquid). (The density of sewage fluent is very nearly equal to that of fresh water.) The experiments inclu both horizontal and vertical jets and from them the following algebraic expr sion was obtained for estimating the dilution:

$$(S_s - 1)\, Q^{0.61} = 1.18\,(10^{-2})\,(L_s + 3)^{2.35} \quad \ldots\ldots\ldots$$

in which S_s is the dilution factor at the point at which the jet axis meets air-liquid surface, defined as the ratio of the total volume of the sample to volume of the primary liquid in the sample; Q refers to the quantity of f of primary liquid, in cubic feet per second; and L_s denotes the length of rising jet measured from the point of discharge to the air-liquid surfa equal to the depth of liquid over the discharge point for vertical jets (H_s) feet. The location of the point at which S_s is measured is shown in Fig. 1. writers also found the diameter of the rising cone at the surface to be betw $L_s/4$ and $L_s/3$ and the thickness of the spreading field to be about L_s

Rawn, F. R. Bowerman, M. ASCE, and Norman H. Brooks, M. ASCE,[3] investigated the original Rawn and Palmer[2] data using dimensional analy techniques. They suggested that the dilution in the jet can be represente

$$S_s = f_1\left(\frac{H_s}{D},\ F,\ R\right) \quad \ldots\ldots\ldots\ldots\ldots$$

in which H_s denotes the depth of liquid over the discharge point, in feet; the diameter of the jet at discharge (equal to the orifice diameter if ther no contraction), in feet; F describes the Froude number, define $u_o/\sqrt{Dg\,(\rho_n - \rho_o)/\rho_o}$; u_o is the velocity of discharge of the jet, in feet second; g denotes the acceleration due to gravity, in feet per second squa ρ_n refers to the density of secondary liquid, in grams (mass) per cubic ce meter; ρ_o represents the density of primary liquid, in grams (mass) per c centimeter; R is the Reynolds number, defined as $u_o D/\nu$; and ν denotes Kinematic viscosity of the primary liquid, in square feet per second. (I literature both density and specific gravity (referred to water at 4°C) are u Because the two are equivalent only the former is reported here.) T analysis showed that the Reynolds number, which varied from 5,000 to 40, was not related to the observed dilutions. The expression therefore reduce

$$S_s = f_1\left(\frac{H_s}{D},\ F\right) \quad \ldots\ldots\ldots\ldots\ldots$$

The investigations examined so far have considered ocean outfalls The vertical discharge of effluent into the ocean follows the behavior patte gases or liquids discharging into another gas or liquid of approximatel

2 "Pre-determining the Extent of a Sewage Field in Sea Water," by A M Raw H. K. Palmer, Transactions, ASCE, Vol. 94, 1930, p. 1036.

3 "Diffusers for Disposal of Sewage in Sea Water," by A M Rawn, F. R. Bowe and Norman H. Brooks, Proceedings, ASCE, Vol. 86, No. SA 2, March, 1960, p. 65

me density, as summarized by G. Abraham.[4] He divided the problem (as ggested by R. H. Schmidt, A. M. ASCE,[5]) according to the magnitude of the oude number. In the case of a submerged jet the effects of gravity forces are ered by the difference in density between the jet fluid and the surrounding id. The inertia forces are due to velocity. Velocities are high and inertia ces predominate in the zone near the point of discharge. The result is a h Froude number. At points distant from the discharge point the velocities e low and the buoyancy forces predominate. In this case, the Froude number es on moderate values. Abraham further divided the area near the jet into zone of flow establishment, $z/D < 5$ or 6, and the zone of established flow, > 5 or 6. (z is the vertical distance from the discharge point to the point der observation, Fig. 2.) In the case of turbulent jets with high initial velocis (such as are often found in outfalls), the zone of flow establishment is irely within the high Froude number range. Abraham presented formulas ected from the works of others for these four regimes of flow and verified choices with experimental data. The equations are summarized in Table 1 d plotted in Fig. 3.

Jets discharging into homogeneous media have been discussed. During summer months the sun often heats the surface waters of the ocean using a temperature gradient which results in a density gradient. Robert Arthur[6] reported thermoclines (sudden changes in temperature with depth) sting from the surf zone to 800 meters offshore in the oceanside and ipps areas. The Allan Hancock Foundation[7] reported thermocline activity areas between Oceanside and Laguna and at Santa Barbara. Representative adings for one summer showed specific gravities (in the Oceanside-Laguna ea) of 1.02254 at the surface and 1.02535 at 200 ft. R. E. Stevenson, Richard Tibby and Donn S. Gorsline[8] reported existence of thermoclines in Santa nica Bay. D. H. Caldwell, F. ASCE, C. G. Hyde, Hon. M., and Rawn[9] esented many temperature-depth curves for locations at Oceanside, Point ma and Imperial Beach. All these areas experience the phenomenon of rmocline occurrence during at least a portion of the spring and summer nths. Some of the data of Caldwell, Hyde, and Rawn[9] are reproduced in g. 4.

The thermal gradients and thermoclines previously described can influence establishment of sewage fields. Stevenson, Tibby, and Gorsline[8] gave calations to show that it may be possible for the less dense upper layer to se a submerged field to occur. Caldwell, Hyde and Rawn[9] also mentioned s possibility and related experiences of observing the phenomenon. Rawn, werman, and Brooks[3] cited three possible behavior patterns of effluent

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4 "Jet Diffusion in Liquid of Greater Density," by G. Abraham, Proceedings, ASCE, . 86, No. HY 6, June, 1960, p. 1.

5 "On the Diffusion of Heated Jets," by R. H. Schmidt, Tellus, Vol. 9, No. 3, p. 378.

6 "Oscillations in Sea Temperature at Scripps and Oceanside Piers," by Robert S. hur, Deep Sea Research, Vol. 2, p. 107.

7 "Oceanographic Survey of the Continental Shelf Area of Southern California," Allan ncock Foundation, 1958, p. 155.

8 "The Oceanography of Santa Monica Bay, California," by R. E. Stevenson, Richard Tibby, and Donn S. Gorsline, Final Report Submitted to the Hyperion Engrs. by the logy Dept. of the Univ. of Southern California, Vol. 1, 1956, p. 32.

9 "Report on the Collection, Treatment and Disposal of the Sewage of San Diego unty, California," by D. H. Caldwell, C. G. Hyde, and A M Rawn, San Diego County wage Survey, 1952, p. 187, p. 194. p. 203.

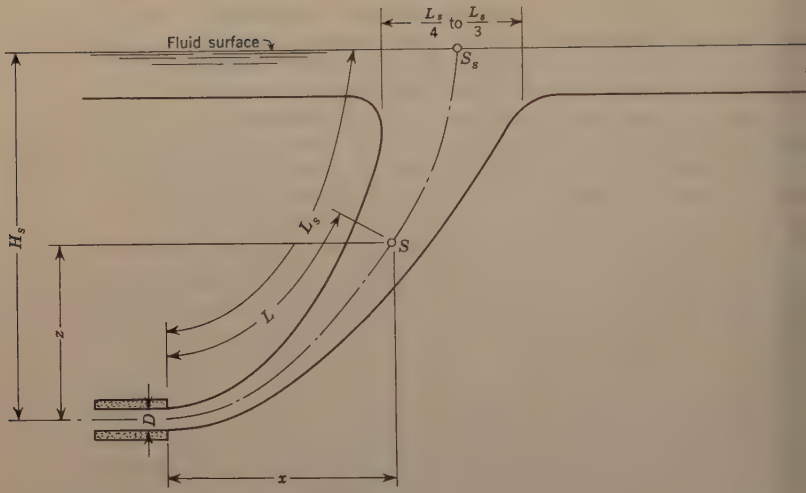


FIG. 1.—DEFINITION SKETCH FOR HORIZONTALLY DISCHARGING JET

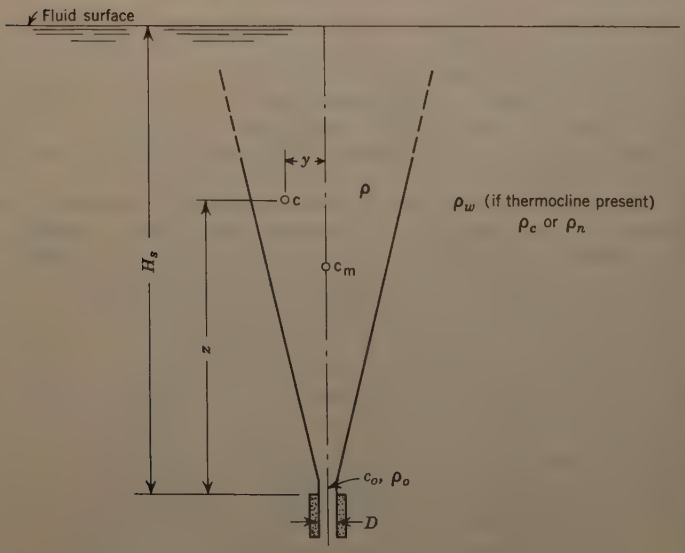


FIG. 2.—DEFINITION SKETCH FOR VERTICALLY DISCHARGING JET

charged from an ocean outfall. In the first instance (which is the most mmon), the sewage rises to the ocean surface and spreads as a surface field. the second case, the jet rises to the surface and then plunges below to the rmocline and spreads as a submerged field. This occurs when the upper er of water is less dense than the rising sewage - sea water mixture but the sidual energy of the jet (that energy in excess of what is required for the to reach the upper layer) is enough to drive it through the upper layer. The rd case occurs when the field rises to the thermocline and spreads at this el.

TABLE 1.—FORMULAS DESCRIBING DISTRIBUTION OF CONCENTRATIONS AND VELOCITIES IN A TURBULENT WATER JET DISCHARGED VERTICALLY UPWARD INTO WATER OF HIGHER DENSITY

Non-buoyancy Case, $F = \infty$

A. Zone of Flow Establishment: $z/D < 6.4$ for velocities, $z/D < 5.2$ for concentrations.

1. For $y < D/2 - 0.078z$: $u/u_o = 1$
   For $y > D/2 - 0.078z$: $8/u_o = \exp(-77(y + 0.078z - D/2)^2/z^2)$
2. For $y < D/2 - 0.096z$: $c/c_o = 1$
   For $y > D/2 - 0.096z$: $c/c_o = \exp(-62(y + 0.96z - D/2)^2/z^2)$

B. Zone of Established Flow: $z/D > 6.4$ for velocities, $z/D > 5.2$ for concentrations.

1. $u/u_m = \exp(-77(y/z)^2)$, $u_m/u_o = 6.4D/z$
2. $c/c_m = \exp(-62(y/z)^2)$, $c_m/c_o = 5.2D/z$

Buoyancy Case, $F \rightarrow 0$

A. $u/u_m = \exp(-80(y/z)^2)$, $u_m/u_o = 3.65\ F^{-2/3}(z/D + 2)^{-1/3}$

B. $c/c_m = \exp(-80(y/z)^2)$, $c_m/c_o = 9.7\ F^{-2/3}(z/D + 2)^{-5/3}$

Intermediate Case, moderate values of F

A. For velocities: $(z_t/D)^2 \approx 5.94\ F^2 \dfrac{g\int_o^{z_t} dz \int_o^{\infty} 2\pi y\,(\rho_n - \rho)dy}{u_o^2\,(D^2/4)}$

B. For concentrations: $(z_t/D)^2 \approx 5.94\ F^2$

(1) The concentration, c, is defined as $(\rho - \rho_n)/(\rho_o - \rho_n)$.

(2) z_t is the value of z for which the equations of the non-buoyancy and buoyancy cases coincide. F must be greater than about 10.

Rawn, Bowerman, and Brooks[3] analyzed the cases of submerged fields. r these to occur they stated the following conditions as being necessary.

$$\frac{\left(S_p - 1\right)\rho_c + \rho_o}{S_p} > \rho_w \qquad (4)$$

which S_p denotes the dilution factor at the point at which the jet axis crosses center of the thermocline; ρ_c refers to the density of cold secondary fluid er, in gravity (mass) per cubic centimeter; and ρ_w is the density of warm

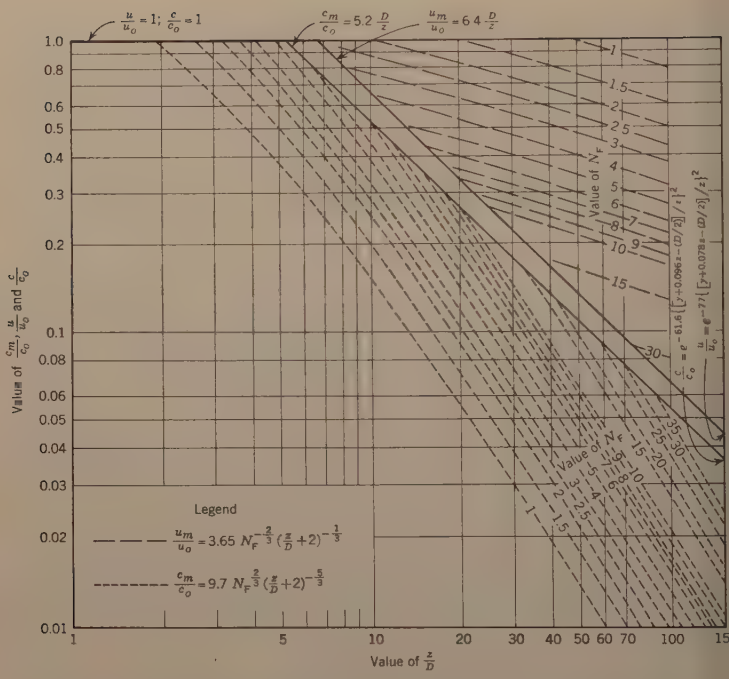


FIG. 3.—CONCENTRATION AND VELOCITY ALONG JET AXIS

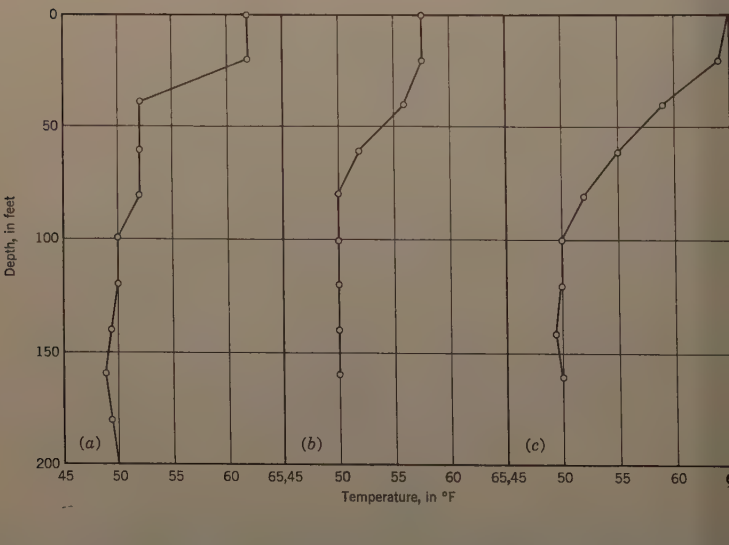


FIG. 4.—TEMPERATURE-DEPTH RELATIONSHIPS IN SOUTHER
CALIFORNIA COASTAL WATERS, SPRING AND SUMME

condary fluid layer, grams (mass) per cubic centimeter. The left expression the density of the jet where the jet axis crosses the thermocline. If this expression is much larger than that on the right, the jet will not rise to the surface and the field will be entirely submerged as in case three. When the two sides of this inequality (Eq. 4) are nearly equal the height of the field over the discharge point will depend on the inertia of the jet. Eq. 4 and the inertia factor

TABLE 2.—TYPICAL DIMENSIONS OF CALIFORNIA OCEAN OUTFALLS[a] AND CONDITIONS OF SOUTHERN CALIFORNIA OFFSHORE WATERS

Item	Range of Values		Diffuser	Source[b]
	High	Low		
pth of outlet	210	6	yes	3
low air-water surface, (feet)	54	0	no	3
ameter of discharge	0.83	0.33	yes	3
ening, D (feet)	2.5	0.33	no	3
locity of discharge, u_o	19.55	4.01	yes	3
et per second)	5.92	0.44	no	3
tio of depth to diameter, /D	343	7.2	yes	3
	21	0	no	3
oude number, **F** or F_c	21.35	5.67	yes	3
	5.28	0.33	no	3
mperature of effluent, (°F)	83	63		2
linity of receiving waters (o/oo)	34.3	33.5		4, 2, 1
mperature of receiving ters, (°F)	76	49		4, 2, 1
nsity of receiving waters, ma-t (gm per cc 10^{-3})	25.9	22.5		4, 2, 1
nsity difference, surface and tom waters, (gm per cc)	0.0018			4, 2, 1

[a] Extreme ranges are not always given, but the majority of outfalls fall within the nes quoted.

[b] 1. Allan Hancock Foundation[7]
2. Caldwell, Hyde and Rawn[9]
3. Pearson[10]
4. Stevenson, Tibby, and Gorsline[8]

represented by the Froude number) can be combined in the following nner:

$$H_f = f_2 (P, F_c) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

which H_f is the height (over the discharge point) at which the field is ablished, feet, $P = (\rho_j - \rho_w)/\rho_o$ F_c denotes the Froude number, defined as

10 "An Investigation of the Efficacy of Submarine Outfall Disposal of Sewage and dge," by E. A. Pearson, Report to the State Water Pollution Control Bd., Sacramento if., 1956.

$u_o/\sqrt{D\,g\,(\rho_c - \rho_o)/\rho_o}$, and ρ_j is the density of the jet where the jet axis cros
the center of the thermocline, grams (mass) per cubic centimeter. Eq. 5
be examined in the section entitled "Experimental Results - Grouping of Ru

EXPERIMENTAL EQUIPMENT AND PROCEDURE

In the work reported herein, a jet of fresh water (the primary liquid)
discharged vertically upward into salt water (the secondary liquid) which h
slightly (about 2-1/2%) greater density. In addition, the secondary liquid
sometimes stratified with a slightly less dense layer on top. It was desire

TABLE 3.—VALUES OF MODEL AND PROTOTYPE VARIABLES FOR INVESTIGA
OF JET DISCHARGE INTO A FLUID WITH A DENSITY GRADIENT

Item	Model	Prototype
Runs 1 through 10: Length ratio = 35		
Jet diameter, feet	0.0235	0.787
inches	0.282	9.44
Discharge depth, average, feet	3.54	124
Discharge velocity, ft per sec, maximum	3.90	23.1
minimum	0.73	4.3
Runs 11 through 33: Length ratio = 25		
Jet diameter, feet	0.0318	0.796
inches	0.382	9.55
Discharge depth, average, feet	3.46	89
Discharge velocity, ft per sec, maximum	5.56	27.8
minimum	0.534	2.67
All Runs		
Initial density difference in secondary fluid, gm per cc, maximum	0.0023	same
minimum	0.0000	same
Density difference between primary fluid and cold secondary fluid, average, gm per cc	0.0271	same

find parameters that located the vertical position, the thickness, and the de
of the spreading primary-secondary liquid mixture (the field).

The problem was investigated with a model designed according to Fro
similarity law. Conditions were similar to those existing at Southern Calif
outfall locations (Table 2). The actual model conditions and the correspo
prototype conditions are given in Table 3.

The experimental layout is shown in Figs. 5 and 6. Left to right in F
are the manometer, primary fluid tank, second fluid tank, and refriger
compressor. The primary liquid, domestic water with a blue dye added
tracer, passed through the metering orifice and then discharged verti
upward from the center of the tank bottom. Discharge was through a 0.0
(0.360 in.) diameter flat plate sharp edge orifice for Runs 1 through 1

ough a 0.0318 ft (0.382 in.) diameter brass tube for Runs 11 through 33. . 7 shows the bell-mouthed entrance to the tube (to reduce undesirable lling eddies) and the screen for triggering turbulence. The diamensions are th (R. L. Daugherty and A. C. Ingersoll, F. ASCE,[11]) that turbulent condi-ns will prevail at discharge whenever the Reynolds number in the tube is ve critical. Adjustment of the primary liquid flow rate was made with the

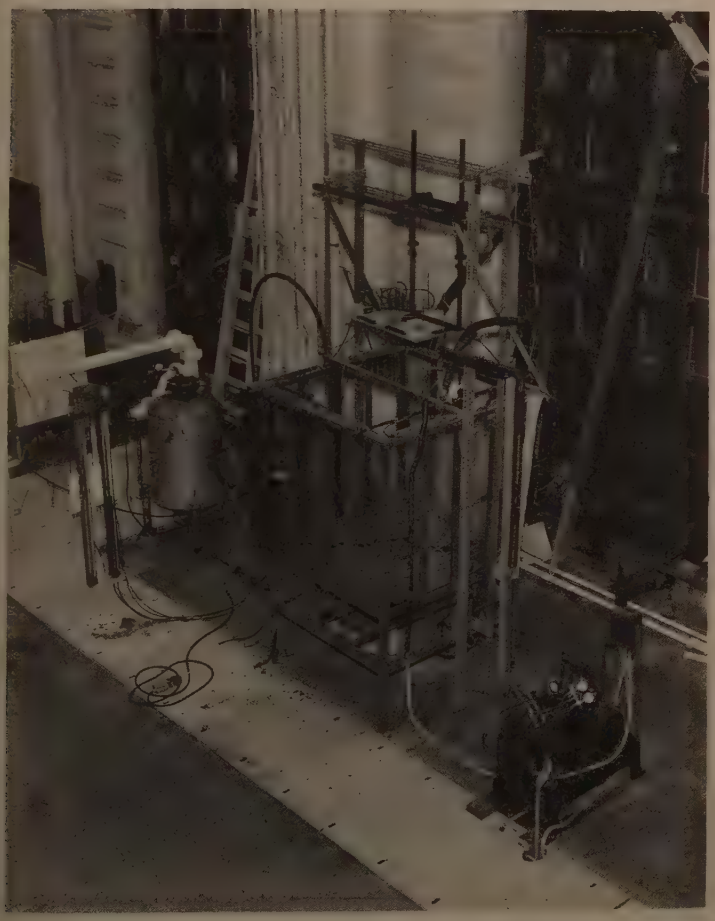


FIG. 5.—MAJOR EQUIPMENT COMPONENTS

dle valve (Fig. 6). The primary liquid flow was started and stopped with quick acting valve. A constant air pressure of 21.5 psi was maintained at surface of the primary liquid in the supply tank. With the flow rates used total head on the primary liquid system varied by less than 1% during a run.

[11] "Fluid Mechanics," by R. L. Daugherty and A. C. Ingersoll, McGraw-Hill Book , Inc., New York, 5th edition, 1954, p. 174.

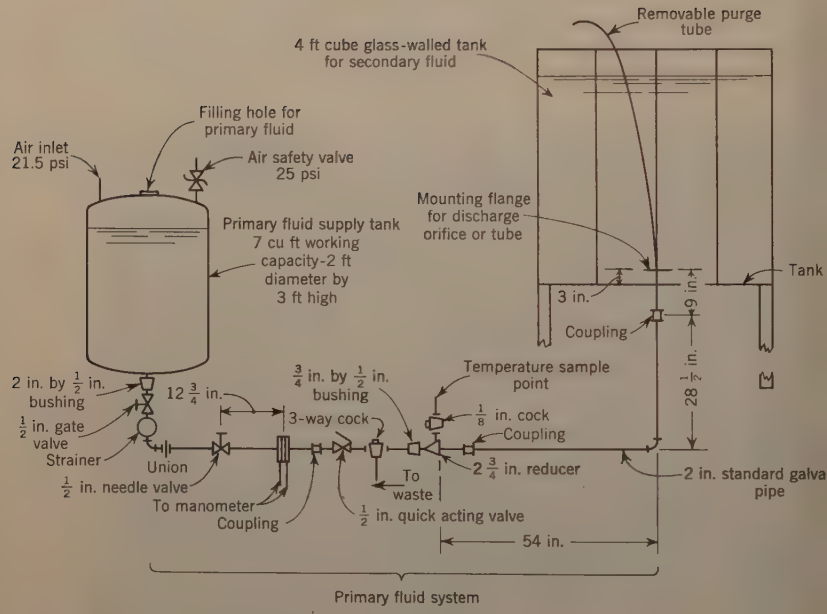


FIG. 6.—EXPERIMENTAL ARRANGEMENT

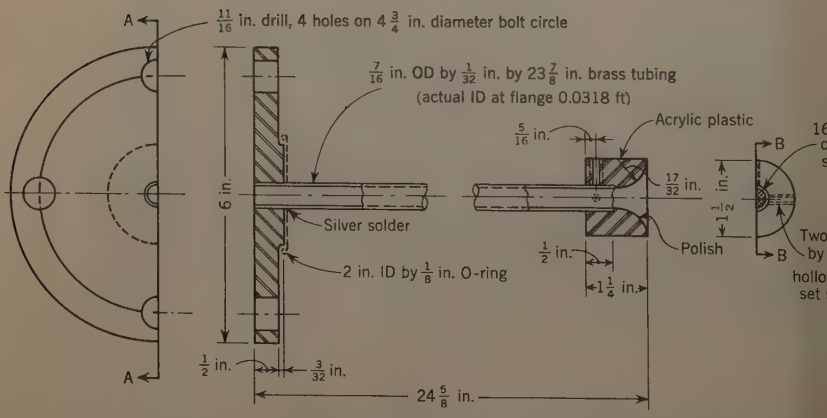


FIG. 7.—DISCHARGE TUBE FOR RUNS 11 THROUGH 33

The secondary liquid was contained in the 4 ft cubical glass walled tank nown in Fig. 8 with grid, probes, sample tubes, pipettes and coil in place r run. Purge tube is shown in position for removing primary fluid sample and ust be removed before run starts. The temperature of the secondary liquid as adjusted with a Freon 5 refrigeration unit. The cooling surface consisted 50 ft of 3/8 in. copper tubing formed into a square flat coil. It was mounted prizontally in the tank with an arrangement that allowed it to be raised and wered. The Freon 12 was brought into and out of the coil through tubes lead- g down from the water surface. Each tube was insulated with a dead air space nclosed by a 3 in. diameter surrounding case.

Initial temperatures of the primary and secondary liquids were measured ith a mercury filled glass thermometer. This unit was calibrated against a andard thermometer (calibrated by the National Bureau of Standards).

Primary and secondary fluid densities were measured with a Westphal alance. The unit, with a least count of 0.0001 gm per cc, was calibrated with 50 mm pycnometer bottle and standard weights. The temperature measure- ents made at the time of the density determinations incorporated an alcohol lled glass thermometer that was also calibrated against the standard thermo- eter.

Concentration of secondary liquid in the spreading field was determined by olorimetrically analysing samples taken during the run. (See Fig. 9 for the rrangement of sample tubes. The probes are shown at level when field is nticipated submerged. Probes are approximately 1 ft higher if field is antic- ated at surface.) Colorimeter was used to measure the optical density of the amples. A solution containing 1% primary liquid (99% secondary liquid) had peak optical density of 655 (± 5) microns and this wave length of light was sed in all measurements.

In situ temperature measurements were made during some runs with ermistors and thermocouples mounted on probes (located according to ig. 9).

Data describing the location and thickness of field were obtained from olored motion pictures. A 1/2 ft grid made from 3/16 in. diameter brass rod as used as a reference. This was placed in the tank perpendicular to the line sight of the camera and within 0.10 ft of the exis of the jet. (See Fig. 9.) ack lighting made the secondary liquid appear nearly clear, the primary quid deep blue and the reference grid as a shadow.

The secondary liquid was prepared for each run by weighing water and odium chloride (commercially available butter salt or rock salt) to make the esired concentration (usually about 3.5% salt by weight). The mixture was ampled at four random locations. The samples were analyzed with the West- hal balance and the average of the concentrations so found was considered he concentration of the mixture.

The primary liquid was prepared by mixing dye and water in the primary quid supply tank. The dye was "telegraph blue," wireless telegraph signal ecorder ink. The concentration ranged from 140 ppm to 280 ppm (by weight).

In most of the tests an attempt was made to have a difference in density etween the primary liquid and the cold layer of the secondary liquid of approx- nately 0.025 gm/per cc, a case which is commonly found at ocean outfalls. he primary liquid was at room temperature. The secondary liquid tempera- re was adjusted by the use of the cooling coil. If there was to be no density hange within the secondary liquid, the coils were raised to the air-liquid rface and the compressor started. Convection currents cooled the entire

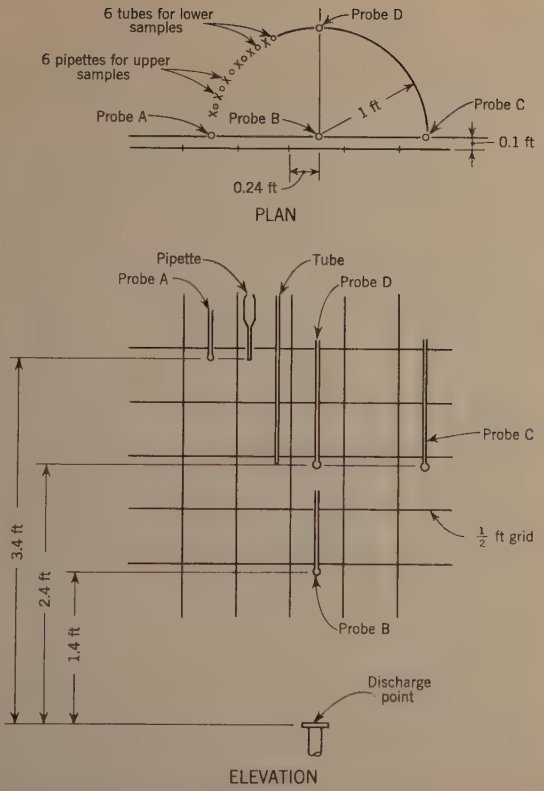


FIG. 9.—ARRANGEMENT OF CONDUCTIVITY, TEM-

dy of secondary liquid. When the required 0.025 gm per/cc density difference tween primary and secondary liquids was obtained (as indicated by the tem- rature of the secondary liquid) the refrigeration unit was shut off and the coil wered to the bottom of the tank. If a density stratification in the secondary uid was desired one of two possible courses were followed. When a high nsity difference was desired between the warm and cold secondary liquid all oling was done with the coil at approximately 2.3 ft above the jet discharge int. If only a small density difference was desired the coils were raised to e surface where all the secondary fluid was cooled for a while, and then wered to the 2.3 ft mark for final cooling of the cold bottom layer.

As in the case when no density difference was required, the convection rrents effectively caused complete mixing of the liquid below the coils, and water mass with temperatures varying by less than 1°F was usually obtained. or a distance extending about 0.1 ft above the coils the secondary liquid had proximately the same temperature as that below the coils. At this point the mperature began to change rapidly with height until a fairly constant rate of ange was reached. This marked the bottom of the thermocline. A similar ange occurred at the upper edge of the thermocline. A method for determin- g these two points is presented later. The severity of these changes and the ngth of the thermocline varied somewhat for different runs.

The reference grid was placed in the tank after the secondary liquid had en cooled. If spreading field samples were to be taken the appropriate uipment was lowered until the C and D probes and the sampling tubes were the expected field level. The sampling tubes and pipettes were cleared of y contaminating liquid by blowing a bubble of air out of each. The amplifiers r the thermistors were adjusted and the bridge circuits given a preliminary lance. At this time the primary liquid system was flushed by forcing about cu ft (the approximate volume of the system's piping) of primary fluid through e system and outside the tank by use of the purge tube. The last 750 mm of is was collected in three samples. The Westphal balance was used to deter- ine the density of these. If they showed no consistent change in density it was sumed that the primary liquid was homogeneous. The temperature of the imary liquid was determined by taking a small sample at the temperature mple point (Fig. 6).

The next step was to determine the temperature distribution of the sec- dary liquid. This was done with the mercury filled thermometer that was wered at the center of the tank. Readings were taken from the water surface wn to the discharge point. The intervals of measurement were 0.1 ft until the ermocline was passed and 0.5 ft thereafter. (For one run two additional ofiles were taken at 1-1/2 ft radius from the jet axis. No significant differ- ce was noted between these and the one at the center.) Finally, the photo- aphic equipment was set, the purge tube removed, and the bridge circuits balanced.

The run was started by opening the quick-acting valve. The preset needle lve controlled the flow of primary liquid. Just as the primary liquid became sible at the discharge point the camera equipment and an electric clock (in e camera's field of view) were started. Field samples were taken after the reading field reached the sampling radius. Samples were taken every 3 sec 6 sec depending on the field spreading rate. An attempt was made to fill the t tubes (or pipettes) before the field reached the sides of the tank. After each n the probes were immersed in known temperature baths for the purpose of

standardizing the thermistor readings. The thermocouple readings were st
ardized by feeding several known voltages into the recording potentiom
The preceding is a description of the usual procedure for the tests. G
variations from it necessitated the discarding of some runs. Runs were el
nated on the basis of experimental conditions at the time of the test and n
the basis of results obtained.

EXPERIMENTAL RESULTS

General.—The most important behavior under consideration in these st
was the vertical location of the spreading primary-secondary liquid mi
(or field). Analysis of the movies showed that a given run could be plac
one of the following three groups.

Group A: The field was established at the air-secondary liquid sur
Group B: The field was being established between the air-secondary l
surface and a lower level. The lower limit was not found during the runs
was never observed to be lower than the bottom of the thermocline.
Group C: The field was established submerged and was located at or
the center of the thermocline.

Figs. 10, 11, and 12 give examples of runs falling in each of these groups.
field profiles were traced from movies. Fig. 10 is for Run 33 for whic
20.3; height to water surface = 111 orifice diameters and which
$\rho_w/\rho_o)$ $(10^4) = -4$. Fig. 11 is for run 29 for which F = 15.0; height to cent
thermocline = 81 orifice diameters and $(\rho_n - \rho_w/\rho_o)$ $(10^4) = 5$. Fig. 12 i
run 31 for which F = 5.8; height to center of thermocline = 83 orifice diam
and $(\rho_n - \rho_w/\rho_o)(10^4) = 16$.

A method of predicting into which group a given run will fall will be e
ined. Each group is then considered separately as to field height (distance
jet discharge point to the center of the field), field thickness and field de
The field density values were obtained from colorimetric samples and i
temperature measurements.

Grouping of Runs.—Eq. 5 gives the variables that were anticipated as a
ing field location. Analysis of the experimental data does not show depen
of field location on Froude number except as the latter affects P. It is
found that P is a more reliable guide if ρ_h is used in place of ρ_j, where
the average density in the jet at H_t. The term H_t describes the distance
the point of discharge to the center of the thermocline. The method of c
lating P will be presented using the equations of Abraham.[4] (Alt
Abraham's equations were developed for use with jets discharging into h
geneous media, they are used with some success in the following discus
The writer is not aware of comparable equations for the discharge of jet
heterogeneous secondary fluids.)

The definition of the concentration, c (Table 1), provides the followi
lationship.

$$\rho = \rho_n + c\,(\rho_o - \rho_n) \quad \cdots\cdots\cdots\cdots\cdots$$

The term ρ is the density at any point in the jet. In the current calcul
the density of the surrounding fluid is taken as that of the cold wate
the corresponding value of ρ_c is used in place of ρ_n. The term ρ_o des

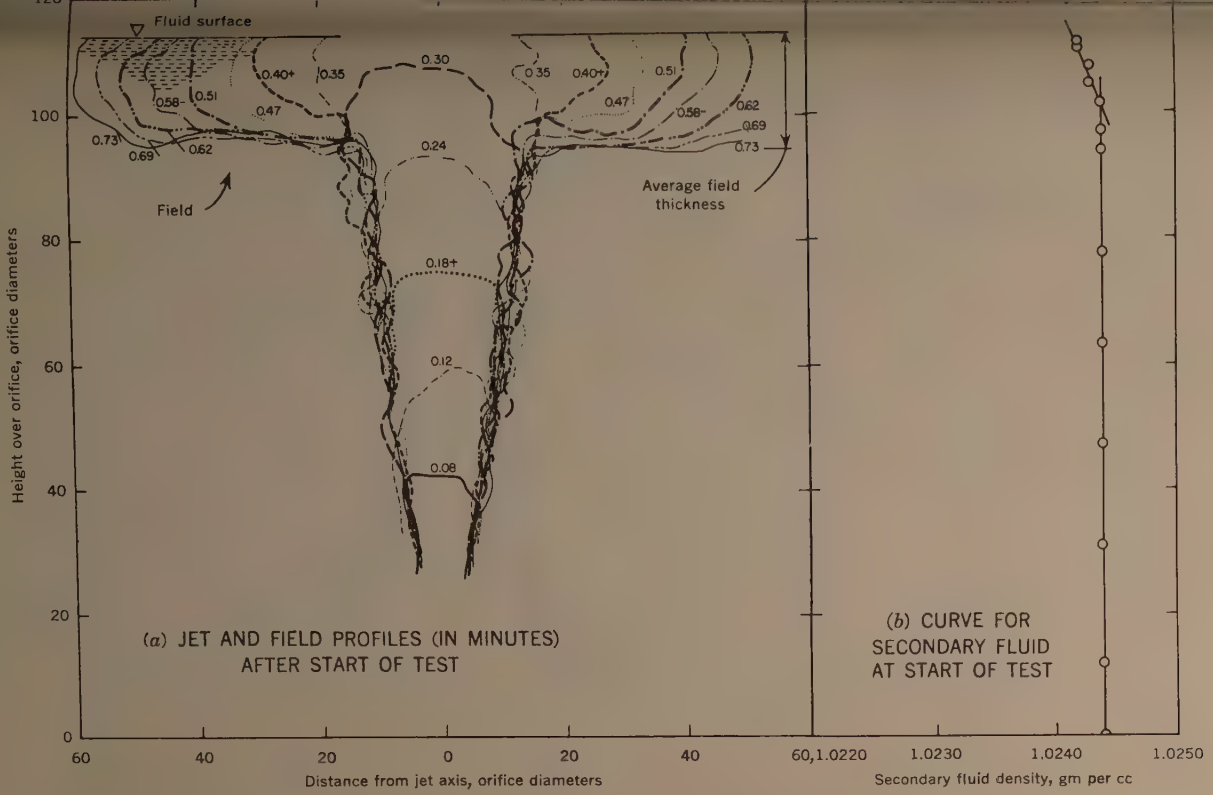
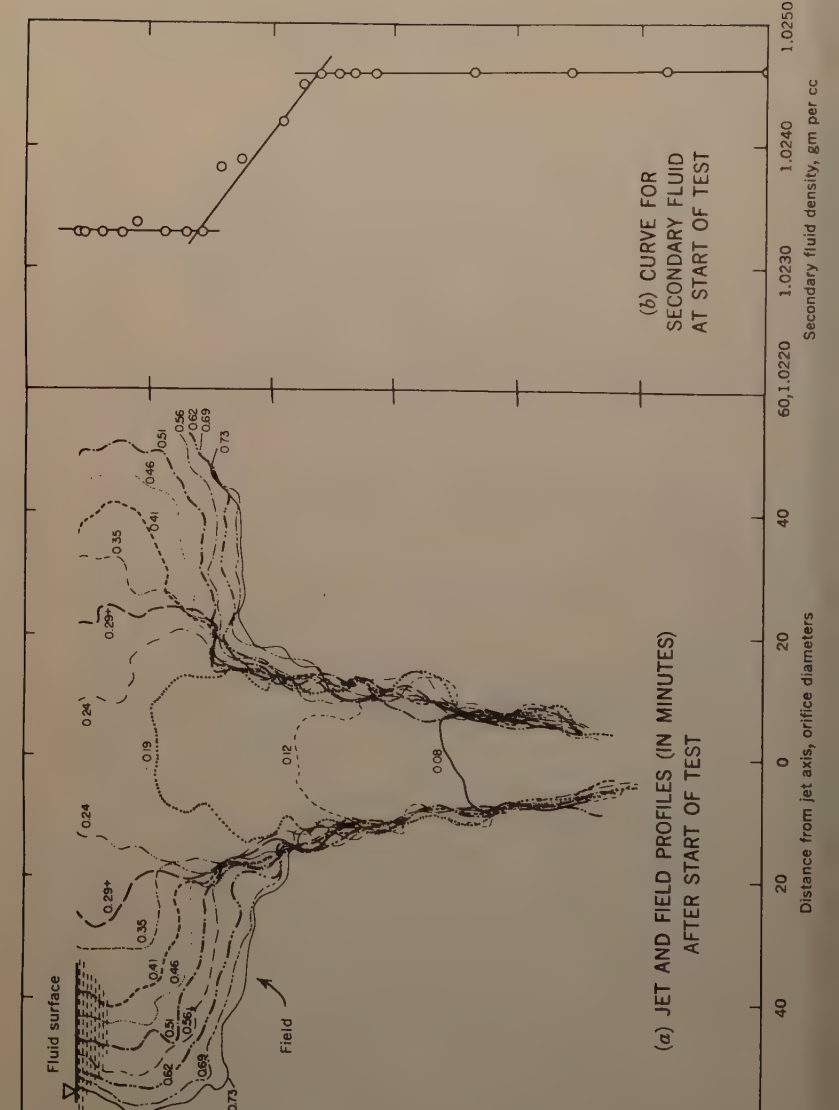


FIG. 10.—JET AND FIELD PROFILES AND DENSITY-HIEGHT CURVE FOR RUN 33


Height over orifice, orifice diameters
120
100
80
60
40
20
0
Fluid surface
0.24
0.29+
0.35
0.41
0.46
0.51
0.56
0.62
0.69
0.73
0.19
0.12
0.08
Field
(a) JET AND FIELD PROFILES (IN MINUTES) AFTER START OF TEST
60
40
20
0
20
40
60
Distance from jet axis, orifice diameters
(b) CURVE FOR SECONDARY FLUID AT START OF TEST
1.0220
1.0230
1.0240
1.0250
Secondary fluid density, gm per cc

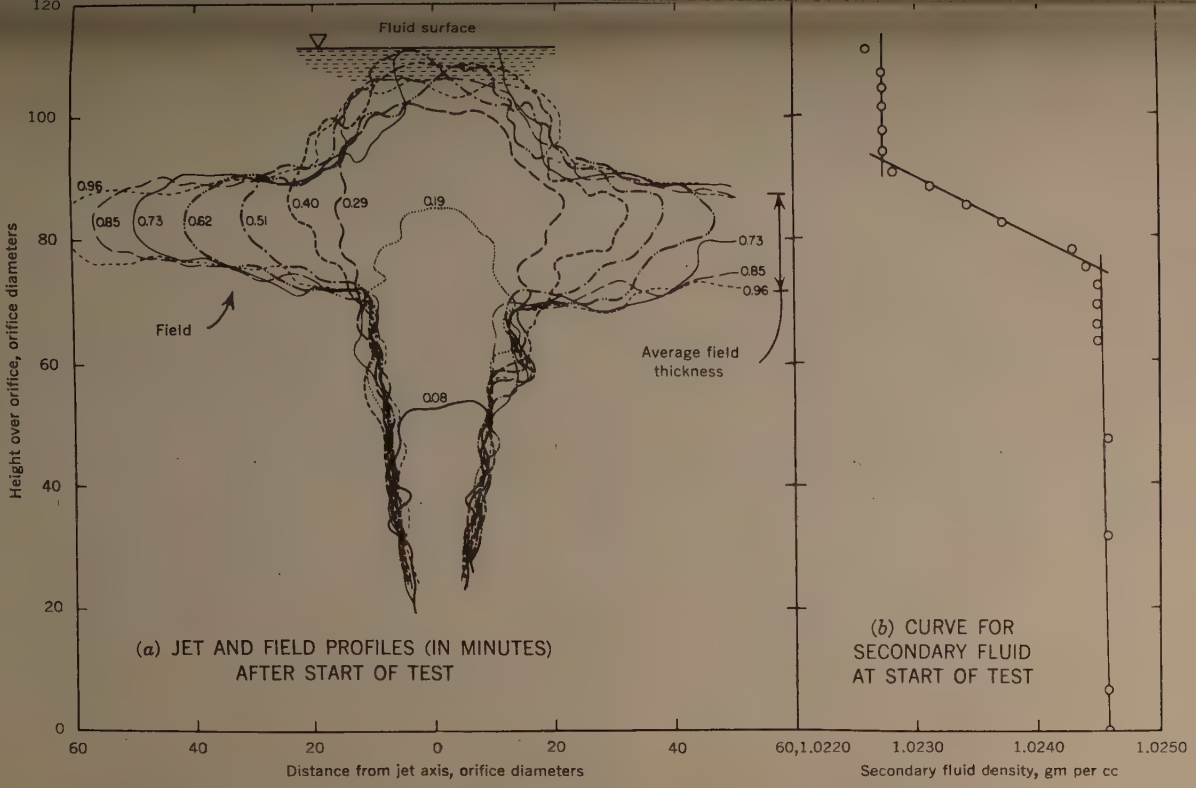


FIG. 12.—JET AND FIELD PROFILES AND DENSITY-HEIGHT CURVE FOR RUN 29

the density in the jet at the discharge point. Both ρ_o and ρ_c can be measu prior to the run. The concentration factor c remains to be evaluated.

The conditions in the jet of interest in this study are all in the buoy case (small values of (F). The following relationships are from Section Table 1:

$$c = c_m \exp\left(-80\left(\frac{y}{z}\right)^2\right) \quad \ldots\ldots\ldots\ldots\ldots$$

and

$$c_m = c_o\, 9.7\, F^{2/3}\left(\frac{z}{D} + 2\right)^{-5/3} \quad \ldots\ldots\ldots\ldots$$

The concentration of the primary liquid, c_o, will have the value of 1 in paper. The concentration on the jet axis at distance z from the point of charge is c_m.

Eqs. 7 and 8 can be correctly applied to the jet as it extends from the charge point to the lower edge of the thermocline. From this point onv there is a varying relationship (not considered in the equations) betwee surrounding fluid density and the height. In many cases the jet rises throug thermocline and falls back to some level within the thermocline. In orde use the equations, constant values of z and ρ_n must be chosen. In the cur analyzis, z was chosen as the height (H_t) from the jet origin to the cente the thermocline and ρ_n was chosen as having the constant value ρ_c, tha

$$c_m = c_o\, 9.7\, F_c^{\ 2/3}\left(\frac{H_t}{D} + 2\right)^{-5/3} \quad \ldots\ldots\ldots\ldots$$

Density-height curves were plotted from recorded temperature-height files assuming constant salinity in the secondary fluid. The distance H_t determined in two ways. In the first method, the area under the density-he curve was measured with a planimeter, and this value was divided by the dif ence between the maximum and minimum density in the profile. The result the average height of the density-height profile. The procedure for the se method was to take the plotted density-height points and draw three str lines through them (referred to in subsequent portions of this paper as "density-height curve"), estimating the upper (warm) water density (ρ_w) lower (cold) water density (ρ_c) and the location and slope (A) of the ther cline. The height from the jet origin to the midpoint of the thermocline s line was taken as H_t. Figs. 10, 11. and 12 show the application of this l method. Both heights were computed but only the values from the the l (tri-line) method were used in Eqs. 7 and 9. The maximum difference obt with the two methods was 5D (Run 11). The average difference was less 2D, which would cause a density difference in Eq. 6 of less than 0. gm per cc under the conditions investigated. If the secondary liquid was constant temperature, H_t was taken as the distance to the air-liquid sur and the symbol ρ_c was applied to the entire secondary liquid mass. S density changes near the air-secondary liquid surface were not consider otherwise uniform density profiles.

The quantity y in Eq 7 is the distance from the jet axis to a point in th cross section at height z. (See Fig. 2.) E. K. Rice, A. M. ASCE,[12] has aver

12 "Discharge from Submerged Outfalls," by E. K. Rice, thesis presented to the of California, in Berkeley, Calif., in 1951, in partial fulfilment of the requiremer the degree of Master of Science.

data from observations of other investigators and determined that the ope of an element of the containing cone of a turbulent jet is 1/4.53 or 0.221. ing the continuity principle the average density of fluid passing the cross ction at $z = H_t$ can be expressed as

$$\rho_h = \rho_c + 9.7\, F_c^{2/3} \left(\frac{H_t}{D} + 2\right)^{-5/3} (\rho_o - \rho_c)\, k \quad \ldots\ldots \quad (10)$$

which

$$k = \frac{\int_o^{0.221} \exp\left(-160 \left(\frac{y}{H_t}\right)^2 \left(\frac{y}{H_t}\right) d\left(\frac{y}{H_t}\right)\right)}{\int_o^{0.221} \exp\left(-80 \left(\frac{y}{H_t}\right)^2 \left(\frac{y}{H_t}\right) d\left(\frac{y}{H_t}\right)\right)} \quad \ldots\ldots\ldots \quad (11)$$

d

is the average density of fluid passing the jet cross section at height H_t, rams (mass) per cubic centimeter. The fractional term k can be evaluated ing numerical methods. It has a constant value of 0.510. It is convenient to tain the notation of c_m for the group of factors preceeding $(\rho_o - \rho_c)$. q. 10 now becomes

$$\rho_h = \rho_c + 0.51\, c_m\, (\rho_o - \rho_c) \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

ne value of c_m can be estimated from Fig. 3.

The parameter used to determine the group into which a run will fall com- nes the density of the diluted jet and the warm water above the thermocline. ne use of the undiluted primary fluid density in the denominator makes P_1 di- ensionless.

$$P_1 = 10^4 \left(\frac{\rho_h - \rho_w}{\rho_o}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

was found that values of $P_1 \leq 0$ resulted in fields that were established at e surface (Group A). For $P_1 \geq 8$ the field was established submerged roup C). When the parameter took on values such that $4 \leq P_1 \leq 7$ fields were tained which continued to thicken from the surface downward and were not lly established by the time they reached the sides of the tank (Group B). alues of P_1 between 0 and 4 were not investigated. Table 4 lists the calculated lues for each run.

Two additional parameters based on the density at the axis of the jet, as q. 5 suggests, were investigated. The first of these uses the equations of braham.[4]

$$\rho_a = \rho_c + c_m\, (\rho_o - \rho_c) \quad \ldots\ldots\ldots\ldots \quad (14)$$

which ρ_a denotes density of the jet, as computed by the equations of Abra- m,[4] where the jet axis crosses the center of the thermocline, grams (mass) r cubic centimeter. Eq. 14 is identical to Eq. 12 except that the averaging ctor 0.51 is not included. The terms c_m, ρ_c and ρ_o are determined as before. ne estimating dimensionless parameter (P_2) is similar to the previous one.

$$P_2 = 10^4 \left(\frac{\rho_a - \rho_w}{\rho_o}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

The second parameter was computed according to formulas of Rawn, Bo
man, and Brooks,[3] and Rawn and Palmer.[2] The left side of inequality Eq.
as mentioned previously, the density of the fluid in the jet at the jet axis.
therefore, possible to write

$$\rho_b = \frac{(S_p - 1)\rho_c + \rho_o}{S_p} \quad \ldots\ldots\ldots\ldots$$

in which ρ_b is the density of the primary-secondary mixture at the jet
grams, (mass) per cubic centimeter; and S_p denotes the dilution in th

TABLE 4.—PARAMETERS CALCULATED FROM OBSERVED EXPERIMENTAL
DITIONS, USED IN ESTIMATING FIELD LOCATIONS [a]

Run	P_1	P_2	P_3
4	17.4	14.9	15.2
5	17.2	13.5	14.4
7	21.7	20.5	20.4
9	18.9	17.0	17.6
11	8.2	6.5	6.5
16	-1.7	-3.3	-3.7
18	16.8	12.8	13.7
19	8.2	5.5	5.8
20	0.5	-7.7	-5.1
21	3.8	-0.3	-0.4
22	-5.3	-10.3	-10.1
23	-2.4	-4.7	-5.0
24	6.7	-1.7	2.8
25	7.1	4.3	4.6
26	-2.9	-5.6	-5.8
27	-4.0	-7.9	-7.9
28	12.6	7.4	8.9
29	4.7	-0.4	0.9
30	14.0	8.2	9.5
31	16.1	13.7	13.8
32	-3.6	-7.1	-7.2
33	-3.8	-7.5	-7.5

[a] (P_1, P_2, and P_3 are defined by Eqs. 19, 20 and 21 respectively.)

where the jet axis crosses the thermocline, grams (mass) per cubic centir
The value of S_p is determined from Eq. 1 using S_p in place of S_s and,
L_p, the distance from the jet origin to the thermocline, in place of L_s. In
of the examination of P_1, H_t is an estimate of L_p. Eq. 1 is now written a

$$S_p = \frac{1 + 1.18\left(10^{-2}\right)\left(H_t + 3\right)^{3.25}}{Q^{0.61}} \quad \ldots\ldots\ldots$$

he corresponding dimensionless parameter is

$$P_3 = \frac{10^4 (\rho_b - \rho_w)}{\rho_o} \quad \ldots\ldots\ldots\ldots \quad (18)$$

The relationship between the three parameters just developed can best be emonstrated by inserting the values of ρ_h, ρ_a and ρ_b into the corresponding quations for P_1, P_2 and P_3. This results in the following three expressions.

$$P_1 = \left(\frac{\rho_c - \rho_w}{\rho_o} - 0.51\, c_m \frac{\rho_c - \rho_o}{\rho_o} \right) 10^4 \quad \ldots\ldots\ldots \quad (19)$$

$$P_2 = \left(\frac{\rho_c - \rho_w}{\rho_o} - c_m \frac{\rho_c - \rho_o}{\rho_o} \right) 10^4 \quad \ldots\ldots\ldots\ldots \quad (20)$$

nd

$$P_3 = \left(\frac{\rho_c - \rho_w}{\rho_o} - \frac{1}{S_p} \frac{\rho_c - \rho_o}{\rho_o} \right) 10^4 \quad \ldots\ldots\ldots\ldots \quad (21)$$

qs. 20 and 21 are based on the axial density of the jet as it crosses the thermocline. The two parameters have similar values for the current investigations (Table 4). This is reasonable, because Rawn and Palmer's[2] experiments ncluded conditions similar to those reported herein. Abraham's[4] equations ere based on dimensionless parameters and his coefficients were chosen rom the work done by Walton Forstall and E. W. Gaylord[13] on aqueous olutions. Eq. 19 is based on an estimate of the average density in a cross ection of the jet.

An important consideration for those concerned with the condition of sewage in the spreading field is the dilution, S. The reciprocal of the coefficients $(\rho_c - \rho_o)/\rho_o$ in Eqs. 19, 20 and 21 are dilutions. The first of these, $/0.51\, c_m$, is an estimate of the average dilution in the jet cross section and he other two, $1/c_m$ and S_p, are estimates of the dilution at the jet axis. The elation of $1/0.51c_m$ to observed dilutions in the spreading filed is examined n the following sections.

Group A.—The runs in this group all had fields established at the surface of he water. The field height would then be the distance to the water surface inus one-half the field thickness. In the current studies, the field thickness as determined by measuring the average thickness of the dye image made on olor movie film as the spreading field reached the boundaries of the tank. See Fig. 10.) The transition from a very dark dye area to clear water was apid, of the order of a few percent of the field thickness. Rawn and Palmer[2] xamined the thickness of spreading sewage when the discharge is into a homoeneous medium. All but one of the runs (number 20) of Group A are of this ase so it is reasonable to apply their results to the current investigations.

13 "Momentum and Mass Transfer in a Submerged Water Jet," by Walton Forstall nd E. W. Gaylord, Journal of Applied Mechanics, Vol. 2, No. 2, 1955, p. 161

Two expressions were given by them. In the first, they assumed no ene loss where the jet changes direction at the surface.

$$h = \frac{L_s}{16} \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

in which h is the field thickness, in feet; and L_s denotes the length of the (equivalent to the distance to the surface for the conditions investigated h estimated by H_s), in feet. They obtained the second expression from ac measurements.

$$h = \frac{L_s}{12} \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

In the current tests of Group A, the distance to the water surface varied f 111 to 113 jet diameters (the diameter at discharge) with an average of diameters. The field thickness varied from 18 to 21 with an average valu 19 diameters. The ratio L/h is therefore 6, considerably less than the va suggested by Rawn and Palmer.[2] The following appears to be an adequate pression for the current investigation:

$$\frac{h}{D} = 19 \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

Note that Eq. 24 can only be applied to depths of about 112 jet diamet

Three runs in Group A provided sufficient colorimetric and tempera data to allow analysis of field densities. The observed densities are pl in Fig. 13 at the times of their occurrences. Run 23 [Fig. 13 (a)] us probe height of 107 diameters and a surface height of 112 diame $(\rho_h - \rho_w/\rho_o)(10^4) = -2$. Run 27 [Fig. 13 (b)] used a probe height of 107 meters and a surface height of 115 diameters; $(\rho_h - \rho_w/\rho_o)(10^4) = -4$. run 33 [Fig. 13 (c)] the probe height was 102 diameters and the surface h of 111 diameters; $(\rho_h - \rho_w/\rho_o)(10^4) = -4$. The initial density in the liquid a point where the sampling occurred, as obtained from the density height cu is plotted at time 0 as ρ_p. Also plotted at this time is the estimated densit

Dilutions in the spreading field were determined for the averages o colorimetric determinations. Table 5 lists these along with the calcu values of $1/0.51\, c_m$, the probe heights, and the upper and lower field hei

The fluctuations in the observed densities can be attributed to at leas causes. First, the six samples (maximum) taken during any one run at a g level were obtained at various angular positions on a radius of 1 ft abou jet axis. The temperature measurements were taken at a single location, 1 ft from the jet axis. (See Fig. 9). If the jet was in any way unsymmet different conditions would be expected at the different observation points. ond, the nature of the flow is one of rolling interfaces and non-uniform mi This can easily be confirmed by viewing the movies taken during the ex ments. Either of these situations could cause large variations in the obse densities.

One can suggest at least two causes, in addition to the preceding ones the differences in calculated and observed values of density. First, the exp sion $1/0.51\, c_m$ gives an estimate of the average density in the jet (at the chosen for calculation) and does not consider any additional dilution that

take place as the field spreads. Thus, the samples are not taken from the location for which the dilution was computed. Second, the sample tubes and probes give single point values (in the vertical dimension) rather than average ones. Therefore, the vertical location of the sampling equipment in the field has a pronounced effect on the value obtained, higher dilutions occurring near the bottom of the field where the mixture is most dense. This is borne out by the high values of observed dilution in Run 33.

Group B.—This group of runs had no established field height; the field was in the process of growing from the surface down. Due to this continually

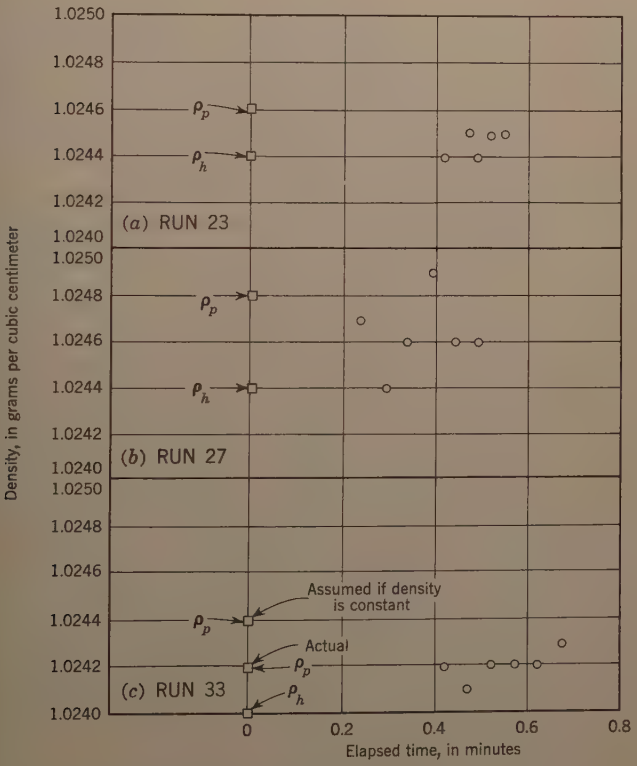


FIG. 13.—FIELD DENSITIES FOR RUNS IN GROUP A

changing situation, it is impossible to assign a specific height to it. Its upper limit was at the secondary fluid surface and the lower limit did not extend below the thermocline during the run. Thickness measurements were made at time intervals of approximately 0.05 min during these runs but no equation was found which would enable the final thickness of the fields to be predicted.

Run 24 was the only one in this group yielding field density data. Samples were taken at heights of 79 diameters [Fig. 14(a)] and 110 diameters [Fig. 14(b)]. The temperatures recorded at the 110 diameter level were obtained

TABLE 5.—SPREADING FIELD DATA

Group	Run	Dilutions		Heights in Diameters		
		Sample Data[a]	$\frac{1}{51c_m}$	For Samples[c]	Top of Field	Btm of Field
A	23	206	115[b]	107	113	95
	27	114	66[b]	107	111	93
	33	340	72[b]	102	111	93
B	24	251	53[b]	110	113	>75
	24	872	53[b]	79	113	>75
C	28	87	51[b]	79	93	75
	31	260	103[b]	85	90	75

[a] Calculated from measured values of sample dilutions and temperatures

[b] Calculated from Eq. 9

[c] Height of sample tube (or pipette) opening

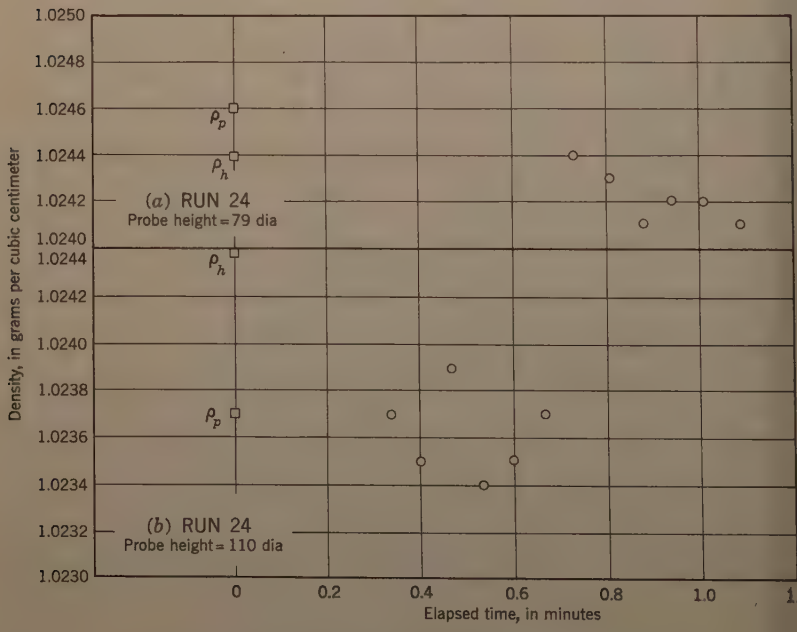


FIG. 14.—FIELD DENSITIES FOR RUNS IN GROUP B

th the thermistor and are not considered as reliable as those at the 79 ameter level (and of other runs) which were determined with a thermocouple. The values of ρ_p and ρ_h, described previously, are plotted (Fig. 14) along th the observed values of density within the field. For run 24 (Fig. 14) the rface height was 113 diameters, the thermocline height 84 diameters and $h - \rho_w/\rho_o)(104) = 7$. The 110 diameter data have no trend with time. pparently the portion of the field at this level had reached its equilibrium nsity. Those at the 79 diameter level decreased with time and then

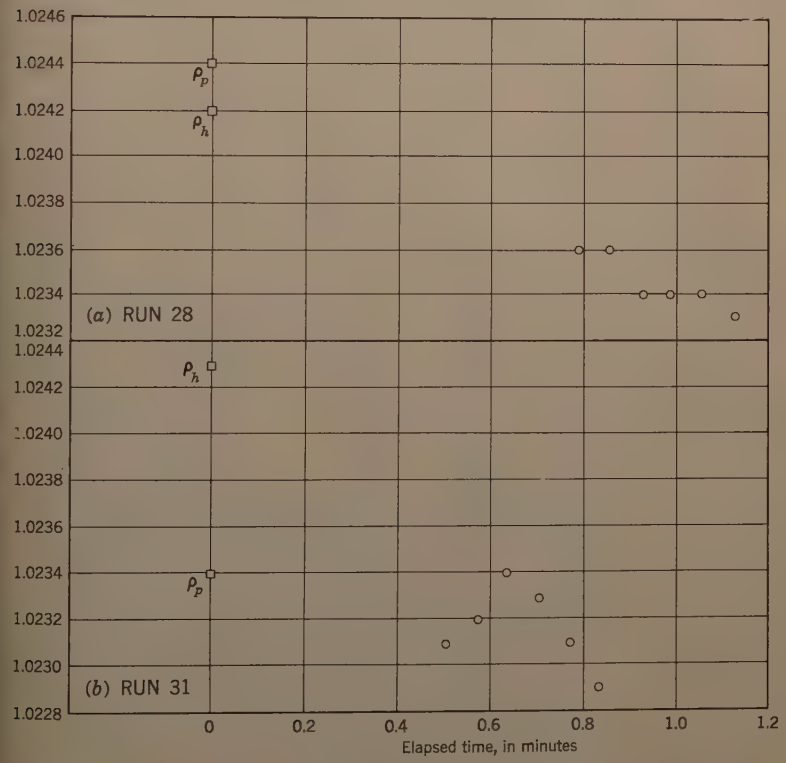


FIG. 15.—FIELD DENSITIES FOR RUNS IN GROUP C

velled out. Observation of the movies shows that the field was just reaching ese sampling tubes at about 0.8 min. At less than 1.0 min the field had eached the sides of the tank and this may have caused a trend toward further ensity lowering. The previous examination of observed density fluctuations nd calculated density discrepancies applies to this run also.

Group C.—In attempting to estimate the height of the center of the field everal parameters were investigated. A good relationship was found when the eight corresponding to ρ_h on the density height curve was determined. The

TABLE 6.—SUMMARY OF RESULTS[a]

Group	Run	P_1 $\frac{\rho_h-\rho_w}{\rho_o(10^{-4})}$	$\frac{\rho_c-\rho_w}{\rho_o}$	$\frac{H_t}{D}$[b]	$\frac{H_h}{D}$	$\frac{h_t}{D}$	F_c	R	Loc. Surf	Loc. Tran	Loc. Subm	Hgt. to Ctr. Fld., H/D Estimates by Eq. 25 Actual	Estim.	Deviation Percent	Field Thickness, h/D Estim. by Eqs. 24 and 26 Actual	Estim.	Deviation Percent
	22	-5	0.0000	112			33.3	18000	x						18	19	+6
	27	-4	0.0000	111			22.6	12000	x						18	19	+6
	33	-4	0.0000	111			20.3	11000	x						19	19	0
	32	-4	0.0000	113			19.2	10000	x						20	19	-5
A	26	-3	0.0000	113			13.5	7200	x						18	19	+6
	23	-2	0.0000	113			10.2	5400	x						18	19	+6
	16	-2	0.0000	111			5.9	3100	x						21	19	-10
	20	0	0.0009	83	87	29	31.9	17000	x						20	19	-5
	21	4	0.0008	82	82	32	10.9	5800		x							
	29	5	0.0010	81	81	30	15.0	8000		x							
B	24	7	0.0012	84	81	29	16.3	8700		x							
	25	7	0.0010	82	79	30	6.4	3400		x							
	19	7	0.0011	82	78	30	5.9	3100			x	85	84	-1	20	17	-15
	11[c]	8	0.0010	83	79	26	3.2	1700			x	82	85	+3	14	15	+7
	28	13	0.0018	82	79	30	15.9	8500			x	85	85	0	16	17	+6
	30	14	0.0020	84	81	29	18.9	10000			x	88	87	-1	18	16	-11
	31	16	0.0019	83	76	28	5.8	3100			x	82	82	0	15	16	+7
C	18	17	0.0021	82	76	29	11.2	5900			x	82	82	0	15	16	+7
	5	17	0.0021	118	110	42	29.2	9200			x	120	114	-5	21	20	-5
	4	17	0.0020	120	115	40	15.8	5100			x	120	119	-1	17	20	+18
	9	19	0.0021	113	106	36	14.0	3900			x	110	110	0	17	18	+6
	7[c]	22	0.0023	121	112	39	5.1	1700			x	110	116	+6	21	19	-10

[a] See Appendix for an explanation of symbols used.
[b] H_t/D is given for runs with no thermocline.

resulting linear equation is

$$\frac{H}{D} = 9.79 + 0.950 \frac{H_h}{D} \qquad r^2 = 0.983 \quad n = 10 \qquad \ldots\ldots\ldots \quad (25)$$

In which H denotes the height of the center of the spreading field, in feet; H_h is the height on the density-height curve corresponding to ρ_h, in feet; r refers to the product-moment correlation coefficient; and n is the number of observations. Table 6 lists actual and estimated H/D values. The data and estimating Eq. 25 are shown in Fig. 16. The maximum deviation of the estimated from the actual was 6%. It should be emphasized that Eq. 25 is applicable only to the

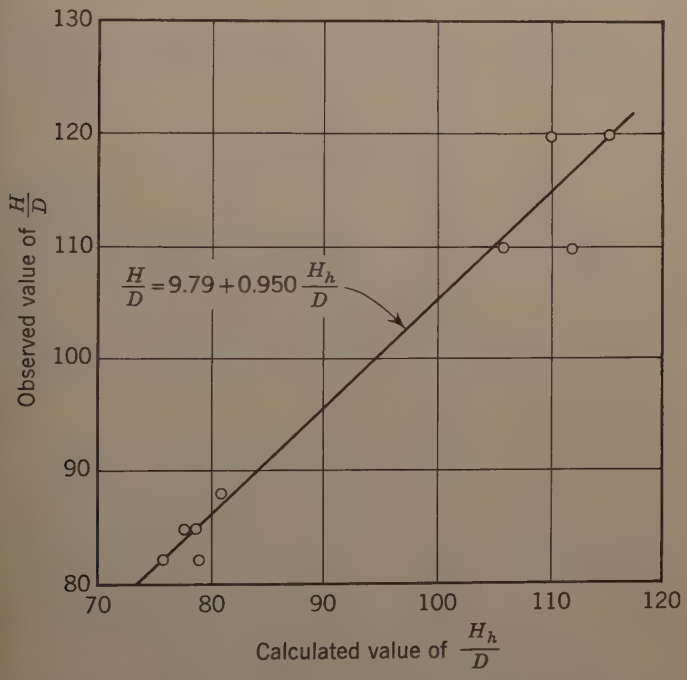


FIG. 16.—HEIGHT TO CENTER OF SPREADING FIELD AS A FUNCTION OF HEIGHT ON DENSITY-HEIGHT CURVE CORRESPONDING TO ρ_h (GROUP C)

range of values given. There are strong groupings around H/D equal to 115 and 82, with no values between.

An attempt was made to find a measurable quantity that would allow prediction of field thickness from original conditions. Linear regression techniques were used on many variables, including the Froude number, Reynolds number, thermocline angle, field grouping parameters, and the height over the center of the thermocline. Logarithms of many of these and the dependent variable were also considered. No completely satisfactory parameter was

found. The equation for the best one is

$$\frac{h}{D} = 7.65 + 0.229\ \frac{h_t}{D} \qquad r^2 = 0.677 \qquad n = 10 \quad \ldots\ldots\ldots$$

in which h_t is the height of secondary fluid over the center of the thermoc as determined from the density-height curve, in feet. The actual and estim values are listed, along with percentages of deviation from actual, in Tab The data and estimating Eq. 26 are shown in Fig. 17. Again, the range of va considered is limited and should be taken into consideration when usin equation.

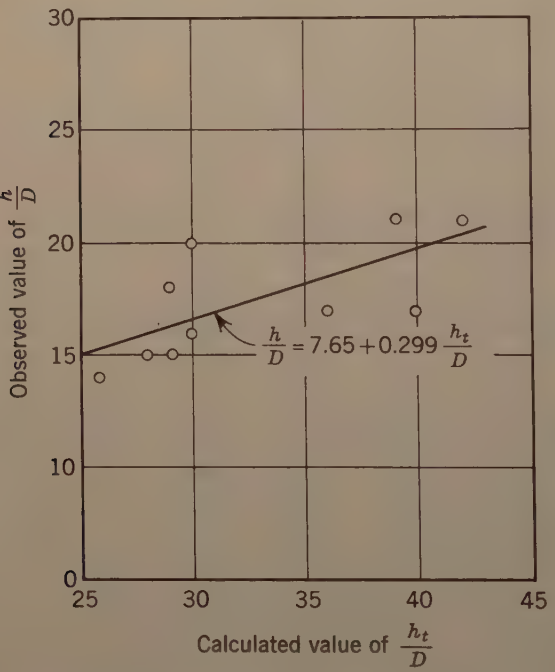


FIG. 17.—THICKNESS OF SPREADING FIELD AS A FUNCTION OF HEIGHT OF SECONDARY FLUID OVER THE CENTER OF THE THERMOCLINE (GROUP C)

Two runs in Group C yielded adequate field density data for analysis. A values of ρ_p, ρ_h and observed densities were plotted (Fig. 15). For r [Fig. 15 (a)] the probe height was 79 diameters, the thermocline heigh 82 diameters and $(\rho_h - \rho_w/\rho_o)(104) = 13$. For run 31 [Fig. 15 (c)] the height was 85 diameters, the thermocline height was 83 diameters and $\rho_w/\rho_o)(104) = 16$. Previous remarks regarding fluctuations and discrepa between observed and calculated densities apply. In addition, the densit

leases in Run 28 were probably contributed to by the action of the field aching the tank at about 1.0 min. It should also be noted that the probes (at t from the axis) were in the dome of the secondary-primary fluid mixture ther than in the spreading field.

CONCLUSIONS

1. The type of a field (surface, submerged, or intermediate) caused by discharging a turbulent liquid jet vertically upward into another liquid of slightly gher density having a density gradient can be predicted within the range of e variables tested. The equations developed for jets discharging into homogeneous fluids, as modified in this paper, are used as indicators.

2. In the case of a submerged field, an accurate prediction can be made f the height of the center of the field above the discharge point. A less accurate prediction of the thickness of the field can be made.

ACKNOWLEDGMENTS

This investigation was conducted by the writer and reported in a Master's nesis in Mechanical Engineering at the University of California, Berkeley, alif. The study was partly supported by a grant from the United States epartment of Health, Education, and Welfare. The writer wishes to thank . L. Wiegel for suggesting the study and A. D. K. Laird for giving valuable dvice during the course of the study.

APPENDIX — NOTATIONS

Symbols used in this paper are listed here for convenience of reference nd for the aid of discussers. (The engineering system of units is used for all uantities except densities. In these cases the centimeter-gram-second econd system is used, which is common in the literature.)

Δ = Slope of the thermocline, defined as the thickness of the thermocline in jet diameters, D, (as found from the density-height curve using the tri-line method) multiplied by 10^{-4} divided by $(\rho_c - \rho_w)$, 10^{-4} cc per gm (mass);

c, c_m, c_o = concentration in the jet, defined as $(\rho - \rho_n)/(\rho_o - \rho_n)$ or $(\rho - \rho_c)/(\rho_o - \rho_c)$: general term, at a point on the axis of the jet, of the jet at discharge, respectively;

D = diameter of the jet at discharge, feet;

F = Froude number: general term $u_o/\sqrt{Dg(\rho_n - \rho_o)/\rho_o}$, referred to cold water layer in secondary fluid $u_o/\sqrt{Dg(\rho_c - \rho_o)/\rho_o}$, respectively;

g = acceleration due to gravity, feet per second;2

h = thickness of the spreading field, feet;

h_t = height of the secondary fluid over the center of the th ocline as determined from the density-height curve

H_f, H_s, H_h, H_t = height over jet discharge point, feet: of field estal ment, to air-liquid surface, as determined by ρ_h density-height curve, to center of thermocline (by the planimenter or tri-line method), respectively::

k = averaging factor in the equation for ρ_h;

L, L_s, L_p = length measured along the jet axis, feet: general ter the surface, to the center of the thermocline, respect

M = mass flow rate through a cross section of the je (mass) cu ft per cc sec;

n = number of observations;

P, P_1, P_2, P_3 = parameters for the estimation of field location: ge term, using ρ_h, using ρ_a, using ρ_b, respectively;

Q = flow rate of jet at discharge, cfs;

R = Reynolds number, $u_o D/\nu$;

r = product-moment correlation coefficient;

S, S_s, S_p = dilution factor, defined as the ratio of the total volu a sample (primary and secondary fluid mixture) volume of primary fluid in the sample: general ter the point where the jet axis meets the air-liquid su at the point where the jet axis crosses the center thermocline, respectively;

u, u_o, u_m = axial component of velocity in the jet, fps: at any at the point of discharge on the jet axis, respect

y = radial distance from the jet axis to a point in the jet

z = vertical distance from the discharge point to a po the jet, feet;

ρ_n, ρ_c, ρ_w = density of the secondary fluid, gm (mass) per cc: ge term, cold layer, warm layer, respectively. (See

ρ, ρ_o, ρ_j, ρ_a, ρ_b = density in the jet, gm (mass) per cc: general term; charge; on axis by any equation; on axis by equati Abraham; on axis by equations of Rawn, Bowerm Brooks and Rawn and Palmer; respectively;

ρ_h = average density through a cross section of the j (mass) per cc;

ρ_p = density at height p as determined from the density- curve, gm (mass) per cc; and

ν = kinematic viscosity of the primary fluid, sq ft pe

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DISCUSSION

Note.—This paper is a part of the copyrighted Journal of the Hydraulics Division, oceedings of the American Society of Civil Engineers, Vol. 87, No. HY6, November, 61.

DICTING STORM RUNOFF ON SMALL EXPERIMENTAL WATERSHEDS[a]

Closure by Neal E. Minshall

EAL E. MINSHALL,[12] M. ASCE.—As Amorocho has pointed out, the unit rographs of Fig. 6 were compiled using storms having approximately equal tions. A study of the relation between rainfall intensity and unit hydroh peak, on the 290-acre watershed, indicated duration of rainfall excess only of minor importance so long as it was of uniform intensity and less the time to peak.

morocho comments that, in the case of large areas, the outflow hydroh becomes rather insensitive to inflow intensity changes, and the shape s to reflect the catchment characteristics. The writer concurs in this and eves the relation between rainfall intensity, unit graph peak and size of nage area (shown in Fig. 9) bear this out as the curves representing difnt intensities are converging toward the larger drainage areas.

Dougal notes that the selection of a k value of 0.95 for the recession factor cates a lasting effect of previous precipitation because API values recess e slowly. This is true for the claypan soils which are slowly permeable have apparent infiltration rates as low as 0.02 in. per hr after the surface r becomes saturated. This is not the true infiltration capacity of the sur- soil but rather the permeability of the claypan layer.

he duplicate API values for a particular date (as for August 14, 1946) are for independent storms beginning at different times but rather for different m durations beginning at the same time. This was possible for a few ms which had short periods of high intensity to which a definite part of the ff hydrograph could be attributed. The method was used only for those ms in which there were short periods of little or no rainfall and the sur- detention is assumed not to have been completely depleted. Fig. 10(a) d have been readily divided in this manner into two periods both beginning :20 with an API of 3.16, the first period ending at 1:45 and the second or e storm ending at 3:10. Under these conditions the abscissa in Fig. 3 d not have been properly labeled "Duration of excess rainfall."

o initial API was assumed and only the 30 days immediately preceding the m were considered. This is not entirely correct, but, as shown by Dougal able 5, the initial API after so long a period has only a minor effect on the ay API. The writer concurs with Dougal that because of the large number orms used in the analysis, it would have been simpler to carry the API ard continuously.

August 1960, by Neal E. Minshall (Proc. Paper 2577).

[12] Hydr. Engr., Cornbelt Branch, Soil and Water Conservation Research Div., Agric. arch Service, U. S. Dept. of Agric., Madison, Wis.

Although there are variations in the seasons from one year to the nex records of cover were not sufficiently complete to justify making any a ment for this when compiling data for the time retention curves of Fig. 3

Dougal comments that identical API values for two storms may result different rainfall distributions and that there exists a considerable diffe in soil surface condition depending on whether the API is rising or fall the time of the storm. The use of a shift curve as he suggested would no have reduced the scatter of points in Fig. 3.

Regarding Dougal's question as to whether the synthetic hydrograph tained from Fig. 9 were compared with the Soil Conservation Service m the writer wishes to point out that no such comparison was made. It w the purpose of the paper to either approve or contradict existing metho rather to present his findings based on actual precipitation and runoff d various size areas for a particular soil type. In the Soil Conservation S method, the design hydrograph, for example, is based on a 6-hr point r modified for size of watershed; computation of triangular hydrographs crements of direct runoff for uniform time intervals; and summation triangular hydrographs. The method, usually applied to drainage ar larger size than the experimental watersheds discussed in the present assumes one unit hydrograph for a particular area. On these small e mental watersheds, the single peak hydrograph generally represents s having lower rates and quantities of runoff than the multiple peak hydrog No frequency analysis of these data was made, but a method was present determining the runoff hydrograph from the rainfall pattern and antecedent ture condition. On W-1 (27.2 acres) only about 15% of the storms that runoff rates of over 0.5 in. per hr had durations of 6 hr, and 50% of this had durations of less than 3 hr. For these small watersheds and with the times to peak, as shown in Fig. 5, the maximum rates and quantities of usually result from a combination of several periods with varying inten Because of the effects of rainfall intensity on the peak rate and time to single unit hydrograph would not give a good reproduction of the actual hydrographs.

With regard to storm pattern, a study of 6-hr storm periods generally the highest intensities early in the storm. However, considering peri 10 min to 30 min, which are the times involved in Fig. 5, the maximum sity does not occur at any particular position but may be anywhere in the period. In computing the hydrograph of Fig. 10(c) the only part to which hydrograph for high intensity early in the storm would apply is between and 10:20. Because this period was assumed as producing no runoff, t hydrographs for highest intensity late in the storm were applied to all periods.

In view of the difference in soil characteristics, topography and c practices between the claypan soils at Edwardsville, Ill. and the mode permeable soils at Monticello, Ill., the writer was surprised at the reas good agreement shown by McFall and Jones in Fig. 12.

McFall and Jones comment that total runoff is a weaker comparis than peak discharge. Both comparisons were presented because of the portance in the design of small dams and the relative importance depe whether the dam is intended primarily for flood retardation or channel ization.

t may have been feasible to construct a coaxial correlation to include par-
ters for season and soil characteristics for use instead of time retention
ves. Figs. 10(a), 10(b) and 10(c) were subdivided into 14, 11 and 6 runoff
ducing periods. Under these conditions, the writer felt that a group of time
ntion curves would be less cumbersome than a diagram of coaxial rela-
s.

VIBRATION PROBLEMS IN HYDRAULIC STRUCTURES[a]

Discussion by David W. Appel and Charles L. Sanford, and H. L. Uppal

DAVID W. APPEL,[23] M. ASCE, and CHARLES L. SANFORD.[24]—The review esented by the author of experiences with vibration of gates in conduits il- trates the increasing importance of understanding the causes of secondary tions and pulsations in flows with various boundary configurations, and the dequacy of existing information. This paper should provide a stimulus for ditional basic investigations and for further detailed observation of flows in ototype structures.

The author focuses attention on several sources of excitation of vibrations hydraulic structures. One of these, the von Kármán vortex trail, is recog- ed as important in flow past two-dimensional bluff bodies. The possibility this being a cause of vibration of culvert gates, as in Fig. 5, needs to be ful- explored. The writers wish to present some pertinent information and give asons for not attributing this case of vibration to a von Kármán vortex trail.

In the alternate shedding of vortices from two sides of a bluff body (such as lat plate or circular cylinder) to form a von Kármán vortex street, there is interaction between the vortices on the two sides and a cyclic cross-flow er the centerline of the wake.[25]

When a splitter plate was placed behind a stationary circular cylinder to minate interaction between the two sides of the flow, Roshko found that al- nate shedding of vortices was eliminated and a steady, symmetrical flow s obtained.[26,27] Also, in studies at the University of Kansas, Lawrence, ns., of flow through abrupt two-dimensional expansion,[28,29] no vortex pat- n comparable to the von Kármán vortex trail was observed. Thus, there is idence that von Kármán vortices occur behind certain two-dimensional, sym- trical, rigid bodies only if there is free communication of flow between the o sides of the wake.

In the study at Kansas, photographic records of what takes place in flow ough abrupt expansions were obtained using two methods of making the flow

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[a] March 1961, by Frank B. Campbell (Proc. Paper 2772).

[23] Prof., Dept. of Engrg. Mechanics, Univ. of Kansas, Lawrence, Kans.

[24] Engr., Fluid Mechanics Lab., Kimberly-Clark Corp., Neenah, Wis.

[25] "Elementary Mechanics of Fluids," by Hunter Rouse, McGraw Hill Book Co., Inc., w York, N. Y., 1946.

[26] "On the Drag and Shedding Frequency of Bluff Cylinders," by A. Roshko, NACA ch. Note 3169, 1954.

[27] "Flow Past a Circular Cylinder at High Reynolds Number," by A. Roshko, Journal Fluid Mechanics, Vol. 10, Part 3, May, 1961.

[28] "A Study of Flow Through Abrupt Two-Dimensional Expansions," Progress Report Mean Characteristics of Flow, by Neel K. D. Sharma, Report No. 2, Studies in Engi- ring Mechanics, Univ. of Kansas, Lawrence, Kans., June, 1960.

[29] Ibid, Progress Report III, Formation of Vortices, by Charles L. Sanford, Report 7, Studies in Engineering Mechanics, Univ. of Kansas, Lawrence, Kans., June, 1961.

visible. In the first, cavitation was induced by lowering the pressure
system. Vapor cavities formed within vortices generated along the surf
separation and served to mark their position,[30] as shown in Fig. 14 for 1
pansion. In this photograph the edge of the enlargement is visible on th
side. The eddy zone in the upper part of the figure contains bubbles
liberated from the flow by the low pressure. The vortices along the e
the eddy zone are essentially two-dimensional except for the horizon
shaped trailing vortices in the boundary layer on the front wall of the expa

High speed motion pictures were taken of flows like this, and the forn
and translation of vortices were observed. (Film with magnetic sound
is available on loan from the Kimberly-Clark Corporation, Fluid Mech
Laboratory, Neenah, Wis.) The generation of vortices was not regular
odic), but the average rate of shedding over a period of a few second
found to be fairly constant and to depend on the expansion ratio b/B, as

FIG. 14.—VAPOR CAVITIES IN VORTICES ON THE SURFACE OF SEPARATION IN AN ABRUPT EXPANSION

in Fig. 15. The frequency parameter $\frac{2f\,(B - b)}{Vo}$ is a Strouhal number c
ponding to the one used by the author. The frequency of vortex formatic
single surface of separation is much greater than for a von Kármán
trail.

To determine whether cavitation affected the frequency of shedding,
were also run with small particles of polyethylene in suspension. These
only slightly lighter than water and therefore followed essentially the
paths as the flow. Photographs with the camera stationary and with the
era moving at approximately the mean speed of translation of the vortic
shown in Figs. 16 and 17. The rate at which vortices were shed was the
without cavitation as with cavitation (Fig. 15).

30 "Cavitation Along Surfaces of Separation," by David W. Appel, ASME pap
60-WA-265, Annual Meeting, November, 1960.

To simulate more closely the case of a gate in a conduit, a vertical plate th a 45° bevel on the downstream side of the lower edge was mounted in the st section. The gate opening was 1.5 in., the conduit height 6 in., and the conit width 3 in. With an estimated contraction coefficient of 0.62, this case ould correspond closely to that of an abrupt expansion with b/B = 0.16. The equency of shedding of vortices was found to be the same as for a plain exnsion (Fig. 15), and the eddy behind the gate was seen to be stable. No evince of vortices occurring at a rate comparable to that of a von Kármán vorx trail was observed.

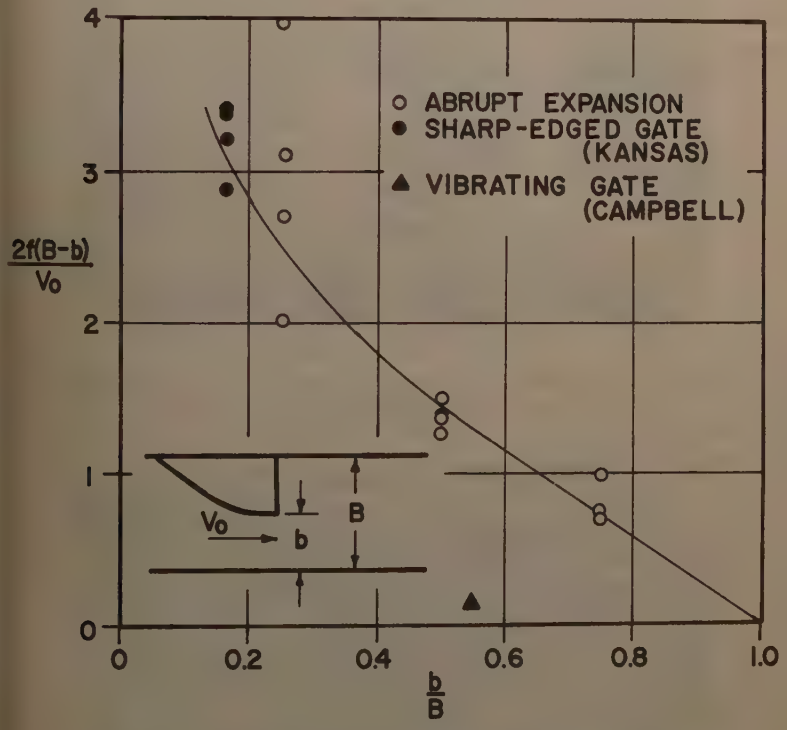

FIG. 15.—FREQUENCY OF VORTEX FORMATION AT EXPANSIONS

The Kansas experiments were performed on flows with rigid boundaries. hen a gate is free to oscillate perpendicular to the flow, as in the experiments escribed by the author, another variable is introduced. In a discussion of ind-induced vibration of antennae,[31] Glenn B. Woodruff, F. ASCE, distinguishes etween two cases of vibration, the first of forced vibration at a rate deterined by the frequency of vortex shedding, and the second of self-induced vi-

31 "Wind Produced Vibration in Antenna Members," by Glenn B. Woodruff, Proceedgs, ASCE, Vol. 87, No. EM3, June, 1961.

bration in which the frequency of shedding is determined by the natural
quency of the number. The discrepancy between the author's observatio
vibrating gates and those of the writers' on flows with fixed boundaries ma
an indication that some form of vortices was induced by the movement o

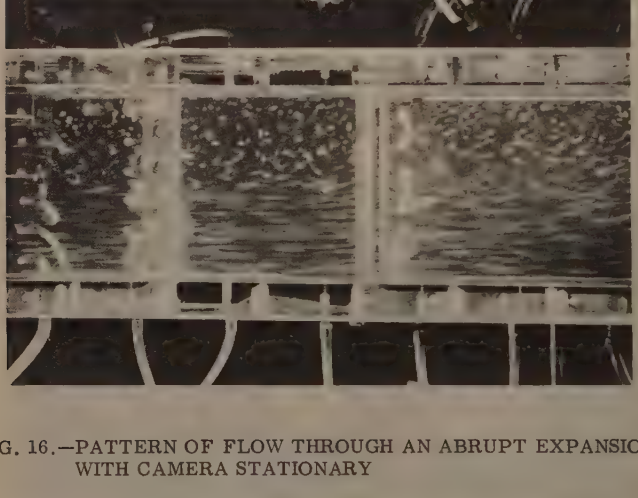

FIG. 16.—PATTERN OF FLOW THROUGH AN ABRUPT EXPANSION WITH CAMERA STATIONARY

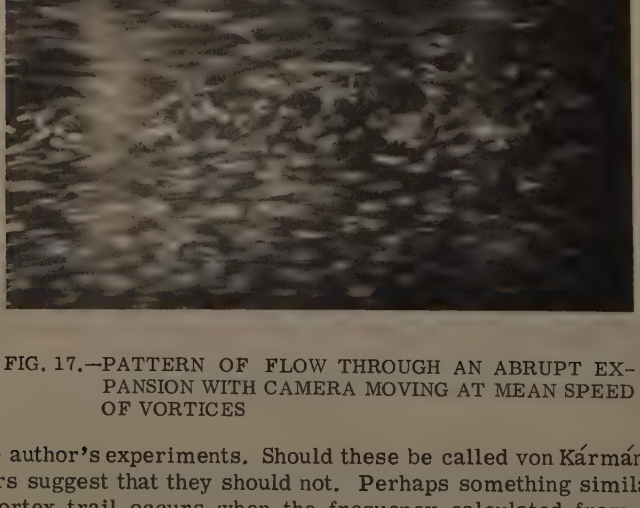

FIG. 17.—PATTERN OF FLOW THROUGH AN ABRUPT EXPANSION WITH CAMERA MOVING AT MEAN SPEED OF VORTICES

gate in the author's experiments. Should these be called von Kármán vort
The writers suggest that they should not. Perhaps something similar to
Kármán vortex trail occurs when the frequency calculated from a Str

umber of 1/7th coincides with the natural frequency of the gate and the gate s caused to oscillate. However, the observations reported herein indicate that o vortex trail should be expected if the gate is rigid. Also, no vortices should e expected if the amplitude of gate movement is small or if the natural frequency of the gate does not coincide with the calculated frequency of a von Kármán vortex trail. In short, the excitation of the gate in the author's experiments must have been dependent on the gate oscillation, whereas the von Kármán vortex trail is not dependent on movement of the boundary for flow past odies.

In the case of a gate which is free to move vertically, changes in vertical orce on the gate may result from variation in size of opening beneath the gate. he pressure on the gate and, hence, the vertical load would vary with change f velocity of flow through the gate opening. This effect would be most pronounced with small gate openings and, presumably, was not the cause of the ibration reported by the author.

Several other possible sources of periodic pulsation in flow through gates n conduits should be considered when a problem of vibration in a prototype is ncountered. If the conduit has symmetrical branches with the flow divided etween them, a periodic fluctuation of headloss associated with the division of low into the two branches should be expected. This type of pulsation was observed in studies at the Iowa Institute of Hydraulic Research[32] and is (in 1961) nder investigation at the University of Kansas, under sponsorship by the National Science Foundation.

Zones of separation also may be unsteady and may cause intermittent change n pressure distribution and dynamic loading on hydraulic structures. In the ase of the gate with lip extension, Fig. 6(b), separation of flow from the gate egins at the upstream corner of the gate and ends on the upstream face of the ip extension ahead of the point at which the final separation, shown in the figre, takes place. Turbulent fluctuations in the flow ahead of the gate will ause small changes in size of the separation pocket in the corner beneath the ate and corresponding fluctuations in downpull.

Secondary flows also will introduce fluctuations in loading on a gate. Vertically oriented corner vortices ahead of a gate trail downward through the gate pening intermittently and can have an appreciable effect on the pressure drop cross the gate. In the study at Kansas, a similar pair of vertical vortices ere observed in the corners of the eddy zone immediately downstream of an brupt expansion of a gate.

The pressure fluctuations shown in Fig. 12 are representative of the simplest case of reflection of an elastic wave on one side of a gate. There would e a corresponding transmission and reflection of the wave on the downstream ide which, generally, would introduce other frequencies and possibly beat henomena. Also, wherever there is a change in cross-sectional area of the onduit or there is a branch point, additional reflections would occur and a ultitude of frequencies of pressure fluctuations at the gate would result. hese pressure waves, particularly if they interact with air on vapor cavities n the vicinity of the gate, could cause large variations in loading. The fluctuations would not necessarily be periodic. It would be interesting to know if intermittent vibrations have been observed in any of the structures operated by he Army engineers.

32 "Mechanics of Manifold Flow," by John S. McNown, Transactions, ASCE, Vol. 119, 954, p. 1103.

The primary sources of pressure fluctuations in regulated conduit f have been identified and the possibilities of encountering a variety of freq cies seldom cause vibration problems in gates, because of their massive struction. To minimize the chance of low-frequency vibrations, the prin frequencies of vortices and surges can be calculated and compared with natural frequency of the gate. Even when these do not coincide, there is a sibility that vibration may be encountered, at least intermittently. Thus, must be built to withstand stresses that vibration would impose.

Further study is needed before vibration-free gates can be designed confidence. In particular, self-induced vibration (flutter) of gates and its e on the characteristics of outflow need to be investigated.

Acknowledgments.—The experiments described herein were made a University of Kansas under sponsorship of the Kimberly-Clark Corpora Neenah, Wis., and the Tennessee Valley Authority, Norris, Tenn. Their port of this work is gratefully acknowledged.

H. L. UPPAL.[33]—Vibrations of the emergency and regulation gates of outlets on high dams is an important problem. This demands much atte during design and operation. Due to heavy vibrations in gates, the hoist ch ber in the right diversion tunnel at Bhakra Dam collapsed.

At Bhakra, the 50.0 ft diameter central portion of the right diversion t was converted into two conduits, 24.0 ft-by-11.75 ft. These were fitted emergency and regulation gates of about 24.0 ft-by-14 ft for regulation of plies to Bhakra areas 3 yr before the completion of the dam. This wa first time that a diversion tunnel of a dam was provided with regulation g

Model tests were carried out to examine the conditions of flow at diff stages of the lake level and determine zones of negative pressures in the duits, housing chamber and so on, and also to determine whether or not velocity jets caused sealing of conduits. Hydraulic studies were carried the Malakpur Hydraulic Research Station and in the Irrigation and Powe search Institute, Punjab, Amritsar, on large scale plexiglass models.

During the 1959 monsoon season the lake level at Bhakra went up to 1440.0 on July 21, 1959 when the right diversion tunnel was operating. vibrations started and were followed by a very heavy rattling noise of the g Cracks occurred in the concrete of the housing chamber, water started ing out suddenly, and the hoist chamber collapsed immediately thereafter ing a great disaster. All inspection galleries leading to power units running full and the left power house under construction was soon subm under several feet of water.

Water in the lake got out of control with the failure of the hoist cha Uncontrolled water went into the right diversion tunnel and into several leries there. The right diversion tunnel was later closed at the mouth great difficulty by dumping in quantities of heavy stones and plastic crates the help of army personnel. As a result of this failure, construction wo Bhakra Dam was set back several months. After closing the right dive tunnel at the mouth, repairs to the Hoist Chamber were undertaken. found that the gates had been badly damaged. The tunnel was closed by m of a concrete plug. The commission which was instituted to inquire int causes of failure of the hoist chamber reported that failure was due to th brations of the gates.

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33 Dir., Irrig. and Power Research Inst., Punjab, India.

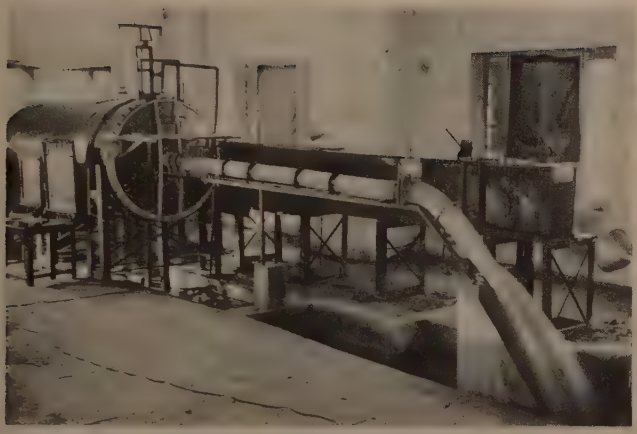

FIG. 19.—SIMPLE AND ACCURATE ARRANGEMENT FOR THE MEASUREMENT OF DOWNPULL

FIG. 20.—DEVICE FOR MEASURING DOWNPULL

The frequency of the vibration of gates largely depends on the frequency o the formation of vortex trail shed from each edge of the gate. If the vorte trail produces an exciting force on the bottom of the gate that would produc reasonance with the natural frequency of the gate, the vibrations will be o large amplitude, causing serious damage and cracks in the concrete.

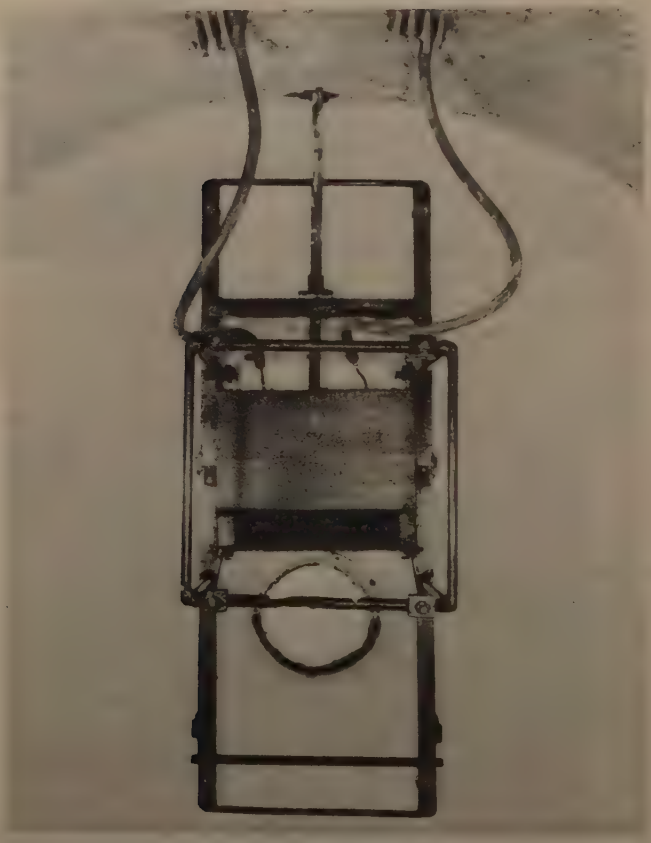

FIG. 21.—ARRANGEMENT FOR THE FREE MOTION OF GATE CARRIAGE ON RAILINGS

Work is being done (in 1961) on the plexiglass model to design a gate wi lips of suitable angles, that would shed vortices with lesser frequency resul ing in minute vibrations.

Moreover, friction in the gate railings and the downpull (measuring devi shown in Fig. 20) is to be reduced,[34] which will help in reducing the strain

34 "Investigations on Hydraulic Downpull Forces on Emergency Gate Under Hi Heads with the Help of Models," by H. L. Uppal, Journal, Central Bd. of Irrig. and Powe India, Vol. 18, No. 4, April, 1961, p. 315.

the concrete in which the gate and the railing are fixed. Fig. 21 depicts an arrangement developed for the free motion of the carriage of a gate on railings.

In order to avoid further occurrence of disaster of this nature, the decision was made to perform a number of tests on vibration of gates. This is being done; high head steel tanks have been constructed and a model of gates fabricated.

STREAM GAGING NETWORK IN THE UNITED STATES[a]

Discussion by Eduardo Basso

E. BASSO,[15] A. M. ASCE.—Problems similar to those presented by McCall exist in Chile on a smaller scale.

The Chilean stream gaging network consisted of 323 stations on July 30, 1961. This amounts to 1.10 stations per thousand square miles. This density is indicated in Fig. 7 which is otherwise identical to the author's Fig. 2. Thus,

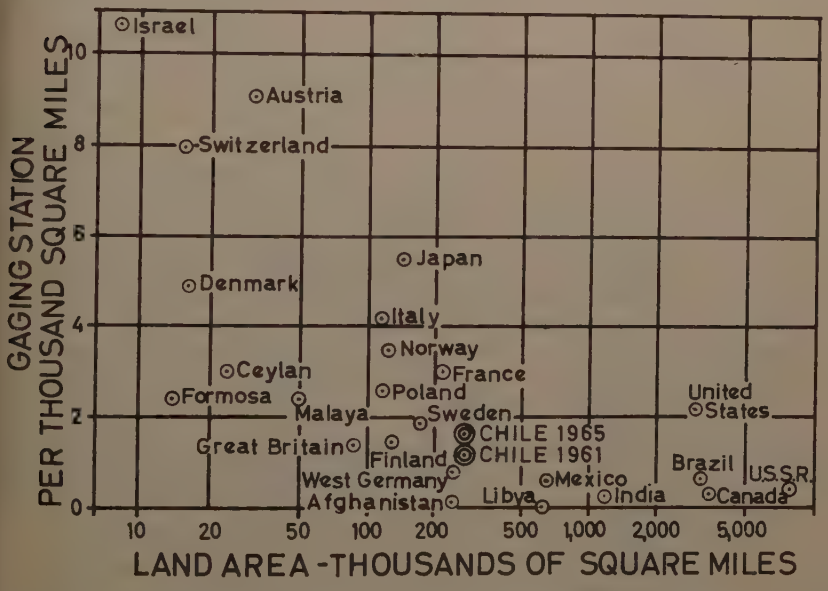

FIG. 7.—DENSITY OF THE CHILEAN STREAM GAGING NETWORK BEFORE AND AFTER THE PLAN

the density of stations in Chile corresponds to a median standard. However, there is great diversity in geographic configuration within the country; thus, although some areas are adequately gaged, others are almost completely ungaged. Such is the case in the two southernmost provinces. They are precisely the parts that contain the greater potential for hydro-electric energy.

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[a] March 1961, by John E. McCall (Proc. Paper 2776).

[15] Acting Chf., Hydrol. Div., Empresa Nacional de Electricidad S. A., Santiago, Chile.

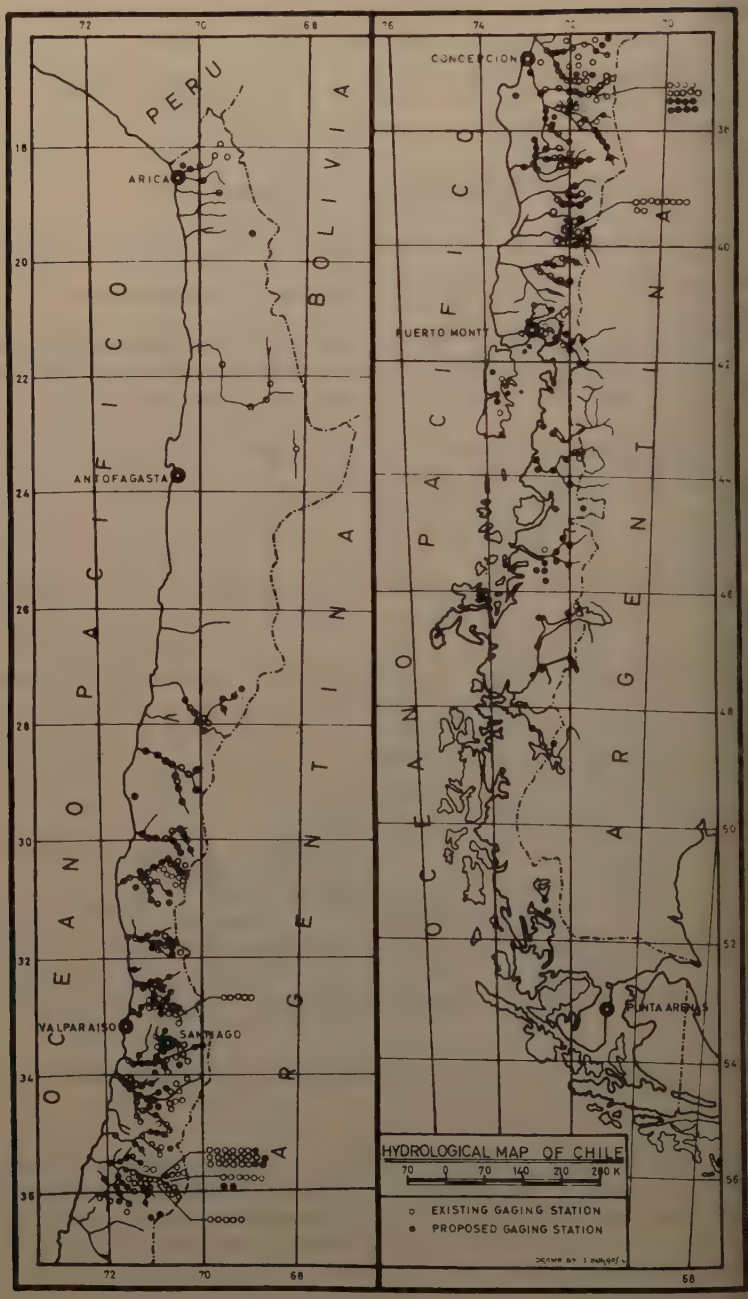

FIG. 8.—APPROXIMATE LOCATION OF ACTUAL AND FUTURE STREAM GAGING STATIONS IN CHILE

In view of this deficiency, the government of Chile has obtained technical and economic assistance from the Special Fund of the United Nations for a project to expand the networks of stream gaging and meteorological stations. With this expansion the stream gaging network will have a minimum of 458 stations (5¢ stations per thousand square miles) after 4 1/2 yr. The government will contribute $1,220,000 to the project and the Special Fund $612,500, mostly in equipment imported to the country.

Three agencies will undertake the work: The Chilean Irrigation office of the Public Work Ministry; the National Electric Corporation; and the Chilean Weather Office.

In Fig. 8 the stations at the beginning of the project and the provisional locations of the new stations are indicated. The areas of concentration of stations are those of great utilization of hydro-electric energy or of intensive irrigation.

The main criterion used for the location of new stations has been the attempt to complete the gaging on the principal rivers above the intakes of irrigation canals and below the junctions of the larger tributaries. Also, some stations will be located in the power parts of rivers in order to measure the water which passes to the sea. The varying density of the proposed network corresponds roughly to the density of population. The almost complete absence of precipitation and stream flow in the desert north implies that gages are necessary only in those beds that contain water permanently or at least usually.

In the central zone of the country, the greater density of stations is rather closely linked with the development of irrigation. In the south there exist enormous quantities of streamflow and potential hydro-electric energy, but the difficulty of access, attributable to the rugged topography and the rigorous climate has resulted in scant population. This scarcity explains the absence of stations. Nevertheless the development of an intercommunicated power pool, currently (1961) effective from latitude 30 to latitude 42 will, in the future, be extended to the Baker and Pascua river (latitude 48°) where there is great power potential. Thus, it is imperative to initiate stream gaging as soon as possible. Similar projects of the Special Fund are being executed in Peru and in Ecuador.

There is no doubt that McCall's recommendations will be useful guides in performing this fundamental work.

AERATED FLOW IN OPEN CHANNELS[a]

Discussion by Jan-Inge Kveisengen; N. Rajaratnam; and Mikio Hino

JAN-INGE KVEISENGEN,[2] A. M. ASCE.—The members of the Task Committee on Air Entrainment in Open Channels are to be congratulated for their summary of the status of present knowledge of the phenomena of aerated flow in open channels. No doubt this paper will serve as an excellent guide for engineers working with design of open channels on steep slopes.

The Progress Report states that much has been learned in this field about the mechanism of air entrainment and the amount of air entrained under various flow conditions. This is of great importance, as design criteria have been established from these data.

Little or no light, however, has yet been shed on the complex relationship of variables that determine the amount of air entrained; that is, of the basic mechanics of the generation of turbulence and of the actual phenomenon of the air entrainment.

Warren DeLapp of the University of Colorado received a National Science Foundation grant in January, 1959, to conduct basic research on the mechanics of air entrainment. The purpose of the study was to relate the total air content and the distribution and size of the bubbles to the fluid properties and the degree of turbulence. The actual entrainment process was also to be investigated by taking moving pictures. The turbulence was generated by a grid agitator moving vertically with harmonic motion through the fluid contained in a 12 in.-by-24 in. transparent plastic cylinder. The physical arrangement was similar to that used by Hunter Rouse,[3] F. ASCE, and others a number of years ago to study sediment dispersion.

This research has been conducted in the Hydraulic Laboratory of the University of Colorado, Boulder, Colo. The following discussion is a report of the main results obtained from this investigation to date.

The apparatus used to determine the air concentration at any one level in the agitated fluid was similar to the electrical measuring device previously developed by Lamb and Killen (2). Because the value of the turbulence intensity of the air water mixture in the cylinder could not be evaluated easily, the following relationship was assumed:

$$C_M = f(v, l, \mu, \sigma, \gamma, \rho) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which

C_M = Mean air concentration

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[a] May 1961, by Task Committee on Air Entrainment in Open Channels (Proc. Paper 2814).

[2] Graduate Research Asst., Dept. of Civ. Engrg., Univ. of Colorado, Boulder, Colo.

[3] "Experiments on the Mechanics of Sediment Suspension," by H. Rouse, Proceedings, 5th Internatl. Congress on Applied Mechanics, 1938.

v = Average agitator velocity assuming harmonic motion

l = Amplitude of agitator

μ = Dynamic viscosity of the fluid

σ = Surface tension of the fluid

γ = Specific gravity of the fluid

ρ = Density of the fluid

In order to systematize the collection of data obtained from the experi tal program and to reduce the number of variables which were to be inv gated, dimensional analysis was used. Tap water, ethyl alcohol, and m alcohol were the fluids used in the investigation.

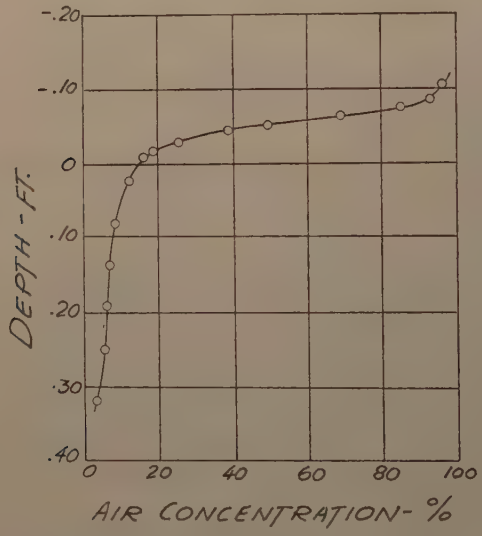

FIG. 6.—TYPICAL AIR-CONCENTRATION CURVE

Based on the preceding assumption, dimensional analysis and the use perimental data, the final equation was

$$C_M = 0.013 \left(\frac{v\ l\ \rho}{\mu}\right)^{0.21} \left(\frac{v^2\ l\ \rho}{\sigma}\right)^{0.58} \left(\frac{v}{\sqrt{g\ l}}\right)^{0.00} \quad \ldots\ldots$$

The terms are forms of the familiar Reynolds, Weber, and Froude bers. It should be noted, however, that Reynolds, Weber, and Froude nu in Eq. 3 are not the same as those derived for open channel flow.

If applied to an open channel, the equation could be written in the foll form for a given degree of turbulence:

$$C_M\ 2^2\ (\mu)^{-0.21}\ (\sigma)^{-0.58}\ (\rho)^{0.79} \quad \ldots\ldots\ldots$$

Because viscosity and surface tension of a fluid are sensitive to small changes in temperature and the density is not, the mean air concentration is affected to some extent by the viscosity and the surface tension, but the density is not important even if it carries a relatively large exponent in the preceding equation.

For illustration, the air content in a steep channel should be about 13% greater at a water temperature of 80°F than at a water temperature of 40°F.

The distribution of the bubbles or the general shape of the air concentration curve, as shown in Fig. 6, seems to be independent of the fluid properties. The number and size of the bubbles at any level, represented by numerical and volumetric indices (obtained from high speed photographs taken through the cylinder wall), are affected by the fluid properties. The larger the value of the viscosity and the surface tension of the fluid (at least within the range of val-

FIG. 7.—SURFACE OF AIR-WATER MIXTURE IN THE TURBULENCE JAR

ues obtained in this investigation), the larger the bubbles and the smaller the number of bubbles.

Fig. 6 shows a typical air-concentration curve. The surface jets were sometimes as high as 5 in. to 6 in., at an average agitator velocity of about 3.5 fps and at a mean air concentration of about 10%. This means that some of the water particles were leaving the surface with a velocity greater than 5 fps. This should give some indication as to the energy of the surface waves at a particular mean air concentration (Fig. 7).

Fig. 7 indicates the surface of the air-water mixture in the turbulence jar. The moving pictures, run at slow speeds, showed that as the transverse kinetic energy of the surface waves becomes large enough to overcome the stabilizing effect of the surface tension, air bubbles are entrained into the fluid body

by the breaking of numerous waves. The bubbles are then carried deeper the fluid column by the action of eddies in the high turbulent air-water ture.

N. RAJARATNAM.[4]—The entrainment of air in chutes and spillways, in dition to increasing the side wall height requirements, might also modify design of the energy dissipators at the foot of such structures. Furtherm the laws of flow resistance of aerated flows are likely to be different those for non-aerated flows. Hence, for evolving a theoretically sound economical design of channels carrying aerated flows, it is necessary to un stand precisely the mechanics of aerated flows.

The mechanism of "the inception of air entrainment" or "air inceptio quite complicated. Of all the theories put forward by different investiga to explain the mechanism of air inception, the most rational one seems that of E. W. Lane.[5] In this theory, for air inception, the flow must bec turbulent, that is, the turbulent boundary layer developing along the solid bo ary must come to the free surface. But it was felt that even if the flow is bulent, unless the transverse fluctuations are severe enough to project "lumps" of water into the air above, air inception might not occur. This pect has been investigated[6] by N. S. Govinda Rao, F. ASCE and the writer was found that Lane's hypothesis was also a sufficient condition for air in tion.

Secondly, a possible similarity could be noticed between the disintegra of liquid jets (in the atomization range) and the disruption of the water sur just prior to air inception. For the disintegration of the liquid jets, it has established[7] that turbulence is essential for initiating the disruption. Howe once it is initiated then the air resistance is mainly responsible for fur disintegration. The air resistance can be represented by a dimensionless p meter

$$F_a = \frac{V}{\sqrt{g\, d\, \frac{\rho_L - \rho_a}{\rho_L}}} \quad \cdots\cdots\cdots\cdots$$

in which V is the mean velocity of the jet, g denotes the acceleration du gravity, d refers to the diameter of the jet, ρ_L describes the density o liquid of the jet, and ρ_a is the density of the surrounding medium. In the of open channel flow, the parameter takes the form

$$F_a = \frac{V}{\sqrt{g\, y\, \frac{\rho_w - \rho_a}{\rho_w}}} \quad \cdots\cdots\cdots\cdots$$

in which y is the depth of flow, ρ_w is the water density and ρ_a is the densi air.

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[4] Senior Scientific Officer, Civ. and Hydr. Engrg. Sect., Indian Inst. of Science, alore 12, India.

[5] "Entrainment of Air in Swiftly Flowing Water," by E. W. Lane, Civil Engine February, 1939.

[6] "On the Inception of Air Entrainment in Open Channel Flows," by N. S. Govind and N. Rajaratnam, Paper for the 9th I.A.H.R. Conf., Belgrade, September, 1961.

[7] "Mechanism of Disintegration of Liquid Jets," by P. H. Schweitzer, Journal of Ap Physics, Vol. 8, August, 1937.

The term $\frac{\rho_w - \rho_a}{\rho_w}$ can be written as unity because $\rho_w \gg \rho_a$ and hence

$$F = \frac{V}{\sqrt{g\,y}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

which is the familiar Froude number. For air inception in open channel flow, the flow has be be turbulent and the critical Froude number will have a certain value. This critical value might vary to some extent with the nature of the boundary that controls the level of turbulence which in turn will affect air inception. To verify this concept, the writer calculated[8] the value of F from the available data on air inception. It was found that F = 7.5 for Werribee weir, 10.6 for Glenmaggie dam and 9.7 for Gickox's collected data on Norris and Douglas dams. From a systematic experimentation it is possible to find F for different boundaries.

Next, for a rational development of the theory of aerated flows, it is felt that the following classification suggested by the writer might be useful.

Uniform Aerated Flows.—Uniform aerated flow is that in which the air concentration and velocities remain constant from section to section. Uniform aerated flows can be established in steep channels of sufficient length. In fact, most of the available data on aerated flows pertain to this type of flow only.

The fundamental concepts of uniform aerated flows have been developed by Straub and Anderson.[9] The aerated flow has been divided into two regions, a lower region of depth y_T in which the air bubbles are distributed by the turbulent fluctuations and an upper region in which the water drops move in a stream of air. The term $\overline{y}$ denotes the depth of the corresponding non-aerated flow, had there been no air entrainment, moving with a velocity of $\overline{V}$, equal to that of the aerated flow. It has been shown[10] that the water discharge carried in the upper region is practically negligible and that the mean velocity of the lower region is $\alpha\,\overline{V}$, where α has been found to be 1.12 from available experimental data. The term $\overline{c}_T$ describes the mean air concentration in the lower region.

The writer has introduced[10] the concept of the air entrained Froude number F_* written as

$$F_* = \frac{\overline{V}}{\sqrt{g\,y_T}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

which has been found useful in his studies on aerated flows and pre-entrained jumps. It is believed that this Froude number might be as useful for aerated flows as the normal Froude number for non-aerated flows.

An energy equation can now be written for the uniform aerated flow. The total energy will be the sum of the energies of the two regions.

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8 "A Critical Froude Number for Air Inception in Open Channel Flows," by N. Rajaratnam, to be published in the Journal, Central Bd. of Irrig. and Power, New Delhi, India. (Also "Some Studies on the Hydraulic Jump," by N. Rajaratnam, Chapter 9, thesis presented to the Indian Institute of Science, at Bangalore, India, in 1961 in partial fulfilment of the requirements for the degree of Doctor of Philosophy).

9 "Experiments on Self-Aerated Flows in Open Channels," by L. G. Straub and A. G. Anderson, Proceedings, ASCE, Vol. 84, No. HY7, December, 1958.

10 "On the Pre-Entrained Jump," by N. Rajaratnam, Under publication in the Civil Engineering and Public Works Review, London. (Also, "Some Studies on the Hydraulic Jump," by N. Rajaratnam, Chapters 3 and 4, thesis presented to the Indian Inst. of Science, at Bangalore, India, in 1961, in partial fulfilment of the requirements for the degree of Doctor of Philosophy).

The energy of the lower region is written as

$$E_{R-1} = \gamma\ \theta_1\ q \left\{ \left(1 - \bar{c}_T\right) y_T + \frac{1}{2g} \alpha^2 \bar{V}^2 \right\} \quad \ldots\ldots$$

(neglecting the variation of the piezometric head with the depth, that is, d ping out the term $\gamma\ y\ \bar{c}_T$ and also leaving out the Corioli's coefficient fo sake of simplicity), in which $\theta_1\ q$ is the discharge in the lower region and the specific weight of the water.

In the upper region, the water drops are believed to travel with a velo the component of which is parallel to the bed of the channel, equal to the locity of the layers from which they are projected. If it is β, $\bar{V}$, β_1 has found[10] to be 0.88 (from limited data).

The energy of the upper region is written as

$$E_{R-2} = \gamma\ \theta_2\ q \frac{\left(\beta_1 \bar{V}\right)^2}{2g} \quad \ldots\ldots$$

(neglecting the position head of the drops) where $\theta_2\ q$ is the discharge i upper region.

$$\frac{E_{R-2}}{E_{R-1}} = \frac{\gamma\ \theta_2\ q \left\{ \frac{\left(\beta_1 \bar{V}\right)^2}{2g} \right\}}{\gamma\ \theta_1\ q \left\{ \left(1 - \bar{c}_T\right) y_T + \frac{\left(\alpha \bar{V}\right)^2}{2g} \right\}} \quad \ldots\ldots$$

For the purpose of showing that the energy content of the upper regi much smaller even when compared to the kinetic energy of the lower re Eq. 11 can be rewritten as

$$\frac{E_{R-2}}{E_{R-1}} < \frac{\theta_2\ \beta_1^2}{\theta_1\ \alpha^2} \quad \ldots\ldots$$

$$\theta_1 = \frac{1}{q} \alpha\ \bar{V}\ y_T \left(1 - \bar{c}_T\right) \quad \ldots\ldots$$

$$\theta_1 = \frac{\alpha\ \bar{V}\ y_T \left(1 - \bar{c}_T\right)}{\eta \left(1 - \bar{c}_T\right) y_T\ \bar{V}} \quad \ldots\ldots$$

and

$$\theta_1 = \frac{\alpha}{\eta} \quad \ldots\ldots$$

For uniform aerated flows η has been found to have an average value of irrespective of the nature of the flow boundary.[10] When the computed v

θ_1, θ_2, β_1 and α are substituted in Eq. 12, it is found that

$$\frac{E_{R-2}}{E_{R-1}} < 1.07\% \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Hence, for all purposes, the energy of the aerated stream can be taken to the energy of the lower region itself as given by Eq. 9.

Non-Uniform Aerated Flows.—Non-unifrom aerated flow is one in which the air concentration and mean velocities vary from section to section. If the variation of the air concentration and velocities are gradual with the distance, it is known as "Gradually Varied Aerated Flows," but if the variation is rapid then it is known as "Rapidly Varied Aerated Flow."

Gradually Varied Aerated Flows.—Aerated flows in chutes come under this group. Aeration starts at a particular location which has been established for a concrete spillway by Gickox[11] and for a steep smooth chute by Rajaratnam.[12] From this section onwards the mean air concentration continues increasing up to the section at which terminal conditions are established.

In the design of chutes it is absolutely essential to find out whether aeration will occur. If so, will terminal conditions be established? All the formulas suggested by the different investigators including the equation suggested by the Task Committee apply only for uniform aerated flows and, hence, will be an overdesign in cases where terminal conditions are not attained.

If x_t is the distance at which terminal flow conditions will be established, it is believed that

$$\frac{x_t}{y_c} = \phi_1 \left\{ \text{slope, roughness} \right\} \quad \ldots\ldots\ldots\ldots (17)$$

in which y_c is the critical depth of flow.

The nature of the function ϕ_1 remains to be investigated by systematic experimentation. Also the rate of increase of $\overline{c}_T$ for the flow with the longitudinal distance can be studied. For the experimental work on this aspect, a chute with a free flow end and of sufficient length is required. Some work in this direction has been accomplished in the Central Water and Power Research Station at Poona, India,[13] but the published results are not useful for further development because of the rather empirical method of flow division into the three zones of intermittent spray, continuous spray and continuous water. Research work in this direction is urgently needed.

In terminal flow for a given boundary, $\overline{c}_T$ has been shown[9] to be a function of $S/q^{1/5}$ and in Fig. 8 the writer has shown that $\frac{y_T}{y_c}$ is a function of $\overline{c}_T$. Hence, the other fundamental quantities like $\overline{y}$, y_u can be obtained through experimental relations established between $\overline{y}$, y_u, y_T and c_T. Of course, similar data must be collected for the various boundaries in use.

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11 "Air Entrainment on Spillway Faces," by G. H. Gickox, Civil Engineering, December, 1945.

12 "Some Studies on the Hydraulic Jump," by N. Rajaratnam, Chapter 11, thesis presented to the Indian Inst. of Science, at Bangalore, India, in 1961, in partial fulfilment of the requirements for the degree of Doctor of Philosophy. To be published.

13 Annual Reports, Central Water & Power Research Sta., Poona, India, 1952, 1953, 1957 and 1958.

Rapidly Varied Aerated Flows.—Air entrainment in a (normal) hy jump and in a pre-entrained jump (pre-entrained jump is a jump in wh supercritical stream itself is aerated. This type of jump has recent studied both theoretically and experimentally by the writer[10]) are cl

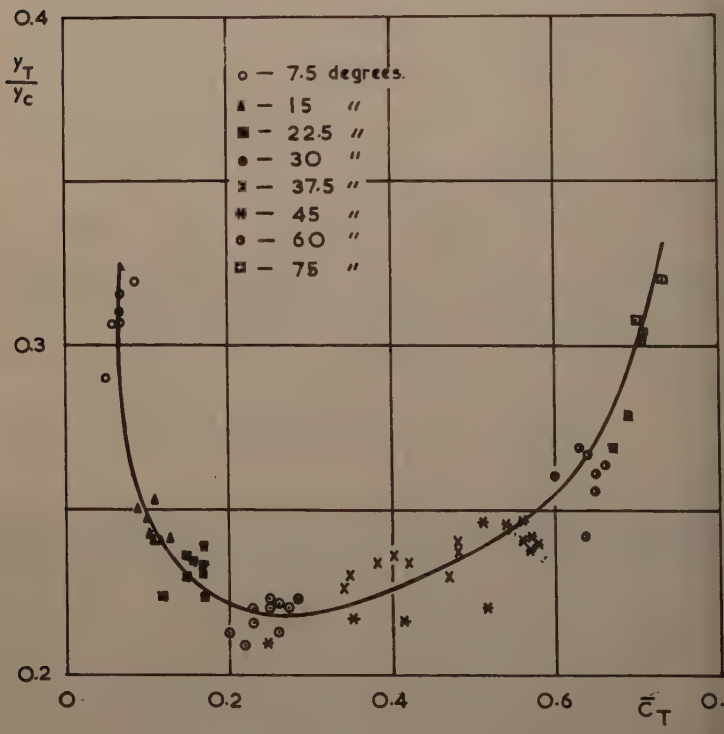


FIG. 8.—CORRELATION BETWEEN $\frac{y_T}{y_c}$ AND $\bar{c}_T$

examples of rapidly varied aerated flows. The variation of the mean a centration $\bar{c}$ with length expressed as a dimensionless ratio is given in for a normal hydraulic jump[14] and in Fig. 10 for a pre-entrained jump

14 "An Experimental Study of the Air Entrainment Characteristics of the H Jump," by N. Rajaratnam, Accepted for publication in the Journal of the Insti Engineers, India. (Also, "Some Studies on the Hydraulic Jump," by N. Raj Chapter 2, thesis presented to the Indian Institute of Science, at Bangalore, 1961, in partial fulfilment of the requirements for the degree of Doctor of Phil

Aeration in the rear of spillways piers also belongs to this class. In the case of flow along a spillway pier, the flow first separates as shown in Fig. 11 at A and B and then the two streams begin expanding and finally impinge upon one another at C causing air entrainment. The pattern of flow for different shapes of spillway pier ends has been studied by the writer in the Irrigation Research Station, Poondi, Madras, India.[15] Hence, it is suggested that two plane jets of water may be directed against each other at different angles and the air entrainment be studied. Then these results may be used for solving the air entrainment caused by the spillway piers. A similar phenomenon can be

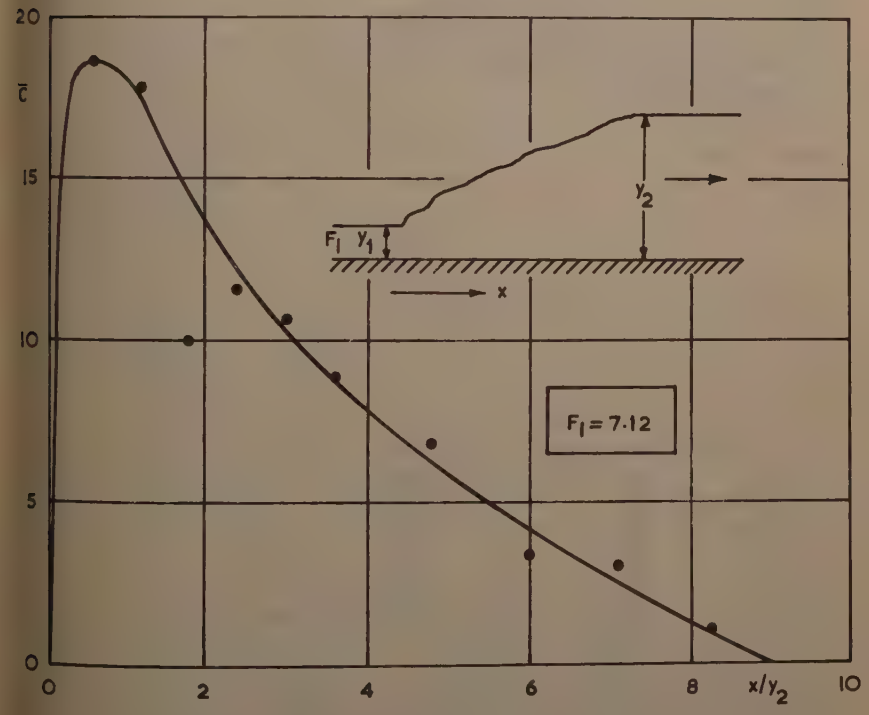
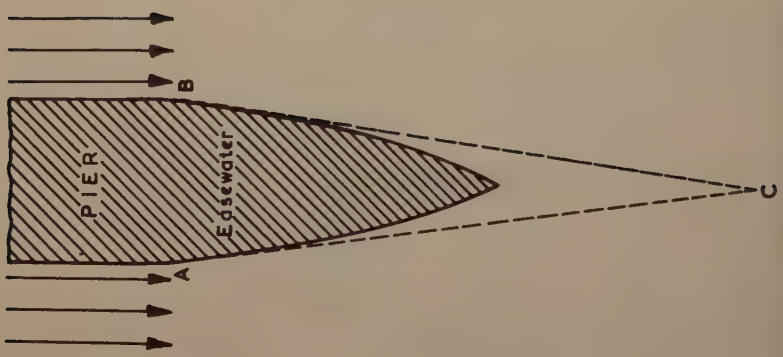


FIG. 9.—AIR ENTRAINMENT IN A HYDRAULIC JUMP

observed in the rear of a battery of river sluices that have a glacis. Air entrainment by a nappe or jet, plunging into water, air entrainment in a shaft spillway are still other examples of rapidly varied aerated flow.

The writer is grateful to N. S. Govinda Rao for his encouragement in the preparation of this discussion.

15 "Studies on Easewaters," Unpublished Report of the Irrig. Research Sta., Poondi, Madras, India, 1957.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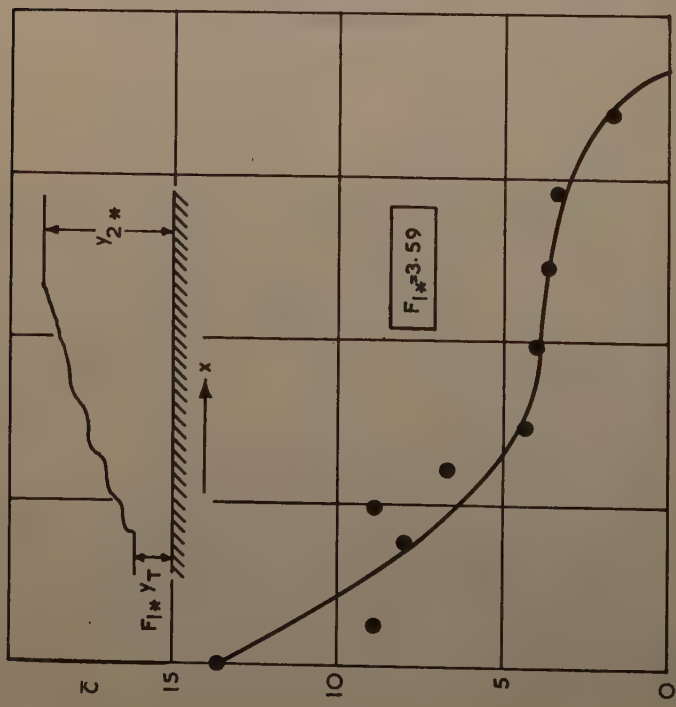
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MIKIO HINO.[16]—The progress report is valuable for researchers in this field, affording complete references and bibliography on the subject and giving advice for the direction of future researches.

Research works in this field has also taken place in Japan. As the progress report has pointed out, the aeration phenomena are closely correlated to the structure of turbulence in open channel flows. The fundamental study on the structure of turbulence in open channels has long been continued by the writer at the University of Tokyo. The knowledge thus obtained is applied to the engineering problems, including that of aerated flow.[17,18,19] In previous studies, the writer, applying the statistical theory of turbulence, has presented, first, a criterion for the initiation of self-aeration; secondly, the air distribution formula for the upper region of equilibrium aerated flows which consists of sprays and globules of water; and thirdly, the mechanism of the air bubble distribution in the lower region.[20,21] The results are believed to be rather successful notwithstanding a preliminary nature of the proposed theory. However, further researches, theoretical and experimental, are felt to be necessary.

F. Noda[22] reported an attempt to examine the effects of entrained air on the decay of turbulence (increase in the energy dissipation rate) in hydraulic jumps and high velocity flow on spillways.

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16 Tech. Lab., Central Research Inst. of Elec. Power Industry, Tokyo, Japan.

17 "The Turbulent Structure of Open Channel Flows, with its Application to Engineering Problems," by M. Hino, thesis presented to the University of Tokyo, at Tokyo, Japan, in March, 1960, in partial fulfilment of the requirements for the degree of Doctor of Philosophy. (In Japanese.)

18 Discussion by M. Hino of "Eddy Diffusion in Homogeneous Turbulence," by G. T. Orlob, Proceedings, ASCE, Vol. 86, No. HY4, 1960, p. 95.

19 "The Structure and Diffusion Coefficient in Turbulent Shear Flows," by M. Hino, to be published as Technical Report No. C6102, Tech. Lab., Central Research Inst. of Elec. Power Industry, 1961.

20 "On the Mechanism of Self-Aerated Flow on Steep Slope Channels—Applications of the Statistical Theory of Turbulence," by M. Hino, 9th Cong., IAHR, Bergrado, 1961.

21 "A Theory on the Mechanism of Self-Aerated Flow on Steep Slope Channels—Applications of the Statistical Theory of Turbulence," by M. Hino, Technical Report No. C6101, Tech. Lab., Central Research Inst. of Elec. Power Industry, June, 1961.

22 "Researches on the Effect of Entrained Air on the Hydraulic Jump—I," by F. Noda, Preprint, 16th Annual Meeting, the Japan Soc. of Civ. Engrs., 1961. (In Japanese.)

FORMS OF BED ROUGHNESS IN ALLUVIAL CHANNELS[a]

Discussion by C. F. Nordin, Jr. and J. K. Culbertson; Alan V. Jopling; and Vito A. Vanoni and John F. Kennedy

C. F. NORDIN, Jr.,[28] A. M. ASCE, and J. K. CULBERTSON,[29] M. ASCE.— he observations of bed configurations and flow resistance recorded for the ume experiments with a 0.28 mm median diameter sand were of special in- rest to the writers because (1) the size of bed material used in the flume ex- eriments is close to the median diameter of bed material for the Rio Grande ear Bernalillo, N. M. (d = 0.29 mm), where characteristics of flow resistance nd bed configurations have been studied for several years, and (2) the changes n resistance coefficients with changing bed configurations observed for the ume are similar to those observed in the Rio Grande.

It has been indicated[30,31] that flow in the Rio Grande near Bernalillo may e divided into flow over a dune bed (lower regime flow) and flow over a plane ed (upper regime flow), with a transition zone between (Fig. 6). Flow for both ower and upper regime is reasonably stable, slope will vary conservatively, nd $V \propto R^{\frac{1}{2}}$; that is, for most practical purposes, either Chezy's $C/\sqrt{g}$ or roude number, **F**, may be assumed constant. Practical applications of this lea are examined by D. R. Dawdy,[31] in which for plane bed flow, the velocity- epth relation defined for any stream-gaging site by existing records may be sed to extrapolate rating curves. He indicates that flow resistance, as meas- red by $C/\sqrt{g}$, is primarily dependent on median diameter of bed material for pper regime flow.

Values of f for upper regime flow in the Rio Grande are lower than those ound in the flume studies, varying from 0.012 to 0.023 with an average value approximately 0.016. In terms of Manning's n, an average value of 0.014 ap- lies for upper regime flows.

For flow over a dune bed, resistance coefficients for the Rio Grande indi- ate a greater range in values than in the flume. Values of f vary from 0.06 0.10, and average approximately 0.07. One reason for this greater range is at dunes apparently grow to greater height, relative to depth, in the case of e Rio Grande.

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[a] May 1961, by D. B. Simons and E. V. Richardson (Proc. Paper 2816).

[28] Hydr. Engr., USGS, Albuquerque, N. Mex.

[29] Hydr. Engr., USGS, Albuquerque, N. Mex.

[30] Discussion by J. K. Culbertson and C. F. Nordin of "Discharge Formula for Straight luvial Channels," by Hsin-Kuan Liu and Shai-Yean Hwang, Proceedings, ASCE, Vol. 86, . HY6, June, 1960.

[31] "Studies of Flow in Alluvial Channels; Depth-Discharge Relations of Alluvial reams—Discontinuous Rating Curves," by D. R. Dawdy, Water-Supply Paper 1498-C, GS, Dept. of the Interior, Washington D. C., 1961.

Although neither velocity nor sediment discharge are related to bed in the transition zone, as stated by the authors, it should be noted that f this zone may be neither transitory nor unstable. In the field case, flow transition zone may persist for many weeks. The channel will tend to toward an equilibrium condition to allow transport of the imposed sedimen at the prevailing discharge. The adjustments that occur are not well u stood. It appears that the maintenance of a state of near-equilibrium for sition flow through a reach of natural channel is conditioned by rather changes in water-surface slope, with a wide variation occurring in mea

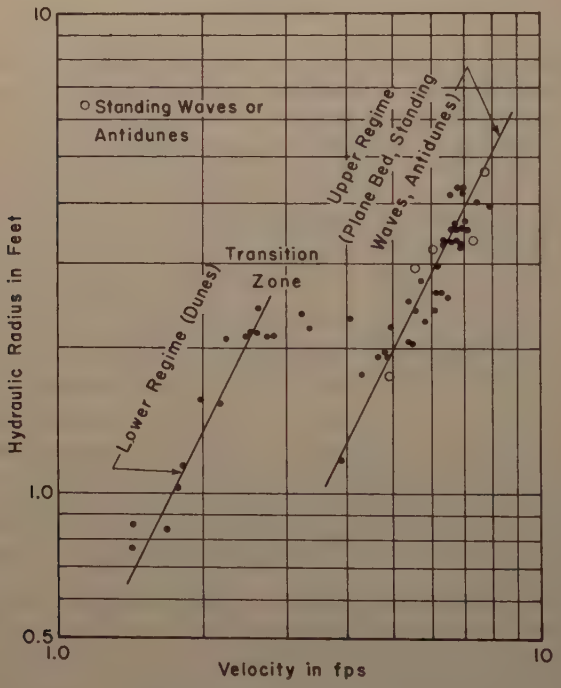

FIG. 6.—RELATION OF HYDRAULIC RADIUS TO VELOCITY FOR RIO GRANDE NEAR BERNALILLO, N. M.

elevation, depth, bed configuration, and flow resistance. The adjustme cur in terms of both lateral variations at a cross section and longitudin iations through a reach.

A factor of major importance in the case of the Rio Grande, not pre flumes, is the ability of the channel to adjust its width. Width devel takes place at the wider sections of a reach by the process of buildi bars across a portion of the channel while scouring in the thalweg of th This process occurs within the active banks of the channel; transition the Rio Grande near Bernalillo always occurs below bankful stage, a the width may be close to bankful width. In this type of adjustment, ant

conditions are important, but the sequence of events is generally the same from year to year.

On a rising stage of a spring runoff event, for example, all sections of the reach will generally have flow over the complete width of the channel between the active banks. At the wider sections, scour may occur over a portion of the channel while concurrent deposition will occur over another portion. If the flow remains in the transition zone for several days without increasing discharge, the area in which deposition occurred may emerge as an island or as a point bar running from the center portion of the channel to one of the banks, and all flow will channelize at about one-half to two-thirds bankful width in the scoured portion of the channel.

If discharge continues to increase through the transition zone to produce upper regime flow through the entire reach, both narrow and wide sections will probably flow at near-bankful width until the receding stage, when bars will

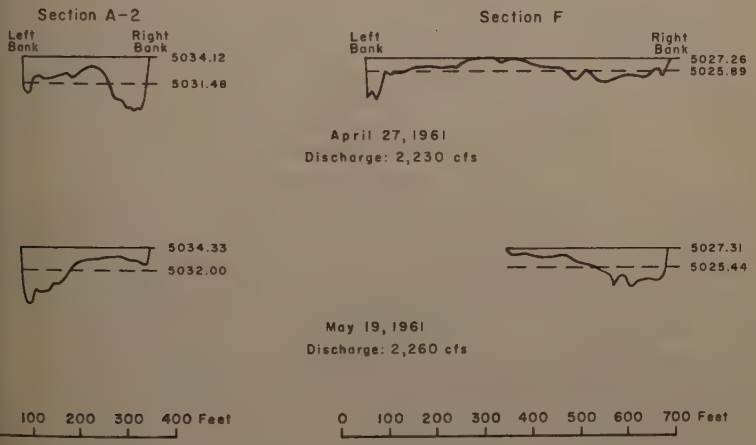

FIG. 7.—CROSS SECTIONS SHOWING WATER SURFACE AND MEAN BED ELEVATIONS, RIO GRANDE NEAR BERNALILLO, N. M.

again form and the width will decrease substantially at the wider sections. At this point, if an increased sediment load is imposed by tributary inflow, this process may be reversed. That is, through the wider sections, deposition will occur over portions at the bed, and the flow will spread again to near-bankful width, with a decrease in depth and of flow resistance.

In general, the narrow or partially confined sections through a reach are not capable of a rapid width adjustment, but will show a greater variation in mean bed elevation and water-surface elevation than the wider sections.

Fig. 7 shows plotted cross sections for observations at two sections of the Rio Grande near Bernalillo, N. M. The water-surface elevations and mean-bed elevations shown on the figure are referred to a datum of mean sea level. Mean-bed elevation were determined by subtracting mean depth of flow from water-surface elevation. The observations were made during the spring of 1961, when flow persisted in this transition zone for several months. Section

A-2 is a narrow section confined on the right bank by a sandstone bluff. left bank is composed of a mixture of clay, silt, and sand partially stab by vegetation. Through the range of discharge from about 2,000 cfs to 1 cfs, the width varies from 268 ft to 273 ft.

It should be mentioned that the behavior at this section, which has one erodible bank, is typical of the behavior for several other narrow sec through the reach where flow width is approximately the same and the fl

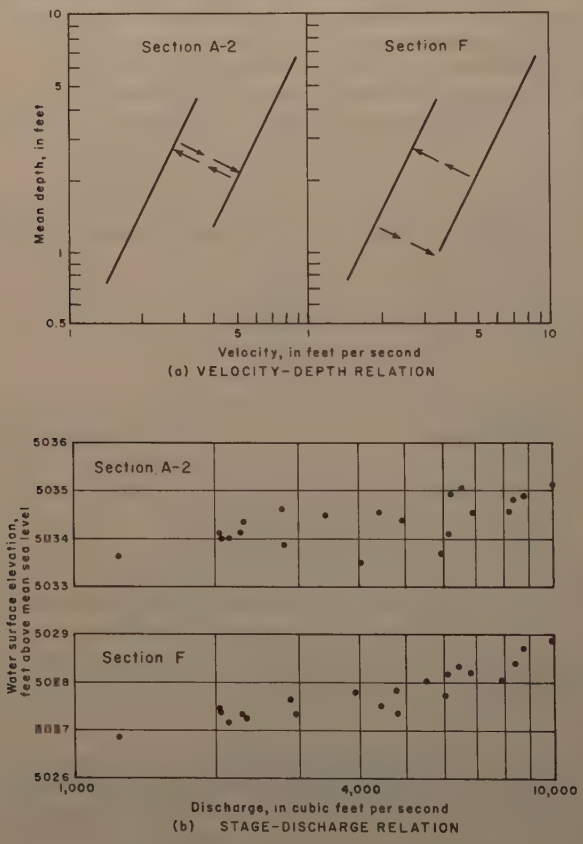

FIG. 8.—VELOCITY-DEPTH AND STAGE-DISCHARGE RELATIONS, RIO GRANDE NEAR BERNALILLO, N. M.

confined only by banks of erodible material. The behavior to be examine sequently for narrow sections is not dependent on the erodibility of the but only on the condition that width be relatively constant for the range charge considered.

Section F, located 8,240 ft downstream from section A-2, is a wide s confined by banks that are of material similar to the left bank of sectio

and that are partially stabilized by vegetation. For the same range of discharge, width of section F varies from approximately 330 ft to 650 ft.

It may be seen that at the narrow section the thalweg shifted from one side of the channel to the other during the three-week period, and that the bed elevation increased about one-half foot, indicating deposition. At section F, the flow channelized along the right bank, and the mean-bed elevation was about one-half foot lower on May 19 than on April 27. The width, however, was reduced by almost half during these three weeks; thus considerably more deposition actually occurred at this section than at the narrow section. It should be noted that a change in mean-bed elevation in this transition zone of flow is not an index of the net scour or fill of a channel, nor does it indicate anything of the nature of channel adjustments, unless width is specified.

The nature of changes for the Rio Grande are shown qualitatively in terms of the velocity-depth relation in Fig. 8(a). The wider sections will proceed to a plane-bed condition at much lower depths than will the narrow sections; that is, at much lower values of bed shear, γ D S. The shear at a wide section which displays a plane bed may be one-half the value of the shear at a narrow section which displays a dune bed. Thus, Fig. 3 is quantitatively significant only for the flume in which the observations were made. The magnitude of the bed shear at which transition flow commences, like the Froude number, appears to be mainly dependent on the scale of the system under consideration. The bed shear at which transition flow occurs in the Rio Grande may be 2 to 3 times as great as the shear at which transition flow occurs in the flume.

The stage-discharge discontinuities generally are more pronounced at confined sections than at wide sections, as indicated by Fig. 8, where water-surface elevations are plotted against discharge for 21 concurrent observations for sections A-2 and F. In connection with Fig. 8, it should be noted that neither the stage-discharge relations nor the velocity-depth relations (or unit discharge-depth relations) give much insight into the nature of channel behavior, especially where the width changes. Fig. 5(a), which shows a break in a rating curve occurring at a greater discharge on the rising stage than on the falling stage, is typical of many narrow or confined sections. On a wide section, this relation is likely to be reversed; that is, the break from dune flow to plane-bed flow will occur at a lower discharge on the rising than on the falling stage, largely due to the tendency for wider sections to channelize on falling stages. The distinction between a looped rating curve and a discontinuous rating curve often may be a matter of degree; however, it perhaps should be mentioned that a rating which shows a distinct discontinuity between dune and plane-bed flow may also show a loop or hysteresis within a given regime of flow.

One of the basic characteristics of natural channel flow is that the flow is generally non-uniform in nature. L. B. Leopold, F. ASCE, and M. G. Wolman, M. ASCE, indicate[32] that natural streams are seldom straight for a distance exceeding ten channel widths. Even where a channel is straight, there are usually variations in width from one section to another along the length of the reach. The preceding discussion shows that the observed behavior at an individual cross section is by no means indicative of average conditions through a reach, especially in the transition zone of flow. Because general resistance and transport relations are least understood for flow in the transition zone, it would seem important to concentrate effort in this area.

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[32] "River Meanders," by L. B. Leopold and M. G. Wolman, Bulletin, Geol. Soc. of Amer., Vol. 71, 1960.

In connection with future studies, it will be important to consider the e of width adjustments in a channel, because this type of adjustment appe be of primary importance in the transition zone of flow. Also, it will be n sary to determine the effect that the scale of the system under conside has on the observed quantitative relationships.

The authors' careful observations, both qualitative and quantitative, ar portant steps in determining the characteristics of bed configuration an resistance in alluvial channels. Further studies of this type, with an ex range of conditions, will contribute substantially to a better understand the nature of alluvial flow.

ALAN V. JOPLING.[33]—[b]The basic researches of Simons and Richa have been directed specifically toward a hydraulic goal, but some of t perimental data have geologic ramifications of particular interest to th dent of stratigraphy and sedimentation. The sedimentologist, for exam probing the depositional environment of many types of clastic geologic posits is often faced with the task of interpreting and reconstructing the parameters of a defunct hydraulic regime. As already pointed out by Brush,[34] the meager field evidence commonly available, but not inva militates against a geologic solution to the problem of reconstruction. ever, the more information available on the mechanics of present-day p ses, the greater will be the feasibility of analyzing past conditions from occurrences in nature.

It is evident, therefore, that the accumulation and syntheses of data type presented here by the authors will eventually establish a reference f work of considerable value to the geological profession. The shape c teristics of ripples and dunes and the ratios of either ripple or dune to depth of flow are pertinent in this connection. The classified forms roughness also afford a useful working scheme for the geologist interes the operation of "process," and the interpretation of the forms of bed ness as a function of Froude number and other parameters carries a me ful connotation in environmental studies.

The writer[35] was indirectly concerned with boundary roughness in a laboratory study of the origin of bedding in cross-bedded deposits. One techniques used in this investigation is mentioned here because it ma some interest to hydraulicians studying the transition from one form roughness to another. The technique in question was based on a photog count of the particles carried over the lip of a 2-dimensional delta adv downstream in a laboratory flume. The distributive intensity of the p cloud was recorded photographically by collimating a thin vertical sh light into the channel in a darkened laboratory. Collimation was facilita means of two thin sheet-metal plates with lower edge immersed a little water level. The beam paralleled the wall of the lucite flume at a dista

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[b] Published by permission of the Director, USGS, Dept. of the Interior, Was D. C.

33 Research Geologist, Surface Water Branch, Water Resources Div., USGS, he Interior, Washington, D. C.

34 Discussion by L. M. Brush, Jr. of "Resistance to Flow in Alluvial Channe Daryl B. Simons and E. V. Richardson, Proceedings, ASCE, Vol. 87, No. HY1, 1

35 "An Experimental Study on the Mechanics of Bedding," by Alan V. Jopling presented to Harvard Univ., at Boston, Mass., in partial fulfilment of the requi for the degree of Doctor of Philosophy.

approximately 1 in., and it illuminated a vertical strip of the cloud of grains settling on the frontal slope of the delta (Fig. 9, inset). R. A. Bagnold[36] used a comparable technique for tracing the path lines of sand grains in a wind-tunnel experiment, and he demonstrated that the ripple wavelength of blown sand was a function of the mean path followed by the grains in saltation.

In the experimental work of this investigation the flow and sediment feed were kept constant during a given run, and 10 or more photographs were taken of the cloud of particles illuminated by the narrow beam of light. The exposure time was $\frac{1}{250}$ sec. Ottawa sands 20-30 and 30-40, both of which had been re-sieved to ensure uniformity of grain size, were used in the study and plane-bed (sheet) conditions prevailed in each of the runs. Even with the heaviest discharges used, almost the entire sediment load was confined to within 5 or 6 grain-diameters of the bed. In each of the runs the heavy-fluid layer flowing along the stream bed "slurried" over the lip of the delta with a tendency to settle as a collective unit, but its entity was partially disrupted by the eddies of free-turbulence shear flow. Depths of streamflow ranged from less than an inch up to several inches, and the depth of the "stilling basin" was approximately 7 in. Supercritical flow with Froude numbers equal to or somewhat greater than one prevailed in all the runs. The average slope of the transport surface was measured over a distance of 5 ft or 6 ft upstream of the delta lip and the bed tractive stress was computed from the conventional formula

$$\tau = \gamma \, R \, S \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which τ is the shearing stress, γ the specific weight of the fluid, R the hydraulic radius, and S the slope of the energy gradient (taken equal to the slope of the stream bed).

A photographic negative was used for the particle count, and the frontal slope region of the delta was divided into equally-spaced vertical zones (Fig. 9, inset). A rough count was made of the number of particles in each zone, extrapolating where necessary for those zones near the lip of the delta where the concentration of particles was high. Various factors militate against an accurate count: Diffraction of light beyond the nominal limits of the beam, high density of the particle cloud near the lip of the delta, disturbances caused by the immersion of the guide plates, and so on. Despite these limitations the particle count method has some advantages over a photometric approach. With relatively low transport rates it is a simple matter to study particle distribution, but at high rates the count degenerates into an estimate.

The center of gravity of the particle cloud was computed for each photograph, and its average distance from the lip (measured in the horizontal) of the delta was recorded for each series of photographs. The plot of average trajectory versus bed tractive stress is shown in Fig. 9, and the results are also plotted in terms of the dimensionless parameter $\frac{V_*}{V_s}$ (ratio of shear velocity to measured fall velocity of the particles) in Fig. 10.

The study reported here was of a reconnaissance nature and was actually designed for the purpose of interpreting the shape of the delta front in terms of the micro-mechanics of particle movement. For example, with a short tra-

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36 "The Physics of Blown Sand and Desert Dunes," by R. A. Bagnold, Methuen and Co., London, 1941.

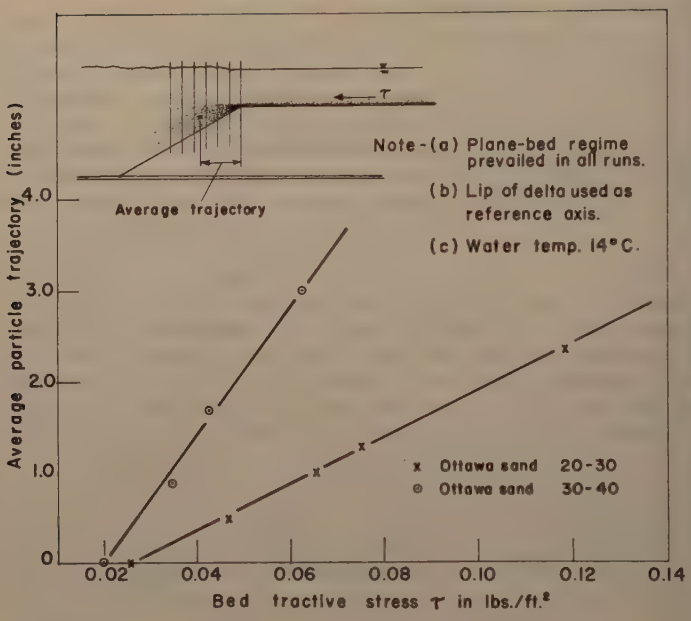

FIG. 9.—PLOT OF AVERAGE PARTICLE TRAJECTORY VERSUS BED TRACTIVE STRESS

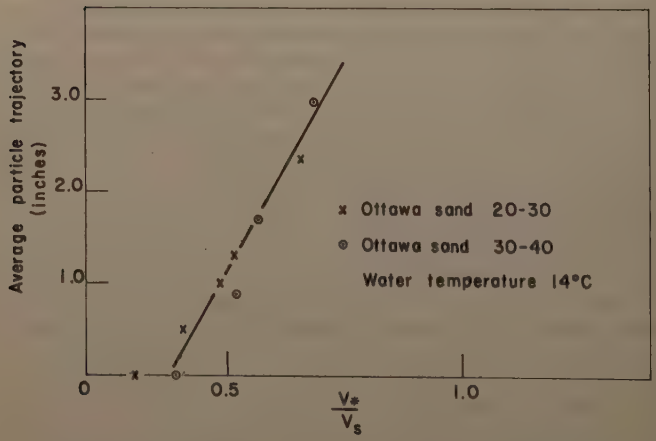

FIG. 10.—PLOT OF AVERAGE PARTICLE TRAJECTORY VERSUS RATIO OF SHEAR VELOCITY TO SETTLING VELOCITY

'ectory or particle "jump" the bulk of the sediment load is dumped on the frontal slope of the delta, and periodic gravitative slip results in redistribution of material down the frontal slope. This in turn results in a planar type of front with an angular basal contact with the underlying pavement (basin floor). However, with an increasing length of trajectory some of the particles lodge in the toe sector of the delta—or beyond—and the angular contact is replaced by an asymptotic or tangential contact within the underlying pavement. Analysis of these points is beyond the scope of this discussion.

It is suggested that a particle count technique may serve as an accessory tool in the study of the stability characteristics of form-drag structures. The transitions in flow regime could conceivably be related to and interpreted as a function of particle trajectory. It is obviously difficult, however, to apportion the total shearing resistance between granular resistance and shape resistance, especially where conditions of repetitive varied flow prevail in a 3-dimensional dune system, and where the intrinsic shape characteristics of the mobile boundary are remoulded by systematic changes in the flow regime.

The general problem of boundary roughness is indeed a formidible one, but detailed researches of the type presently being carried out by Simons and Richardson are paving the way toward a better understanding of the principles that govern the geometric configuration of a mobile boundary.

VITO A. VANONI,[37] F. ASCE, and JOHN F. KENNEDY,[38] M. ASCE.—These new data from the authors' continuing study of sediment transport are a welcome addition to the sedimentation literature. The data from this study are particularly valuable because for each sand they cover the whole range of flow conditions and bed configurations, from no sediment movement through the antidune regime.

In categorizing the different forms of bed roughness, it is important that the classifications be made along rational lines as far as possible. All bed forms that have the same mechanics of formation and that exhibit the same general characteristics should be classified together. The authors have defined eight different bed forms in Fig. 2. In the writers' opinion, there were only five distinct bed forms which are, in the authors' terminology, dunes, transition, plane bed, antidunes, and antidunes with $F > 1$. The features called ripples have been called dunes by other investigators,[39,40,41] and the bed forms referred to as dunes by the authors are apparently the same as those called sand bars by others.[41,42] The lengths and heights of the authors' dunes are several times larger than those of their ripples, and there do not appear to be any

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[38] Research Fellow in Civ. Engrg., W. M. Keck Lab. of Hydraulics and Water Resources, California Inst. of Tech., Pasadena, Calif.

[39] "Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams," by Vito A. Vanoni and Norman H. Brooks, Report No. E-68, Sedimentation Lab., California Inst. of Tech., 1958, p. 99.

[40] "Further Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams," by John F. Kennedy, Report No. KH-R-3, W. M. Keck Lab. of Hydraulics and Water Resources, California Inst. of Tech., January, 1961.

[41] "A Study of the Sediment Transport in Alluvial Channels," by James R. Barton, Report No. 55JRB2, Colorado State Univ., Fort Collins, Colo., 1955.

[42] "The River Klaralven—A Study of Fluvial Processes," by Ake Sundborg, Bulletin No. 52, Institution of Hydraulics, Royal Inst. of Hydraulics, Stockholm, Sweden, 1951.

forms with intermediate sizes. Although the authors' ripples are smalle
their dunes, they are essentially of the same shape and move by the ide
mechanism, that is, by sediment moving up the upstream face and depo
on the steep downstream face. Therefore, the writers believe that thes
forms are merely different sizes of the same kind of form and shou
classed together. The third distinct bed form is the plane or flat bed.
this case, the water-sand bed interface is stable. The authors' antidune
standing waves are the result of an interaction between surface waves a
sand bed, as has been shown by Kennedy,[43,44] and there is no logical r
for differentiating between them. Finally, for antidunes with $F > 1$, the t
port rate is so great that no equilibrium configuration at a uniform de
possible, and chutes and pools result.

The authors' transition zone includes bed forms ranging from dunes to
bed or standing waves. By using this generic term, they have evaded a
cription of the bed form in this important range of depth and velocity whe
configuration changes from one well-defined type to another. A mean
description of the bed could have been given by simply noting which of the
ious bed forms occurred. Also, they state that ripples were superimpos
dunes in some runs, but do not list this configuration for any runs in Ta
The value of their data would have been greatly enhanced had they repo
more complete description of the bed for each run, including measureme
the lengths and heights of the ripples and dunes, and wave lengths of anti
Further, when more than one type of bed form was present, such as ri
and dunes, the dimensions of both forms should have been included. The
ors are urged to include these data in their closing discussion, if possib
make the description of their experiments complete.

The breaking of the stationary waves that accompany antidunes is in n
analogous to a hydraulic jump, and cannot, as the authors state, be analy
such. It has been shown by Kennedy[43,44] that the velocity pattern asso
with the surface waves above antidunes (and the authors' standing waves)
same as that of stationary, deep water waves in a fluid flowing with a ve
equal and opposite to the wave celerity, the flow over the antidunes co
ponding to the flow above an intermediate streamline of such waves. The
file of deep water waves has been investigated by Michell[45] who foun
their maximum height from trough to crest is 0.142L in which L is the
length. This value is in good agreement with observed values[43,44] of the
at incipient breaking of long-crested, two-dimensional stationary wave
companying antidunes.

The only difference between the authors' antidunes and standing wa
that the former become so high that the stationary surface waves break, w
as the height of the latter is less than that at which the surface waves
the critical height for breaking. The height of antidunes is limited in two
In the first of these, the maximum height is reached when the transport

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43 "Stationary Waves and Antidunes in Alluvial Channels," by John F. Kennedy,
presented to the California Institute of Technology, at Pasadena, California, in 1
partial fulfilment of the requirements for the degree of Doctor of Philosophy.

44 "Stationary Waves and Antidunes in Alluvial Channels," by John F. Kennedy,
No. KH-R-2, W. M. Keck Lab. of Hydraulics and Water Resources, California I
Tech., April, 1961.

45 "The Highest Waves in Water," by J. Michell, Philosophical Magazine, V
1893, p. 430.

rest of the antidunes reaches a value such that no more deposition and, , growth is possible. The other maximum height is reached when the sur- waves over the antidunes have grown to the point at which they are un- e and break. The critical height of the surface waves and the correspond- eight of antidunes may be calculated by the theory referred to previously. maximum height of anitdunes imposed by the transportability of the sedi- has been observed[43,44] to increase with increasing velocity and decreas- sand size. This explains why the authors found that standing waves oc- ed with the 0.45 mm sand, but not with the 0.28 mm sand. With the finer , the bed features were still growing in height when the surface waves be- e unstable and broke, and the maximum height controlled by transport near ed was never reached. On the other hand, with the 0.45 mm sand and e depth and velocity, the maximum height of the bed features controlled by sport was reached, and it was not great enough to cause the waves to break.

he authors have defined two regimes of flow in alluvial channels and a sition zone that separates them. The basis they put forth for this classi- ion includes the mode and magnitude of sediment transport, the resistance ow, and the bed and water surface configurations. However, as shown in e 2, the classification is actually based only on the bed configuration, be- e the total load concentration, friction factor, and slope at the divisions een the various regimes differ widely for the two different sands. This in- tes that their classification into upper and lower flow regime and transi- serves no useful purpose. The meaningful classification is according to configuration.

n describing the succession of bed configurations, the authors speak of the ges as occurring with increasing shear. Boundary shear stress is not a parameter to use to describe the bed configuration or to determine the and transport parameters because for a given shear, it is possible to have e than one equilibrium bed form, velocity, and transport rate. This has shown by Norman H. Brooks,[46] and Vanoni and Brooks[39] for flows at con- depths, and by Kennedy[40] for a series of experiments with a constant dis- ge. The authors cite this fact, and observe that it is true only in the tran- n zone. However, as shown in Fig. 3, the transition zone includes approx- ely half of the velocity range investigated with the coarser sand, and over rth of the velocity range covered with the finer sand. Thus, this zone in- es a significant range of flow over which the velocity, bed form, and sedi- concentration cannot be expressed as single-valued functions of bed shear ss, or any combination of depth and slope. This is appraent in Fig. 3 in n, for the constant depth, vertical lines of constant slope are also lines of ant bed shear stress. For large ranges of velocity and sediment concen- on, it is seen that the slope and, hence, the shear stress do not change ap- iably. It is also apparent that over this same velocity range for which the and shear stress are nearly constant, the bed configuration changes from s to plane bed and standing waves. However, for the whole range of ve- investigated, Fig. 3 also shows that there is only one slope, one bed con- ation, and one sediment concentration for each velocity. Therefore, if the rs had selected flow velocity as the independent variable, as suggested by ks,[46] and plotted slope, friction factor, and sediment concentration as in-

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"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks, sactions, ASCE, Vol. 123, No. 2931, 1958, p. 526.

dependent variables, they would have presented a clearer relation amo variables and avoided the difficulty of having more than one value of th pendent variables (concentration, velocity, friction factor, and bed for some values of their dependent variable (slope or shear stress).

The authors have made several other statements that the writers w discuss. They state that for a given width-depth ratio, incipient motion at $F \approx 0.15$ in the large flume, whereas in a small flume, beginning of may occur at Froude numbers greater than unity. They also state tha form in a flume at higher Froude numbers than in a natural stream larger than the flume. Although no doubt true for proper selections of widths, and sand sizes, these statements are confusing. For example, Shields' diagram for critical shear stress,[47] which is fairly reliable fo dicting incipient motion, the width-depth ratio of the flume and Froude n do not appear and are, thus, irrelevant parameters for this problem; i bed shear stress and fluid and sediment properties that determine whet bed particles will move. The Froude number is proportional to the squa of the ratio of the inertia forces to gravity forces[48] regardless of the s the system. It characterizes gravity effects, but it does not relate to t cous forces that move the sediment and is an inappropriate parameter in such relations. Relating beginning of sediment motion and dune for to velocity is much more meaningful than relating them to Froude numb done by the authors.

In Fig. 2, the authors indicate that standing waves and antidunes occ when the Froude number of the flow equals or exceeds unity. This is co to laboratory observations[43,44,46] that antidunes may occur at Froude bers as low as approximately 0.7. The authors' reasoning appears to be on the one-dimensional analysis indicated in Figs. 1(b) and 1(d), in wh surface disturbance is the same or the inverse of the bed distrubance d ing on whether the Froude number is greater than or less than unity. Ho this is an inappropriate model for flow over a wavy bed where the v components of velocity are also important. For this type of flow, the surface waves are in phase or out of phase, according to whether the v is greater or less than the celerity of a simple harmonic wave of the wave length as the bed form.[49] The celerity c of such a wave is give $= \frac{g\,L}{2\,\pi} \tanh \frac{2\,\pi}{L} D$, in which L is the wave length.

The authors attribute the lower friction factor of the plane bed and s wave regimes compared to the no-movement regime to the moving of th roughness in the former case. A more important factor is probably th of the suspended load, which reduces the friction factor by modifying t bulence structure of the flow, as reported by Vanoni,[50] and Vanoni an

47 "Application of Similarity Principles and Turbulence Research to Bedloa ment," by I. A. Shields, 1936, Trans. from German by W. Poh and J. C. van U. S. Soil Conservation Service, Coop. Lab., California Inst. of Tech., Pasadena p. 98.

48 "Engineering Hydraulics," by Hunter Rouse, John Wiley and Sons, Inc., 195

49 "Theoretical Hydrodynamics," by L. M. Milne-Thompson, The MacMil 1961, p. 406.

50 "Transportation of Suspended Sediment by Water," by Vito A. Vanoni, Tran ASCE, Vol. 111, No. 2267, 1946, p. 67.

Nemicos.[51] By investigating the form of the velocity profiles, they found that it is not simply the moving of the roughness, but an actual change in the form of the velocity profile that accounts for the reduced friction factor.

It was observed by the authors that in the antidune regime the resistance increased much more rapidly for the finer sand than for the coarser sand, and they state that this was reflected by increased antidune activity. A more reasonable explanation would seem to be that the finer, more easily transported sand formed antidunes with breaking waves more frequently than the coarser sand. The greater energy dissipation in wave breaking with the finer sand was reflected as increased resistance to flow. This is the inverse of the authors' explanation that the increase in shear was responsible for the increased antidune activity.

With the objective of improving the communication of ideas among people interested in sedimentation, the writers would like to propose a system of nomenclature for bed forms. The proposed nomenclature is presented in Table 3 together with some of the terminology used in the literature which illustrates the difficulties of communicating ideas and information. Most of the terms proposed have been used before, but not always in the sense adopted here. The terms underlined in the right column of Table 3 are those used by the authors.

In selecting the terms, an effort was made to classify forms into types based on the characteristics of the bed features and the particular phenomena involved in their formation or movement, or both. For example, as stated previously, the small peaked forms of Fig. 2(a) and the larger ones of Fig. 2(c) are considered to be of the same kind. According to Table 3, these can be called either dunes or peaked ripples. On the other hand, when the bed configuration contains peaked forms of two different sizes, as in Fig. 2(b), it is proposed that the small ones be called peaked ripples and that the large ones be referred to as dunes. The Froude number for a flow with dunes and peaked ripples is usually low enough so that surface waves do not appear, but if any do form, they will be out of phase with the dune, as in Fig. 2(c). Flat dunes are distinguished from dunes because they are much less peaked than dunes, and appear to be a form associated with the transition from dunes to flat bed. They have profiles somewhat like that shown in Fig. 2(d), except that they often have a sharper peak. There was a temptation to call them bars, because in profile they do resemble the bar that forms a major hazard to river navigation. However, such river bars are rather isolated forms, and appear different from flat dunes which occur in patterns completely covering the bed. The term "flat" bed[46] was selected in preference to "plane" or "smooth" because it is more accurate. It takes cognizance of slight irregularities that do, in fact exist, which the absolute term "plane" does not allow. It also avoids the objection raised to the use of the term "smooth" on the grounds that it already has a special meaning in hydrodynamics. The form called "rounded ripple" was not listed by the authors, and has not been reported in flume studies. It was observed by Ake Sundborg[42] in shallow water near the bank of a stream and was apparently formed by wave motion.

The characteristic feature of antidunes is that they are in phase with the surface waves that always accompany them. Although they occur within the

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51 "Resistance Properties of Sediment-Laden Streams," by Vito A. Vanoni, Transactions, ASCE, Vol. 125, No. 3055, 1960, p. 1140.

higher range of Froude numbers, it is not necessary to so specify in th
nition, because this follows through analytical relations from the requi
that the surface and bed waves must be in phase. In line with the idea
previously, the definition adopted for antidunes includes the authors' "s
waves" and "antidunes" shown in Fig. 2(f) and 2(g). This definition al
not restrict the motion of the forms; they may be stationary or more

TABLE 3.—PROPOSED NOMENCLATURES FOR BED FORMS IN ALLUVIAL STREAMS

Terms and Definitions	Alternate ter in use
1. Bed Form is a generic term used to denote any irregularity of any shape produced on the bed of an alluvial stream by flowing water.	Forms of bed ness Sandwaves Dunes Bars
2. Bed Configuration. A complex of bed forms covering the bed of an alluvial stream.	
3. Rounded Ripple. A bed form with a rounded and approximately symmetrical profile.	Symmetrical
4. Dune or Peaked Ripple. A bed form with a triangular profile with a gentle upstream slope and a steep downstream slope, which occurs in tranquil flows, and is out of phase with any surface disturbance that it may produce. Although the terms dune and peaked ripple may be used synonymously, it is understood that when a bed configuration consists of dunes of significantly different sizes, the large ones shall be called dunes and the small ones shall be termed peaked ripples.	Dune, ripple Sand wave Bar, bank, ri Current rippl
5. Flat Dune. A form with a dune-shaped profile with an upstream slope very little different from the general slope of the stream, and a steep downstream slope.	Washed-out-I
6. Flat Bed. A flat bed is one in which the surface irregularities deviate only a few grain diameters from a plane.	Plane bed Smooth bed Even bed
7. Antidune. A bed form with a rounded symmetrical profile that is in phase with the accompanying surface waves.	Standing wave Antidune
8. Peaked Antidune. A bed form with a dune-shaped profile that is in phase with the accompanying surface waves.	Dune
9. Chutes and Pools. A bed configuration giving flows like mountain torrents, consisting of a series of pools with tranquil flow, followed by a steep section or chute with rapid flow terminating in a hydraulic jump at the head of the next pool.	Antidune

upstream. They often grow in height, causing the surface waves to gr
they become unstable and break. The breaking waves change the flow
in such a way that the bed flattens out, after which the growth cyc
peated. The term peaked antidune, defined in Table 3, describes a
form not reported by the authors. Kennedy[43,44] has observed peaked a
that moved downstream. They were shaped like dunes, moved by t

mechanism as dunes, and produced surface waves in phase with them. In one experiment, these peaked antidunes had wave lengths of 0.25 ft and occurred on a bed of 0.55 mm sand with flow 0.346 ft deep, with a mean velocity of 2.58 fps. A flow with the same depth and velocity over a bed of 0.23 mm sand produced a flat bed.

The surface waves over antidunes and peaked antidunes have been called sand waves.[52] This is a rather unfortunate term, because it is more appropriate for denoting bed forms than surface waves. However, the writers do not have a better term to suggest. In rivers the lengths of crests of sand waves are usually approximately the same or longer than their wave lengths.

In high velocity flows in flumes[43,44] a form similar to the antidune and sand wave, but with crest length considerably shorter than the wave length, has been observed. The surface waves accompanying the forms grow, break, and move like sand waves. These surface waves have been called rooster tails.[43,44] Kennedy[43,44] is of the opinion that rooster tails are formed by a superposition of a cross wave and the longitudinal wave. They are considered as special cases of sand waves, and have not been given a special category in the proposed nomenclature.

The interesting form defined as "chutes and pools" and shown in Fig. 2(b) is something completely different from antidunes. An example of this form was observed by Kennedy[43,44] in a laboratory flume 33 1/2 in. wide with a bed of 0.23 mm sand. The mean depth was approximately 0.22 ft, the mean velocity was approximately 3.29 fps, and the pools were spaced 5 ft to 10 ft apart.

52 "The Measurement of Silt-Laden Streams," by R. C. Pierce, Contributions to the Hydrology of the United States, 1916. Water Supply Paper 400, USGS, 1917.

ROUGHNESS SPACING IN RIGID OPEN CHANNELS[a]

Discussion by Jamil Malaika; T. Blench; Donald R. F. Harleman and Ralph R. Rumer, Jr.; Walter Rand; Jacob Davidian and Rolland W. Carter; and John A. Roberson

JAMIL MALAIKA,[30] A. ASCE.—The authors are to be commended for their valuable experimental contribution toward a better understanding of the roughness problem, in particular, for systematically applying the available theories to their results and, thereby, finely displaying the modern principles underlying the subject of resistance to flow in rough channels.

The problem of roughness, which has baffled the practicing engineer long before the logarithmic distribution hypothesis was ever proposed, has never been quite understood or nearly solved. Not until sufficient data on the combined effects of size, shape, and distribution of roughness are available will any final conclusions as to the exact nature and magnitude of parameters that completely define roughness be possible.

The authors' comments are sought on the following remarks that are made in an effort to acquire a better insight into this intricate problem.

Inasmuch as χ is still an implicit function of roughness, little advantage seems to be gained by plotting the horizontal y_n/χ lines in a generalized resistance diagram (Fig. 10). Were it possible to obtain a unique relation between ψ and the roughness density $\frac{ab}{x\ (e + b)}$ (Fig. 8), then a plot of the lines $\frac{1}{f\left[ab/x\ (e + b)\right]}\ \frac{y_n}{a}$ in Fig. 10 would be significant. Evidently the roughness density remains by far the roughness spacing parameter with the best correlation; yet it is a poor one as Fig. 8 shows. Furthermore, it is greatly influenced by shape and angularity of roughness elements. For a particular shape of roughness one would also expect its variation with the orientation of the elements (for example, whether or not the baffles are at an angle with the flow direction), and with their relative location (that is, whether they are staggered, as in these tests, or aligned).

As the spacing increases and the smooth area between the roughness elements grows in extent, the influence of these elements and their wakes becomes less significant. One might reasonably expect, in the case of sparse spacing, a tendency of the y_n/χ curves (Fig. 10) to vary with the Reynolds number (in a fashion probably similar to that near a smooth boundary) instead of remaining horizontal. Because the little data in Fig. 10 for any particular sparse spacing do not seem adequate for substantiating or disproving the pre-

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[a] May 1961, by William W. Sayre and Maurice L. Albertson (Proc. Paper 2823).

[30] Visiting Research Engr. under Natl. Academy of Sciences sponsorship, Albrook Hydr. Lab., Washington State Univ., Pullman, Wash.; on leave from the Coll. of Engrg., Univ. of Baghdad, Baghdad, Iraq.

vious statement, it is debatable whether the interpolated horizontal lines appear in Fig. 10 for these spacings. In this respect it remains for fut vestigators to draw a limit between a dense and a sparse spacing.

Unlike the case of Nikuradse's packed sand roughness, a logarithm tribution is not to be expected to hold in the close proximity of boundar spaced roughness elements like those tested. In this regard the un scope and thoroughness of H. Schlichting's work, which marks a distinc stone in the course of development of the search, probably deserves mo a passing mention. The results of his investigations, which covered a twenty-one spacings for six different shapes of roughness, indicate a co deviation from the logarithmic law within y/a values ranging from 2.5 t deviation being more pronounced in the cases of the baffle type roughne those of this paper. The entire falling of points of many of the veloci files within the preceding range (in these tests y_n/a ranges from 2 to 8 to account for the failure to obtain consistent values of κ by the veloci file method (Table 1 and Figs. 9 and 12). Noteworthy is the fact that boundary vicinity of his sparsest spherical roughness an inexplicable velocity distribution which in no way is in accord with the logarithmic l observed by Schlichting.

The statement is made that, " . . . in the equation

$$\frac{C}{\sqrt{g}} = \frac{2.3}{\kappa} \log \frac{y_n}{\chi} \quad \ldots\ldots\ldots\ldots\ldots$$

the parameter χ (= ψ a) should completely describe the boundary roug whereas the observed variation of values of κ with spacing within the mental range (0.35 to 0.39) appears to contradict it. Writing Eq. 6(c form

$$\frac{C}{\sqrt{g}} = \frac{2.3}{\kappa} \log \frac{1}{\psi} \frac{y_n}{a} \quad \ldots\ldots\ldots\ldots\ldots$$

it becomes readily apparent that, due to the logarithmic nature of the r when values of y_n/a are small (as in the case of the exceptionally high roughness values of the tests: $y_n/a \approx 8$ to 2) a variation in κ is less sig than a comparable variation in ψ. It should be recalled, however, tha higher values of y_n/a may be encountered in practice where a variatio values of κ is of great significance. The following example, based on perimental results, is introduced for illustration. Were one to disreg sparser spacings and the κ-values that are based on the velocity profil of which being, as tacitly speculated, of lesser consistency, and cons first two of the more dependable dense spacings and the correspondin of κ that are based on the mean velocity (Table 1 and Figs. 9 and 12), on prepare Table 3. Apparently, because the logarithmic equation fails for the preceding ψ values at y_n/a equal to unity, the latter is not liste dently the great variation (55.2%) in the value of $1/\psi$ is of lesser sign inside the log-terms; its significance, furthermore, continuously dimin the latter $\left(\Delta \log \frac{1}{\psi} \frac{y_n}{a} = 22.9;\ 10.4;\ 6.7;\ 5\%\right)$ as y_n/a increases (10; 10 10000). Obviously the variation of κ (= 5.4%) is as significant as tha log-term, particularly within the more practical range of y_n/a, and the these experimental results seem to substantiate, rather than dispro

mounting evidence that κ as well as χ are equally significant in describing roughness.

As done in this work, previous investigators of the "universal constant school" seem to follow a fashion of assuming an "average" value of κ for a particular experimental range. Values of 0.36; 0.37; 0.38; 0.40 and most values were assumed at different times. Actually, in most cases a wide range of variation of κ was observed. A close examination of Schlichting's data, for example in which an average κ of 0.40 was assumed and fitted, reveals after disregarding points on the boundary proximity and central region of his closed conduit, an actual variation of κ with shape and distribution of roughness ranging between 0.33 and 0.41. It is particularly interesting to note that for his short angles (baffles) with three different spacings, κ approximately changes

TABLE 3.—COMPARISON OF THE SIGNIFICANCE OF VARIATION IN THE VALUES OF κ AND Ψ

Runs	Ψ	$\frac{1}{\Psi}$	κ	$\log \frac{1}{\Psi} \frac{Y_n}{a}$			
				$Y_n/a = 10$	100	1000	10000
1-7	1.49	0.67	0.37	0.83	1.83	2.83	3.83
8-11	0.96	1.04	0.39	1.02	2.02	3.02	4.02
	Δ =	55.2%	5.4%	22.9%	10.4%	6.7%	5%

from 0.35 for the sparser spacing to 0.38 for the denser one, a trend seemingly very similar to that in this paper both in magnitude and direction. All evidence seems to lead one to conclude that velocity distribution near a rough boundary is best expressed as

$$\frac{v}{\sqrt{\frac{\tau_o}{\rho}}} = a \log b\, y \quad \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

in which a and b are only constant for a particular roughness.

T. BLENCH,[32] F. ASCE.—To the practical user the "roughness height," as used in formulas that state it explicitly, conveys no more than any other roughness number, such as Manning's n, because this "height" is unmeasurable. Measures are possible of the corrugations of C. M. culverts and of the jags of rock tunnels but there is no present means of translating to "roughness height" for use in a formula. A major trouble with basic investigations of these cases is that the friction factor f, for a known Q, varies as the fifth power of diameter or the third power of broad channel depth, and the difference between internal and external diameter is large. Another problem is that corrugations come in only two sizes and rock jags present an enormous number of patterns. Of course the practical engineer manages quite well with whatever formula he likes, the diameter or depth he finds convenient, and the n or other parameter that experience shows to work for cases of interest. But it would be helpful to

[32] Pres., T. Blench and Assoc., Cons. Hydr. Engrs., and Prof. of Civ. Engrg., Univ. of Alberta, Edmonton, Alta., Canada.

have a fairly reliable rule that would convert the roughness dimensions t could measure into a roughness factor that, inserted in a formula, woul a fairly reliable answer for cases outside his experience. Accordingl writer is most impressed by the authors' initiative and success in inven simple common sense formula that professes to give an "equivalent roug height," χ, for a set of patterns of relatively large baffle blocks in a flum does, in fact, give a consistent relation (Fig. 6) between friction factor, equivalent relative roughness χ/y over a satisfying large range. The flow course, fully turbulent or "rough boundary." The potentialities of the f seem so important for corrugated pipe and rock tunnel work that the v ventures to discuss some basic aspects and give an application to corru pipe.

Scope and Presentation of Data.—The essential diagram is Fig. 6. merely leads to it and Fig. 7 loses its applicability if the ordinates are rected for effective depth—in fact it then shows that y/a needs to be cor

TABLE 4.—RANGES IN FIG. 6 ANALYZED

Function	Range	Remarks
$C/\sqrt{g} = \sqrt{8/f}$	2.45 to 16.3	Data Fig. 6
$1/\sqrt{f}$	0.87 to 5.76	Data Fig. 6
f	1.33 to 0.03	Data Fig. 6
f	0.10 to 0.008	Standard Moody Diagram
f = 0.04 at top of smooth boundary line of Moody Diagram		
$\chi/4y$	0.10 to 0.0005	Data Fig. 6
$\Sigma/4y$	1.37 to 0.005	Data Fig. 6 with $1/\sqrt{f} = \log_{10}(4y/\Sigma) + 1.14$
$\Sigma/4y$	0.07 to 0.0002	Nikuradse experiments
$\Sigma/4y$	0.05 to 0.000,001	Moody Diagram Extrapol of Nikuradse
$e/4y$	28 to 0.014	Data Figure 6 with $1/\sqrt{f} = 2(4y/e)^{1/4}$

to y/χ to have any meaning. For comparison with general data of Fig its legend a standard Moody diagram, rather than Fig. 10, would have tages. Fig. 10 is actually a Moody diagram with the left-hand ordinat instead of the conventional $\sqrt{1/f}$, with the diagram height containing tw conventional log. range, and with the scale shrunk to permit this an some of the data of Fig. 6 to be inserted. Fig. 10 lacks the relative rou lines of Moody, the major portion of Fig. 6 data that penetrate into the diagram, and the last three cycles of ten of the conventional abscissa cordingly Table 4 has been prepared for use with the Moody diagram. It that the Fig. 6 data provide (a) a useful overlap into the sand-roughness where there is little ambiguity about how to measure diameter or depth a wide excursion out of it. Fig. 6 data indicate that χ, the equivalent rou height, conveys an idea of visual reality whereas the Moody and four measures do not because they give relative roughness greater than 1

l by altering the parameters of the corresponding equations); and that the ody range of relative roughness is extrapolated unjustifiably from the Ni-radse experiments that are supposed to form its base whereas the authors' ative roughness range is comparable with Nikuradse's although in a dif-ent position. These general findings are unaltered by the writer's later agreement with use of depth to flume floor instead of to top of equivalent ughness.

The data of corrugated pipes, in the field,[33,34,35] fall neatly inside the auth-s' range of data in Fig. 6, as indicated by Table 5.

The Definition of Depth for Computing f.—In corrugated pipe practice in-rnal diameter is used for flow calculations. The reason, apart from conven-ice, is that the water in the corrugations is observed to circulate, so does take part in the general forward flow. (In small models flow may follow corrugations.) Use of the external diameter, to correspond with the auth-s' use of depth to flume floor, instead of to top of equivalent baffles of tight acing, would give a totally false impression of velocity of flow. Study of the end of Fig. 6 seems to indicate that χ is fairly close to "a" for tight spacing, that it replaces actual roughnesses of large pitch-to-height ratio, x/a, and acing-to-height ratio, e/a, by smaller equivalent ones at tight spacing. Thus, a first approximation, the writer would prefer to calculate the ordinates of

TABLE 5.—PARAMETERS FOR C. M. CULVERTS

	a(ft)	x(ft)	x/a	e/a	$\chi = a^2/x$	$\chi = a/x$	$\frac{y - \chi}{\chi}$
ndard C. M.	0.04	0.22	5.6	0	0.0072	0.18	31
uctural C. M.	0.167	0.55	3.0	0	0.022	0.33	7.5
marks	height	pitch			equiv. roughness height	spacing parameter	D = 5 ft equiv. 4y

g. 6 from $y' = y - \chi$ instead of from y, with the expectation that this might ow a closer linkage with formulas based on data for small relative rough-ss.

Effects of Adjusting Fig. 6 for depth $y' = y - x$.—Because $f = 8gS\,d^3/q^2$ in ich q is discharge intensity and d is depth, the preceding adjustment of Fig. an be made by increasing all ordinates in the ratio $(k/(k - 1))^{1.5}$ in which y/χ. Table 6 shows the calculation for selected points of the fitting curve Fig. 22 plots the result. (The abscissa was not altered to k - 1 as there ms no argument to prove it any better than k; however, points for k - 1 are rked ▭ where they affect the plot). Notice that the plot for $y/\chi > 15$ is

33 "Friction Factors in Corrugated Metal Pipes," by M. J. Webster and L. R. Metcalf, ceedings, ASCE, Vol. 85, No. HY9, September, 1959.
34 "Hydraulic Data Comparison of Concrete and Corrugated Metal Culvert Pipes," by G. Straub and H. M. Morris, Tech. Paper 3B, St. Anthony Falls Hydr. Lab., 1950.
35 "Hydraulic Investigations Related to Large Corrugated Metal Culverts," by C. R. ll, thesis presented to the University of Alberta, at Edmonton, Alta., Canada, in uary, 1961, in partial fulfilment of the requirements for the degree of Master of nce.

straight and has the equation

$$\sqrt{\frac{8}{f}} = 5.6 \log \frac{y}{\chi} + 1.2 \quad \ldots\ldots\ldots\ldots$$

to be compared with Eq. 18. The ▱'s are fitted by

$$\sqrt{\frac{8}{f}} = 5.5 \log \frac{y^1}{\chi} + 1.4 \quad \ldots\ldots\ldots\ldots$$

for $y^1/\chi > 15$. In both cases f is calculated from y^1.

Universal Flow Formula and Universal Velocity Distribution Concep plied to Eqs. 18 and 45a.—Whatever empirical equation one prefers to universal velocity deficiency distribution (universal in the sense that s nondimensional coordinates reduce all but the near-boundary parts of v deficiency distribution curves to a single one) its parameters should unalterable under the conditions to which they apply. If then, the equat one built with fair logic on it—is integrated to obtain a flow formula, th ficients in that formula should also remain constant under the same c stances. The constant stated by L. Prandtl,[36] and used by V. L. Stre

TABLE 6.—ADJUSTMENT OF ORDINATES OF FIG. 6 TO GIVE FIG. 13

$k = y/\chi$	2.5	3	4	6	10	20	30	50	70	100	200	30
$(k/k-1)^{3/2}=\rho$	2.15	1.84	1.54	1.32	1.16	1.07	1.05	1.03	1.03	1.02	1.01	1.
$C/\sqrt{g} = \sqrt{8/f}$	2.45	2.95	3.7	4.8	6.05	7.9	9.0	10.3	11.2	12.2	14.0	15.
$\rho C/\sqrt{g}$	5.3	5.4	5.7	6.4	7.0	8.5	9.5	10.6	11.5	12.4	14.1	15.

F. ASCE, to be applicable near a pipe wall, and therefore presumab channel, to correspond with the 6.06 of Eq. 18, is 5.52 which is quite di but approximates the 5.6 of Eq. 45a or 5.5 of Eq. 45b.

This makes the writer believe that the use of depth to the top of "eq tightly-spaced blocks" to calculate f is more realistic than the author to floor. The writer is not perturbed by the logarithmic relation o breaking down at small y/χ values because (a) the type of flow when y a mates "a" can hardly be imagined to correspond to that when y is consi greater than "a"; and (b) the logarithmic relation, being based on a dyna impossible and wrongly derived (though empirically excellent velocity bution, cannot be expected to extrapolate indefinitely.

Relevant to (b) the writer must disagree with the authors' statement logarithmic velocity distribution . . . "derivation appears in many s fluid mechanics texts." These texts avoid genuine derivation but, by in ing the name of Prandtl, suggest that proof is possible. They do not q specific references to Prandtl, nor do they quote him verbatim becaus

36 "Fluid Dynamics," by L. Prandtl, Blackie and Son, Ltd., London, 1953 Eq. 25.

37 "Fluid Mechanics," by Victor L. Streeter, McGraw-Hill Book Co., Inc., N 1958, p. 159.

not derive what they advocate. Actually he dealt with a case having one less independent variable than required for a canal, so that he could introduce a simple dimensional argument. Then, consistent with his general practice of trying to give physical illustrations to explain the results of mathematical ar-

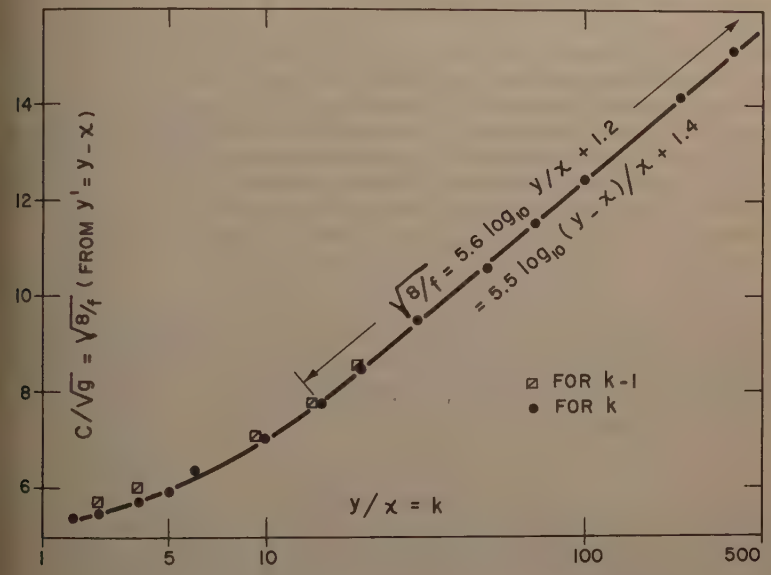

FIG. 22.—FITTING LINE OF FIG. 6 WITH f ADJUSTED FOR y - X

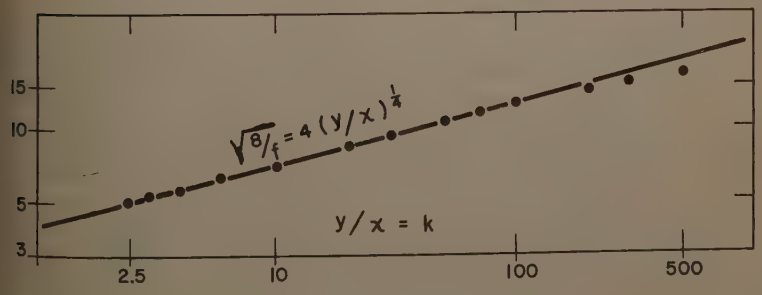

FIG. 23.—FIG. 22 TO DOUBLE LOG SCALE

uments, he made certain plausible hypotheses from which a logarithmic distribution (for his conditions that were not those of a finite channel) could be educed. One hypothesis was that $\overline{\rho\, u'\, v'}$ was constant. However, in the at-

mospheric case,[38] where check is possible, it was found to vary from 3.62 units in the region 1.5 m to 19 m from the ground where the logar fit is excellent. Of course, a "right answer" being obtained from wrong ises should surprise neither the novice nor the expert.

Proponents of the universal flow formula idea[39] accept the fact that th of rigid boundaries (both rough and smooth) and of the highly duned bou condition of canals with mobile beds can be put into a universal flow fo and that this formula dissociates into one sub-formula for each of the boundary conditions when the equivalent roughness height is stated exp in terms of the factors on which it depends. So they would agree, even readily that the universal velocity proponents, that the 6.06 in Eq. 18 that the depth has not been given its proper physical measure.

Universal Flow Formula Concept Used to Analyze Fig. 6.—If the uni flow formula postulate is accepted then its basic relation[39]

$$\frac{1}{\sqrt{f}} \alpha \left(\frac{d}{e}\right)^{1/4} \alpha \left(\frac{\text{channel depth}}{e}\right)^{1/4} \quad \ldots\ldots\ldots$$

should be tested with the adjusted data of Fig. 6 shown in Fig. 22. Acco ly, Fig. 23 replots Fig. 22 to double log scale. The line of slope 1/4 th this new plot has been chosen with the simple equation

$$\sqrt{\frac{8}{f}} = 4\left(\frac{y}{\chi}\right)^{1/4} \quad \ldots\ldots\ldots\ldots\ldots$$

partly to have a simple formula and partly because it is hard to belie the Koloseus data for cubical blocks could really belong exactly to the flat-sheet roughnesses. In fact their fine fit to Fig. 6 is probably fort The irregularity of the Robinson data is more in accordance with norm pectation.

Comparing Fig. 23 with Fig. 22 it seems that the former is distinguis having its points fitted by its theoretical line down to $y/\chi = 5$ against th ter's fit to $y/\chi = 15$. However, the Koloseus points for y/χ are not pe fitted by the fourth root formula though they are by the logarithmic one also interesting to note that Fig. 23 would not be too well approximate straight line of slope 1/4 if y'/χ were used as abscissa. In consider implication of this it should be remembered that index in an exponenti mula corresponds mathematically to the multiplier of the logarithmic o multiplier in the former corresponds to additive in the latter.

Allowing for the facts that the data of the different workers, as plo Fig. 6, are not strictly comparable, and that y' is not claimed to be an than a good approximation to the perfect value of depth, the writer fee Fig. 22 and 23 show no appreciable difference in fitting the facts by eith arithmic or fourth root formulas within the range of data used. This c ponds with his general opinion[39] that the two methods will not give appr

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38 "Atmospheric Turbulence," by O. G. Sutton, Methuen, London, 1955, a Wiley and Sons, Inc., New York, 1955, p. 26.

39 "The Fourth-root n-f Diagram," by T. Blench, Proceedings, ASCE, Vol. HY1, January, 1960, with closure to discussion in Proceedings, ASCE, Vol. 87, N February, 1961, p. 155.

different results unless compared over the entire range of engineering data. How well a logarithmic formula can approximate an exponential one is illustrated by the two equations marked "smooth" in Fig. 10. In fact, using a suitable logarithmic formula to approximate an exponential one is rather like using an arc of a very large ellipse to approximate a straight line—a clumsy but effective way of performing a simple operation, provided it is not carried too far.

An Application to Structural Plate Corrugated Culverts.—C. R. Neill[35,40] working with a 5 ft structural plate corrugated metal culvert in a field set-up, found that data were best represented by n = 0.036. The corrugations were 2 in. high with 6 in. pitch. The n was computed using internal diameter. To find what the authors' data would indicate, assuming that corrugations follow Eqs. 14 and 15, use $\chi = 1/18$ from Eq. 18, to obtain e = 8/9 ft. Then the writers' f - n fourth root diagram[39] or the formula written on it, gives $d^{1/12}\, n = 0.037\, e^{1/4} = 0.038$, whence, for d = 5 feet, n = 0.033; this makes no allowance for the bolt-heads, joints and imperfections of real pipes.

This agreement is not as startling as it may look. Roughness height, being a multiple of the fourth power of an absolute roughness very close to Manning's n, is over-sensitive, so that quite large deviations of it produce relatively little difference in practical flow calculations. Stated another way, flow is fairly insensitive to changes of visual roughness. A corollary of this is that there is no immediate practical need to strain after an esoteric definition of equivalent roughness height in terms of the parameters that really affect it. Hence, the successful conclusion to the authors' bold venture.

An Alternative Way to Relate Roughness Height to Pattern.—The authors invented a common sense formula for χ and then proceeded to investigate the consequences. A more fundamental procedure would have been to accept a type of roughness formula, such as the fourth root one, expressed as

$$\frac{8\, g\, S\, (y - \chi)^3}{q^2} = 2.0 \left(\frac{4\, y}{\chi}\right)^{1/4} \quad \ldots\ldots\ldots\ldots (48)$$

Then for each observed point of Fig. 6 find the value of χ to conform to Eq. 48. Thereafter χ/y could be correlated graphically with x/a and e/a for different patterns. This could be useful when more data for more patterns are obtained.

Conclusions.—The writer believes that, by using depths to the top of equivalent tightly spaced roughness, and adopting the fourth-root type of formula, a little more experimental work or analysis of existing data would bring corrugated pipe and rock tunnel data into line with those of mobile bed canals and ordinary rigid boundaries. This means of estimating equivalent roughness height would be that proposed by the authors, or perhaps a slight modification. The cumbersome, and not dynamically justified, logarithmic formula could then be dropped.

DONALD R. F. HARLEMAN,[41] M. ASCE, and RALPH R. RUMER, JR.,[42] A. M. ASCE.—The resistance to flow in smooth open channels will be a frictional drag. When discrete roughness elements are placed on the channel bot-

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40 "The Hydraulic Capacity of Large Corrugated Metal Culverts," by C. R. Neill, paper presented to Engrg. Inst. of Canada, Annual Meeting (Tech.), May, 1961.

41 Assoc. Prof. of Hydraulics, Massachusetts Inst. of Techn., Cambridge, Mass.

42 Research Asst., Hydrodynamics Lab., Massachusetts Inst. of Tech., Cambridge, Mass.

tom, additional resistance is imposed on the flow. The additional resist is the form drag on the roughness elements. The resulting flow patter comes complex and non-uniform in the vicinity of the boundary. This ty flow has been described[43] as having three possible regimes depending o height and concentration of the roughness elements. The first is classifi isolated roughness, that is, where the wake behind each roughness ele dissipates and does not interfere with the flow on any other roughness ele The second is classified as wake-interference flow. The third is classifi skimming flow; this occurs when the roughness elements are so close stable eddies are formed between roughness elements and the bulk of the moves over the top of the roughness elements and the stationary eddy z

The form drag can be adequately described by the equation

$$D = C_D A \frac{\rho V^2}{2} \quad \ldots\ldots\ldots\ldots\ldots$$

in which D is the force (in pounds) exerted on the element in the direct flow, A is the projected cross-sectional area (in square feet) of the ele perpendicular to the direction of flow, ρ denotes the fluid density (in slug cubic feet), V is the flow velocity (in feet per second), and C_D refers t drag coefficient as defined by Eq. 49. The coefficient C_D must be deter by experiment. For shapes with well-defined separation points (such a roughness elements used by the authors) C_D has been shown to be consta Reynolds numbers greater than 1000. Thus, in the tests by the author should be a function of the arrangement and aspect ratio of the roughnes ments. The form drag experienced by one isolated element or pattern o ments would increase with increasing velocity but C_D would remain a co for Reynolds numbers greater than 1000. For the roughness investigat the authors the preceding restriction on the Reynolds number is not impo

With the preceding analysis in mind the writers suggest that small tests to determine C_D for each arrangement of elements would have pro enough data to adequately predict the resistance coefficients for all valu y_n/a within the range of depths and velocities investigated by the authors. cost of such an investigation would be small compared to the rather exte flume studies that have been made in the past. In 1950, Hans A. Einste ASCE, and R. B. Banks, M. ASCE, reported[44] an approach similar to the ers'. They suggested that the arrangement of discrete roughness ele was important in determining C_D even though the actual number of ele per unit area remained constant. This is in agreement with the preceding ysis. The data of Einstein and Banks was not considered directly appli to the writers' analysis because their flows were non-uniform and thre mensional.

For turbulent flow the velocity at the top of a roughness element c stated according to the Prandtl-von Kármán universal velocity distributio as,

$$\frac{v_t}{V} = 1 + 2.7 \frac{V_*}{V} \left(1 + \ln \frac{a}{y_n}\right) \quad \ldots\ldots\ldots\ldots$$

43 "Flow in Rough Conduits," by H. M. Morris, Transactions, ASCE, Vol. 120, p. 373.

44 "Fluid Resistance of Composite Roughness," by H. A. Einstein and R. B. Transactions, AGU, Vol. 31, No. 4, 1950, p. 603.

in which v_t is the velocity at the top of the element (in feet per second), a is the height of the element, V_* denotes the so-called friction velocity (in feet per second) and V is the average velocity (in feet per second). Using this velocity, v_t, to define the form drag on the elements the writers have developed a drag equation for the resistance in open channel flow.

The total bottom shear will be made up of the frictional drag on the bottom plus the form drag on the elements, thus,

$$\tau_o = \tau_f + \tau_D \quad \ldots\ldots\ldots\ldots\ldots\ldots (51)$$

in which τ_o is the total bottom shear (in pounds per square foot), τ_f describes the bottom frictional drag (in pounds per square foot), and τ_D is the roughness form drag (in pounds per square foot). For two-dimensional uniform free surface flow

$$\tau_o = \gamma\ y_n\ S_e \quad \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

in which γ is the specific weight of the fluid (in pounds per cubic foot), y_n is the normal depth of flow (in feet), and S_e refers to the slope of the energy grade line and the bottom slope. From the Chezy equation

$$S_e = \frac{V^2}{C^2\ y_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (53)$$

in which C is the Chezy resistance coefficient (including both frictional and form drag). Substituting Eq. 53 into Eq. 52

$$\tau_o = \rho\ V^2 \left(\frac{\sqrt{g}}{C}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (54)$$

The drag on each element is given by

$$D = C_D\ A\ \frac{\rho\ v_t^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (55)$$

thus

$$\tau_D = N\ D = N\ C_D\ A\ \frac{\rho\ v_t^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

in which N is the number of roughness elements per square foot of channel bottom. The bottom frictional drag will be dependent on the nature of the flume bottom and also on the size and extent of the separation zones associated with the discrete roughness elements. As roughness concentration increases τ_f will become less important. As a first approximation this bottom frictional drag can be given as

$$\tau_f = C_f\ \frac{\rho\ V^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

in which

$$C_f = 2\left(\frac{\sqrt{g}}{C_b}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

C_b represents the resistance coefficient for the channel bottom without r
ness elements. Substituting Eq. 58 into Eq. 57 yields

$$\tau_f = \left(\frac{\sqrt{g}}{C_b}\right)^2 \rho V^2 \quad \ldots\ldots\ldots\ldots$$

Substituting Eq. 59, 56, and 54 into Eq. 51 yields

$$\rho V^2 \left(\frac{\sqrt{g}}{C}\right)^2 = \rho V^2 \left(\frac{\sqrt{g}}{C_b}\right)^2 + N C_D A \frac{\rho v_t^2}{2} \quad \ldots\ldots$$

Eq. 50 can be simplified noting that $V^* = \frac{\sqrt{g}}{C} V$ and v_t becomes,

$$v_t = V \left[1 + 2.7 \frac{\sqrt{g}}{C} \left(1 + \ln \frac{a}{y_n}\right)\right] \ldots\ldots\ldots$$

Substituting Eq. 61 into Eq. 60 and dividing by ρV^2

$$\left(\frac{\sqrt{g}}{C}\right)^2 = \left(\frac{\sqrt{g}}{C_b}\right)^2 + \frac{N C_D A}{2} \left[1 + 2.7 \frac{\sqrt{g}}{C} \left(1 + \ln \frac{a}{y_n}\right)^2\right] \ldots\ldots$$

This is the equation from which the resistance coefficient C $\left(\text{or } \sqrt{C/g}\right)$ r
computed providing one knows in advance the bottom resistance coeff
C_b, and the drag coefficient, C_D, for the particular roughness pattern.

For a/y_n greater than 0.5, v_t is approximately equal to V. For thi
Eq. 60 can be simplified after dividing by ρV^2 to

$$\left(\frac{\sqrt{g}}{C}\right)^2 = \left(\frac{\sqrt{g}}{C_b}\right)^2 + \frac{N C_D A}{2} \quad \ldots\ldots\ldots\ldots$$

For a/y_n less than 0.5 the roughness elements are in a region or rapidly
ing velocities and Eq. 62 should be used.

The writers have calculated from the authors' data C_D for each rou
pattern. The value for C_b was obtained from the equation $C_b = \frac{1.49}{n} y_n^{1/}$
$n = 0.013$ which seemed reasonable for the plywood flume used by the a
The results are tabulated in Fig. 24. The tendency is for C_D to decre
the spacing of the elements increases. The apparent inconsistency of
C_D for runs 16-19 can be qualitatively explained on the following basis
well known that the coefficient of drag for two disks in tandem[45] can
duced below that value for the two disks side by side. This is due to
tablishment of a stationary separation zone between the disks which p
the flow from impinging upon the second disk. This is probably what oc
in runs 16-19. Stable separation zones between the roughness elements
the flow to skim over the plates thus significantly reducing the form dr

In Fig. 25 Eq. 62 is plotted for appropriate values of C_D with the a
experimental data included for comparison. The agreement in Fig. 25 in

45 "Fluid-Dynamic Drag," by S. Hoerner, Published by Hoerner, 1958.

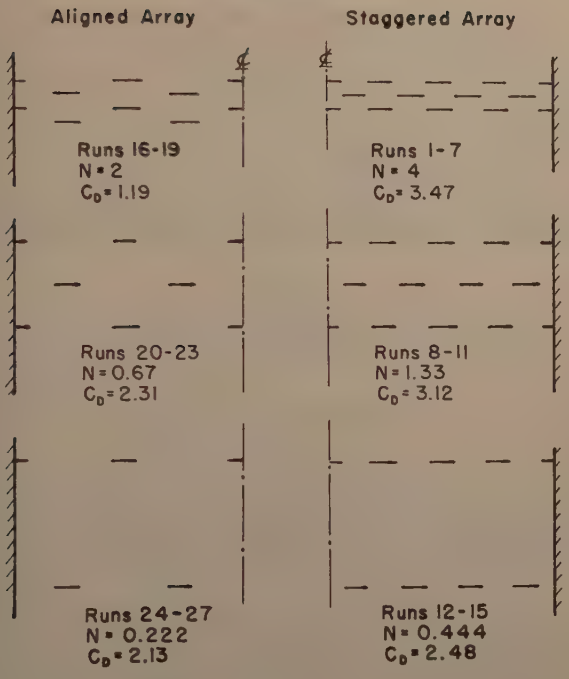

FIG. 24

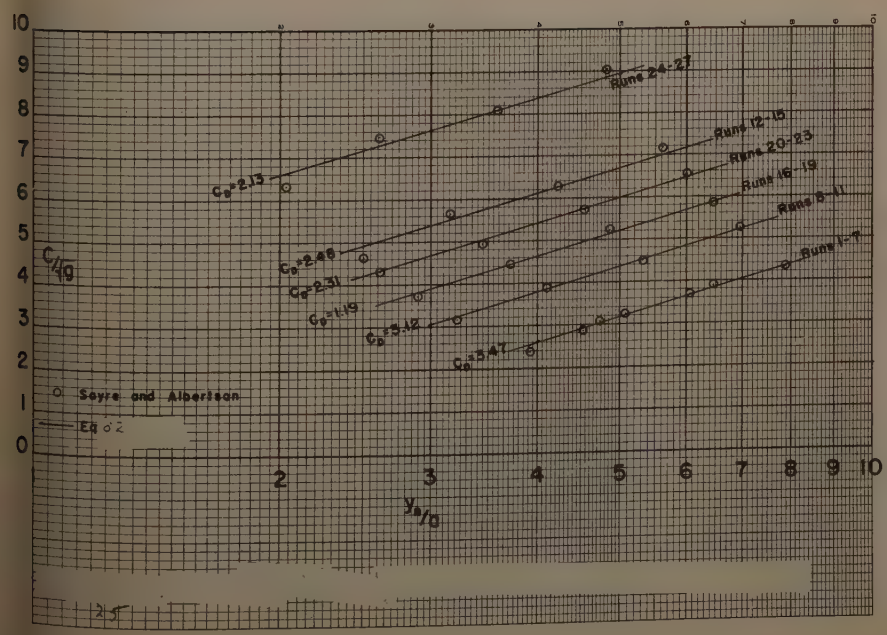

FIG. 25.—COMPARISON OF DRAG EQUATION WITH DATA OF SAYRE AND ALBERTSON

that Eq. 62 is a correct functional relationship for the resistance coeff its quantitative validity depends on a correct C_D.

The writers also analyzed data presented by J. Bryce[46] with Eq. 63 coefficients of drag for flat plates presented by H. Rouse.[47] The resul tabulated in Table 7.

The ability here to use published values of C_D is due to the large he the roughness elements relative to the depth of flow and also because th was in the regime termed isolated roughness. This situation approach case of a flat plate in a uniform flow on which the drag coefficients were

TABLE 7

Run	a/y_n	N	C_D	Bryce Observed n	Writers' Calculate n
1	0.5	2.2	1.2	0.027	0.028
2	0.5	4	1.2	0.033	0.035
3	0.75	9	1.25	0.060	0.061
4	0.75	36	1.25	0.116	0.120

This drag analysis is based on fundamental concepts. The succes which it predicts resistance coefficients for uniform two-dimensiona channel flow is interesting and most encouraging. It is hoped that fo interested in this aspect of fluid dynamics consideration will be given approach.

WALTER RAND,[48] M. ASCE.—The authors have based the analysis o data on the logarithmic velocity distribution law, related to the von K coefficient κ, and to parameters χ and ψ, both of which are their own co tions to this relatively uninvestigated part of fluid mechanics. The s their investigation is broad, and all the forementioned factors are open ther interpretations in accordance with various aspects of the logarithm locity distribution law.

This law implies that a straight line velocity curve in a semi-logar plot will show zero velocity at a distance y' above the datum, from wh flow depth is measured. The most revealing form of this law is

$$\frac{v}{\sqrt{\frac{\tau_o}{\rho}}} = \frac{1}{\kappa \log e} \log \frac{y}{y'} \quad \ldots\ldots\ldots\ldots$$

with $1/\log e$ replacing the constant "2.30," used by the authors, e be base of natural logarithms. A definite integral of this equation over the

46 "Hydraulic Models on the St. Lawrence Power Project," by J. Bryce, Proc ASCE, Vol. 85, No. HY5, May, 1959, p. 147.

47 "Elementary Mechanics of Fluids," by H. Rouse, John Wiley and Sons, In York, 1946.

48 Assoc. Prof. of Civ. Engrg., The City Coll. of the City Univ. of New York

variable (y', y_n) provides an expression for the average velocity V of two-dimensional flow

$$\frac{V}{\sqrt{\frac{\tau_o}{\rho}}} = \frac{1}{\kappa}\left(\frac{1}{\log e}\log\frac{y_n}{y'} - 1 + \frac{y'}{y_n}\right) \quad \ldots\ldots\ldots\ldots (65)$$

By equating Eq. 65 with the Eq. 6c, the meaning of the roughness parameter χ emerges as

$$\chi = y'_e\left(1 - \frac{y'}{y_n}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (66)$$

or if y'/y_n is neglected as being relatively small

$$\chi = y'_e \quad \ldots\ldots\ldots\ldots\ldots\ldots (67)$$

It was the authors' assumption that the coefficient C_2 "must be factor dependent upon the arrangement and shape of the roughness elements" that led to their definition of χ, "dependent on the size, shape, and spacing of the roughness elements," or in other words, dependent on absolute size and geometry of the roughness. This, of course, is correct, because, as shown in Eq. 67, χ actually represents y', is unique for any given roughness, and varies with size and geometry. Consequently, χ can describe only one particular roughness, as shown by the authors in the section "General Resistance Diagrams for Open Channels," and cannot be used to describe geometrically similar roughnesses.

For this last purpose, the spacing parameter ψ is introduced by the authors. This parameter is similar to Nikuradse's m for his sand roughness k_s. The purpose of both m and ψ is to relate the roughness to y'. By using the height of roughness baffle "a" as being equivalent to k_s and combining Eq. 67 with Eq. 14, the similarity between m and ψ is revealed by

$$m = \frac{k_s}{y'} = \frac{a}{y'} = \frac{a}{\frac{\chi}{e}} = \frac{e}{\psi} \quad \ldots\ldots\ldots\ldots\ldots (68)$$

With m = 30 for Nikuradse's sand roughness, and χ from Eq. 67,

$$\frac{k_s}{\chi} = \frac{m\ y'}{y'_e} = \frac{m}{e} = \frac{30}{2.72} = 11 \quad \ldots\ldots\ldots\ldots (69)$$

results. Eq. 22 is based on their empirical data and coefficients. Theoretically, from Eq. 69

$$k_s = \frac{m}{e}\chi \quad \ldots\ldots\ldots\ldots\ldots\ldots (70)$$

The spacing parameter ψ, that might rather be called a roughness geometry parameter, would be a constant for geometrically similar roughnesses, provided y' does not depend on the flow depth y_n. The authors did not study geometrically similar roughnesses; however, the values of y' could possibly be determined from the logarithmic plots of single velocity curves, and eventually used

to determine χ and ψ. Of course, this procedure might not be succes to considerable scatter of data. Nevertheless, the determination of χ f resistance function, as has been done by the authors (Fig. 6), has impl that will be investigated in connection with the von Kármán coefficient

Direct determination of κ from the velocity distribution curves show siderable variation of this coefficient (Table 1 and Fig. 12). The rathe iar trend of the variation, and a considerable scatter of data (Fig. 9) the authors to conclude that, "It cannot be said that the experimental provided an entirely satisfactory answer to whether the von Kármán κ constant . . . or how and why κ might vary if it is not a constant."

The indirect determination of κ from the general resistance curve appeal due to much less scatter of data and straighter curves (Figs. 5 However, it can be demonstrated that κ and χ, determined by the indire od, might have an entirely different meaning. The proof of this state based simply on the analysis of the geometrical properties of the log velocity distribution plot, and does not offer any physical interpretat the logarithmic law is accepted without questioning, as the authors ha the following properties of the law cannot be neglected, even though, u (1961), no attention has been given in the literature to these aspects of

First, the general resistance curve imposes a constant κ^* for a roughness, as there is only one curve for each roughness (Fig. 5). Th tion of κ with the flow depth cannot be determined, and it is somewhat r ing to show κ^* in Table 1 as remaining constant for each run with roughness. Actually, κ^* is experimentally related only to the resistan tion as given in Fig. 5.

To reveal a second, and more important property of the logarithr assume that a hypothetical set of semi-logarithmic velocity distributio exists for a given roughness, with variable slope that corresponds to a κ, and with a constant y' (Fig. 26). The points that correspond to the velocity of these curves ($v = V$) are marked by a circle. It is general that the depth y_V at which the average velocity occurs in a two-dim flow is determined by

$$\frac{y}{V} = \frac{y_n}{e} \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

being independent of κ and y'. This value can easily be derived by Eqs. 64 and 65 and neglecting the value y'/y_n as being small. It might sible to draw a straight line through the points that indicate the averag ity. This line might indicate a distance y'_* on the y-axis. The equatio dashed line in Fig. 26 can be written as

$$\frac{V}{\sqrt{\frac{\tau_0}{\rho}}} = \frac{1}{\kappa^* \log e} \log \frac{y_V}{y'_*} \quad \ldots\ldots\ldots\ldots$$

By replacing y_V from Eq. 71, an equation similar to the general resist results:

$$\frac{V}{\sqrt{\frac{\tau_0}{\rho}}} = \frac{1}{\kappa^* \log e} \log \frac{y_n}{y'_*} - \frac{1}{\kappa^*} \quad \ldots\ldots\ldots\ldots$$

In fact, it is the general resistance function for the set of velocity curves in Fig. 26. However, it demonstrates that the resistance function may have a slope, and κ^* different from the variable slopes and variable κ of the actual velocity curves. A constant $\kappa = \kappa^*$ may result only if $y' = y'_*$. Accordingly, if the logarithmic velocity distribution law is valid, and if the same universal coefficient κ figures in both the velocity distribution law and in the resistance law, experimental results should confirm it. However, the data of this paper

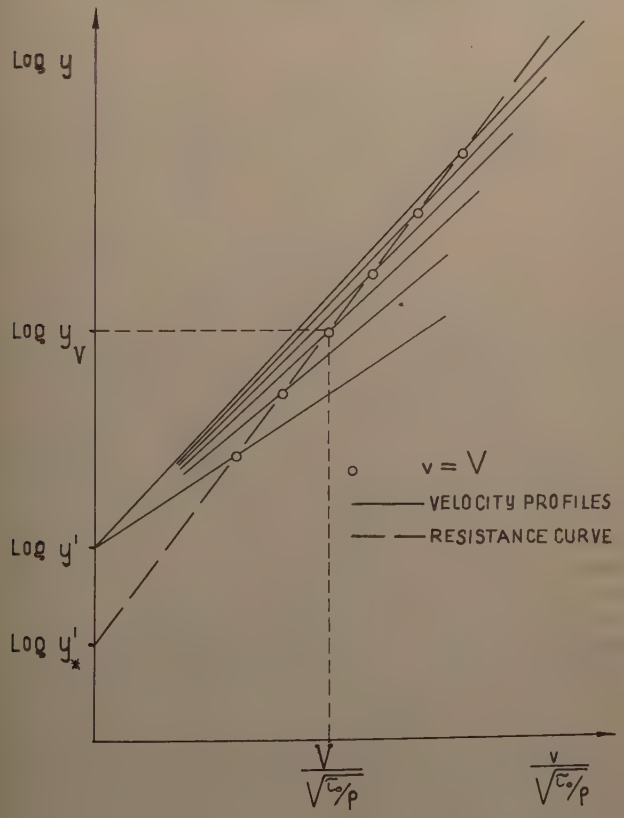

FIG. 26

indicate that $\kappa \neq \kappa^*$. The authors use the resistance function (Fig. 6) to derive $\kappa^* = 0.38$, and to determine their χ and ψ. These are related by y'_* by the following relationships:

$$y'_* = \frac{\chi}{e} = \frac{a}{e}\psi \quad \ldots\ldots\ldots\ldots\ldots\ldots (74)$$

If these values are substituted in Eq. 73, together with $C/\sqrt{g}$ for $V/\sqrt{\tau}$ authors' concluding Eqs. 17 and 18 follow. For example,

$$\frac{C}{\sqrt{g}} = \frac{1}{\kappa^* \log e} \log \frac{y_n}{a} + \frac{1}{\kappa^*}\left(\frac{1}{\log e} \log \frac{e}{\psi} - 1\right)$$

$$= 6.06 \log \frac{y_n}{a} + \left[6.06 \log \frac{e}{\psi} - 2.63\right]$$

$$= 6.06 \log \frac{y_n}{a} + C_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

is Eq. 17 and the values of C_2 will be these given in Fig. 5 if values of Fig. 6 are used in the brackets. Similarly

$$\frac{C}{\sqrt{g}} = \frac{1}{\kappa^* \log e} \log \frac{y_n}{\chi} + \frac{1}{\kappa^* \log e} \log e - \frac{1}{\kappa^*} = 6.06 \log \frac{y_n}{\chi} \quad .$$

is the authors' Eq. 18. Accordingly, their resistance function correspo actly to the dashed line in Fig. 26. However, on the basis of Fig. 26, t no justification to assume that the actual velocity profiles have the sam and the same von Kármán coefficient as for the resistance function, with perimental proof. The authors have experimentally shown that $\kappa \neq \kappa^*$, κ varies considerably, if determined from the actual velocity profiles can mean only that $y' \neq y'_*$ in the sense of Fig. 26, provided the loga plot shows straight experimental lines. Consequently, there is no exper justification to introduce $\kappa^* = 0.38$ into Eq. 65 to derive Eq. 20, and no cation to assume a constant $C_1 = 2.6$.

Eq. 20 agrees satisfactorily with data in Fig. 9, however, these d quite scattered, and affected by the choice of datum. The authors' co under the section heading "The von Kármán Coefficient" that "It is p that the integrated method provides a more reliable index of κ" should taken with caution, although such a view is fairly traditional in the lit There have been no attempts, as far as the writer knows, to contemp possibility of a variable κ for the velocity distribution law, and a con for the resistance law. It might not have any physical meaning. It m only an appearance due to experimental difficulties in producing a tr dimensional flow. It might depend on the geometry of roughness. The ment between κ and κ^* is fairly close in the case of Nikuradse's sand ness, although even in this case $\kappa^* = 0.40$ for the resistance function, pared to κ from 0.32 to 0.42 according to Vanoni.[25]

The writer does not imply that Fig. 26 represents a new law, nor question the logarithmic velocity distribution law at the present time. tension of this discussion is only to show that if the logarithmic law adopted as the basis of data analysis, the mathematical and geometri perty of the law as expressed in Fig. 26 does not allow use of the re function as certain proof in determining the character of κ.

Even though the authors did not distinguish between κ and κ^* in th clusions, the writer feels, in concluding this analysis, that they should be

commended for clearly separating the two aspects in their analysis, and for their refusal to draw conclusions on the basis of rather scattered data of the velocity profiles. Furthermore, their roughness parameter χ is of value in producing a simple and elegant form of the resistance function in Eq. 6c.

JACOB DAVIDIAN,[49] A. M. ASCE, and ROLLAND W. CARTER,[50] F. ASCE.—The authors introduce new parameters to describe the combined effects of height, spacing, and form of roughness elements. It is interesting to compare the authors' analysis with that of Keulegan[7] who used the concept of equivalent

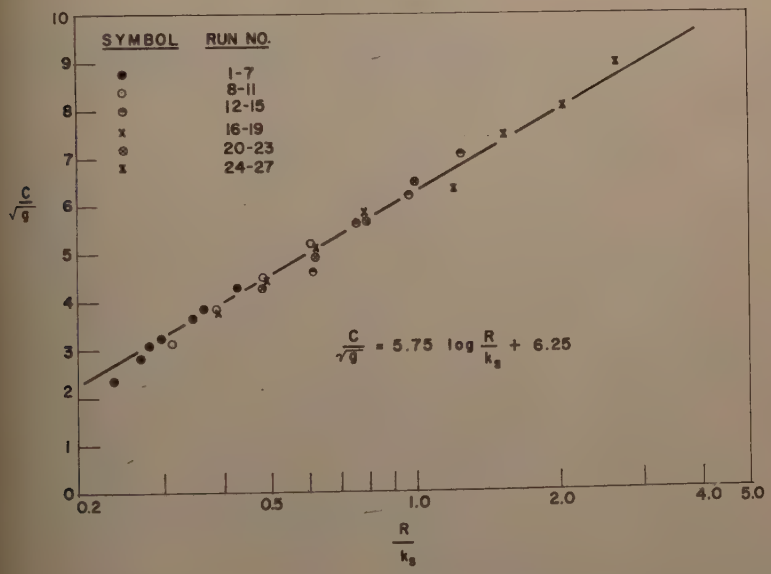

FIG. 27.—COMPARISON OF KEULEGAN EQUATION WITH SAYRE-ALBERTSON DATA

sand grain roughness, k_s. Keulegan's equation for the mean flow in rough channels is

$$\frac{C}{\sqrt{g}} = 5.75 \log \frac{R}{k_s} + 6.25 \quad \ldots\ldots\ldots\ldots\ldots (75)$$

The writers computed the value of k_s from Eq. 75 for each of the authors' experiments, and an average k_s was determined for each of the six roughness patterns. The value of $C/\sqrt{g}$ for each experiment is shown plotted against the hydraulic radius divided by the average value of k_s in Fig. 27. This is es-

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sentially the technique used by the authors in deriving their Fig. 6 fr
equation

$$\frac{C}{\sqrt{g}} = 6.06 \log \frac{y_n}{\chi} \quad \ldots\ldots\ldots\ldots$$

It is appraent from these two equations that χ is proportional to k_s.
assumed that 5.75 is applicable in both equations and that the hydraulic
is equal to the normal depth, then

$$\log \frac{k_s}{\chi} = 1.09 \quad \ldots\ldots\ldots\ldots$$

The authors' parameter Ψ is then equivalent to k_s/a. The variation
with $ab/x\,(e + b)$, which is shown in Fig. 28, follows exactly the same
as the authors' Fig. 8.

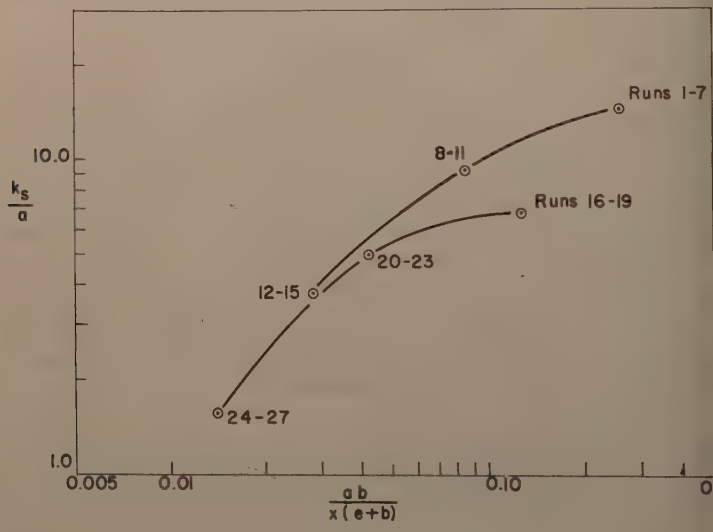

FIG. 28.—VARIATION OF SPACING PARAMETER WITH $\frac{k_s}{a}$

It is difficult to appreciate the merit of the new nomenclature for
concept of relative roughness.

The value of the von Kármán constant derived from a single set of
be misleading because it depends on the value of the additive consta
and in the authors' case, on the definition of the point from which the
measured. Most experimenters report values from 0.37 to 0.41, but t
iation does not seem to be of paramount importance.

Despite the numerous investigations of boundary roughness reporte
literature, there has been little progress in defining the combined effe
height, form, and spacing of roughness elements on resistance to flow
been repeatedly demonstrated that the resistance coefficient varies

logarithm of the hydraulic radius for a fixed roughness geometry, and that a single geometry pattern may be expressed in the resistance equation by a linear dimension such as k_s. This approach, however, appears to be futile if the objective is to predict the resistance coefficient for any given roughness pattern.

JOHN A. ROBERSON,[51] M. ASCE.—The authors' experimental study contributes to the rather sparse fund of data that is available on flow in rough conduits.

In Fig. 8 the authors have plotted the spacing parameter versus the roughness density $ab/x\ (e + b)$. It may be observed that at the higher values of density, the spacing parameter is greater for tests with $e/a = 3.9$ than for tests for which $e/a = 11.8$. This lower resistance with $e/a = 11.8$ undoubtedly results because of the "in-line" arrangement of baffles with a relatively open passage between adjacent rows as opposed to the closed pattern which results when $e/a = 3.9$. This same trend was observed by the writer[52] while analyzing Corps of Engineers'[53] data in a manner similar to that done by the authors. When the roughness elements (cubes) were arranged in lines parallel to the flow, the resistance was less than for the case in which the elements were arranged along lines at a 45° angle to the direction of flow. The writer also observed from results of his analysis that at low densities the effect of pattern was essentially eliminated. This is also shown in Fig. 8 of the authors' paper by the merging of the two curves at low densities. At low densities, the wake of an upstream element is probably diffused so greatly that the downstream element is subejcted essentially to the flow distribution which is produced by the entire array of elements; whereas, at high densities, each upstream element produces a wake that influences the drag on the immediate downstream element.

In Fig. 5, it may be observed that for the two runs having lowest values of y_n/a the reistance is greater than if the points had fallen on the solid curve that was drawn through the remaining points. This abnormally high resistance for large values of relative height is also shown in Fig. 11, where the two points of the authors' data having the highest value of the resistance function $C/\sqrt{g} - 6.06 \log y_n/\chi$ are the same two points having the greatest relative height. This suggests that relative height of roughness beyond a certain limit may also be important in defining the resistance function. The writer also found, by analysis of the Corps of Engineers' data, that the resistance function was abnormally high for a relative height of roughness greater than approximately 0.30, which is comparable to y_n/a less than 3.33. This was observed for high densities as well as low densities. One might deduce that this would be the case for extremely large relative heights. For example, if the roughness elements almost penetrated the water surface, the local turbulence pattern would undoubtedly be distorted. The velocity distribution would, in turn, be expected to deviate from the normal von Kármán distribution.

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51 Assoc. Prof. of Civ. Engrg., Washington State Univ., Pullman, Wash.

52 "Surface Resistance As a Function of the Concentration and Size of Roughness Elements," by J. A. Roberson, thesis submitted to the State Univ. of Iowa, of Iowa City, Iowa, in 1961, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

53 "Study of Finite Boundary Roughness in Rectangular Flumes," Corps of Engrs., U. S. Army, Technical Memorandum No. 2-364, Waterways Experiment Sta., Vicksburg, Miss., June, 1953.

In view of the observations by the writer concerning relative heig
suggested that excessive relative height may be the cause of marked de
from the Kármán-Prandtl distribution rather than the sparsely-space
ness elements, per se, as concluded by Sayre and Albertson. Even th
deviations seem to correlate with sparse patterns, it may be noted th
of the runs at low densities also tend to have high values of relativ

WATER SURFACE PROFILES IN IRREGULAR NATURAL STREAMS[a]

Discussion by Steponas Kolupaila

STEPONAS KOLUPAILA.[2]—The paper is an extension of Mononobe's efforts (16), 1938, to include the effects of velocity change into computation procedure avoiding successive steps. The author introduces local losses—in bends, contractions, enlargements, at bridge piers, and so forth—as a certain percentage of the member representing the change in velocity heads. Two functions, B_1 and B_2, of different dimensions, are derived and applied to computation of backwater profiles. Regrettably, the author refrains from presenting a numerical example of computation, in order to prove his statement that this method is relatively simple and practical. Surprisingly, he concludes that direct procedure is more laborious and difficult than a trial and error method, and suggests preferring a step by step method. The writer would appreciate the author adding an elaborate example in his closure.

The writer has some objections to the author's idea. Local losses are confined to a particular reach or cross-section, such as, at bridge piers. It seems to be incorrect to extend them to the entire length of the channel of consideration. Would it not be more reasonable to split the computation into reaches in which involved factors change uniformly, and to introduce a corresponding additional head loss due to the local non-uniformity? It seems, that in this case the new functions would be useless.

The adequacy of a hydraulic radius in natural streams is also questionable, because it is almost impossible to measure the wetted perimeter. Surface width is usually (except canyons) substituted for the bottom perimeter, thus introducing average depth instead of hydrualic radius. These are some opinions expressed that hydraulic radius is a poor representant of geometrical properties of a cross section.

The writer would like to point out several inconsistencies in the paper: The same value 0.5 for local loss coefficient is proposed for both abrupt contraction and abrupt expansion. The facts from experimental hydraulics show that these two phenomena are quite different.

Eq. 4 for the Boussinesq (not Bazin!) correction factor (called in the paper empirical coefficient of velocity) is assigned to an impulse-momentum equation, whereas in the energy (Bernoulli) equation another, the Coriolis' factor is to be used:

$$\alpha = \int v^3 \frac{dA}{V^3 A} \quad \ldots\ldots\ldots\ldots\ldots (31)$$

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[a] July 1961, by Praxiteles A. Argyropoulos (Proc. Paper 2849).

[2] Prof. of Civ. Engrg., Univ. of Notre Dame, Notre Dame, Ind.

Significance and difference of both factors is explained in textbook
Actual values of the Coriolis' factor are often underestimated. These
are particularly high in overflooded valleys. When this factor is dete
for a compound cross-section, its magnitude, contrary to "weighted a
value," is commonly much larger than for components, the main chann
the valley. The example indicated in Table 1 demonstrates this parado

TABLE 1

	Main channel	Valley	Tot
Surface width, feet	200	1000	120
Cross section area, square feet	4000	2000	600
Average depth, feet	20.0	2.0	
Discharge, cubic feet per second	16,000	800	16,80
Average velocity, feet per second	4.00	0.40	
$\int v^3\,dA$, feet 5 per cubic second	300,000	175	300,17
Coriolis' factor α	1.171	1.367	

The Coriolis' factor in the (simplified) case shown in Table 1 of a
valley at flood was obtained over 2, although its values for separate part
more moderate. This factor tends to decrease in contracted reaches
increase in enlargements.

More literature on this problem was presented in 1961 by the writer

3 "Fluid Mechanics for Hydraulic Engineers," by H. Rouse, McGraw-Hill Bo
Inc., New York, 1938; reprint, Dover Publications, 1961, pp. 51 and 54.

4 "Open-channel Hydraulics," by V. T. Chow, McGraw-Hill Book Co., Inc., Ne
1959, p. 29.

5 "Fluid Mechanics," by R. H. Pao, J. Wiley and Sons, New York, 1961, p. 89,

6 "Bibliography of Hydrometry," by S. Kolupaila, Notre Dame Univ. Press
Dame, Ind., 1961, pp. 804-806.

CONTINUOUS PARABOLIC INTERPOLATION[a]

Discussion by M. D. Lester

M. D. LESTER,[5] M. ASCE.—The method can undoubtedly be useful and, indeed, should be adopted for processing some types of data with computers. However, the article focuses on the application of this interpolation procedure to certain hydrologic and hydraulic measurements, such as stream rating tables, storage volumes, and operating characteristics. Here, the writer disagrees that any real improvement in computer efficiency or in the accuracy of interpolated values is necessarily obtained. In fact, just the opposite may result.

For example, stage-discharge data for the usual stream-gauging station follow the function

$$Q = k\ (G - Z)^b \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

in which Q is the steady-flow discharge, G denotes the gauge reading, Z refers to the zero-flow stage and k and b are station constants. The station rating curve may sometimes be composed of two or more equations of this type, with common tangents at points at which additional channels come into play. In any case, the equation of the curve can be evaluated by a conventional least-squares analysis, after normalizing by taking logarithms. Having determined the best-fit equation, the computer can produce a rating table based on successive solutions for any desired increment of stage. The interpolation function is thus completely by-passed, machine time is saved and the resulting tabulation represents a truer picture than would be obtained by using an interpolation function which bears no relation to the natural law governing the phenomenon.

In the case of storage volume data, the method by which known points on the curve are obtained must be examined. When these are based on planimetric or similar calculations in which all important "breaks" in the function are specifically determined, then is it not readily arguable that intermediate points are best computed on a linear basis? In other words, the true curve is not really "smooth" but rather a set of discrete points. Let us assume, for example, that points 5, 6, 7, and 8 in the author's Fig. 1 represent measured values of elevation versus storage for a hypothetical reservoir. The downward dip between points 6 and 7 in the interpolated curve is then untenable. In general, one weakness of the method is that the computer will not recognize that certain functions must, by their very nature, have a slope which never changes sign.

Regarding operating characteristics of machines or equipment, a different problem arises. Here there is commonly encountered a set of data in which the

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[a] July 1961, by Willard M. Snyder (Proc. Paper 2865).

[5] Senior Engr., Aluminium Labs. Ltd., Montreal, Canada.

measurements or observations are known to have a given possible err in the case, for example, of an efficiency test for a turbine. When co guarantees are involved, it is recognized that a single curve through points, no matter how drawn or computed, has little real value. Instead, a related" curve is determined (analogous to a best-fit calculation) or a b established that encompasses all points within a specified range.

The writer must apologize for elaborating on what the author may hav intended as illustrative applications. However, it is believed that these ples point out three types of data in which smoothing, non-linear interp may yield misleading results:

1. Measurements of a phenomenon whose natural law is in the for known function.
2. Measurements of a relation which may not be represented by a s curve and which, by its nature, cannot dip downward (or bulge upward).
3. Measurements of operating characteristics in which the curve p through all points is not a realistic portrayal of the true relation.

There remains, nevertheless, a large class of data for which the au method seems to have clear advantages in speed and accuracy. It is a g contribution toward the new field of optimizing computer operations.

PROCEEDINGS PAPERS

e technical papers published in the past year are identified by number below. Technical-division oup is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air sport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil anics and Foundations (SM), Structural (ST), Surveying and Mapping (SU); and Waterways and Harbors , divisions. Papers sponsored by the Department of Conditions of Practice are identified by the sym- (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To e papers in the Journals, the symbols after the paper number are followed by a numeral designating ssue of a particular Journal in which the paper appeared. For example, Paper 2703 is identified as (ST1) which indicates that the paper is contained in the first issue of the Journal of the Structural Di- n during 1961.

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EMBER: 2637(ST11), 2638(ST11), 2639(CO3), 2640(ST11), 2641(SA6), 2642(WW4), 2643(ST11), 2644(HY9), 645(ST11), 2646(HY9), 2647(WW4), 2648(WW4), 2649(WW4), 2650(ST11), 2651(CO3), 2652(HY9), 2653 HY9), 2654(ST11), 2655(HY9), 2656(HY9), 2657(SA6), 2658(WW4), 2659(WW4)[c], 2660(SA6), 2661(CO3), 662(CO3), 2663(SA6), 2664(CO3)[c], 2665(HY9)[c], 2666(SA6)[c], 2667(ST11)[c].

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TOBER: 2950(PP2), 2951(PP2), 2952(PP2), 2953(ST7), 2954(SM5), 2955(ST7), 2956(ST7), 2957(ST7), 2958 ST7), 2959(SM5), 2960(EM5), 2961(EM5), 2962(ST7), 2963(ST7), 2964(EM5), 2965(SM5), 2966(SM5), 2967 ST7), 2968(ST7), 2969(ST7), 2970(ST7), 2971(SM5), 2972(SM5)[c], 2973(EM5)[c], 2974(ST7)[c], 2975(PP2).

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Discussion of several papers, grouped by divisions.

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